Abstract

The impact of money supply on the real variables and on utility is an important question in monetary economics. Most previous works study this impact in representative agent economies, often under perfect foresight. With such a framework, however, the use of fiat money as a medium of exchange cannot be endogenously explained. This paper, by contrast, considers an economy where fiat money is intrinsically necessary for exchange, due to the local structure of interaction among agents. It investigates the transitory and permanent impact of local or global injections of money on the dynamics of produced quantities and exchanged quantities, prices, and individual welfare, and the mechanisms that explain this evolution. Furthermore, it examines the impact of real taxation on the above-mentionned variables.

Keywords: economics, spatially differentiated agents, fiat money, decentralized exchange, monetary growth, distribution, taxation.

Real taxation and production in a monetary economy with spatially differentiated agents

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1 Introduction

Standard economics are unable to explain, and in fact negate, what might prima facies be considered the most ubiquitous economic phenomenon: The use of intrinsically useless money (that is, of fiat money) in exchange. Indeed, the mainstay of economics, general equilibrium theory, postulates an anonymous, global, and atemporal market. On this market, all individual buy and sell decisions are instantaneously coordinated by some abstract mechanism which is assumed to guarantee certain efficiency conditions, but remains otherwise unspecified. The only elements of information used by the economic agents are their own primitives (preferences, endowments, etc.) and the relative (not the absolute) market prices – whereby the latter efficiently pool the entire set of the individual primitives. Since every agent simultaneously faces all others, any exchange can in principle be conducted as a barter. Moreover, even if money was used for exchange for some unspecified reason, its quantity would not affect the relative prices and thus the real quantities.

In reality, however, money is a essentially related to the spatial and dynamic character of the exchange process. Real agents make transactions locally and sequentially. Money then facilitates mutually beneficial transactions in the absence of double coincidence of wants, that is, in situations where the buying agent is unable to provide the goods the selling agent wants in return, [1], [2]. It stores value between subsequent transactions. Also, there is strong empirical evidence that changes in the quantity of money do indeed affect real values, at least on the short term, see e.g. [3]. Thus, in the real world, money is needed for a satisfactory functioning of the economy, and changes in the quantity of money may affect this functioning.

To explain the use of intrinsically useless money in payments for commodities and to finely analyze the economic implications of changes in the quantity of money, models are therefore required that take into account restrictions on the operation of markets, such as limited information and/or communication, limited market participation, spatial separation and local interaction [4]. In [5], we presented a bare-bones version of a model with spatially differentiated agents who exchange only with their direct neighbors, and use only the local information directly generated by the exchanges to form their decisions. Such models are fairly rare in the literature – see above all [6], [7] , [8], [4]. To justify the use of fiat money, these models typically assure an absence of double coincidence of wants by making specific assumptions on which goods the individual agents want to buy respectively sell. In [5], by contrast, every agent wants to consume all goods. It is essentially the spatial structure of trade that insures that barter or non-tradable IOUs are impossible. Moreover, the emphasis is on the dynamic properties of the model under changes in the quantity and distribution of money, rather than on the existence and efficiency properties of the equilibrium. This is also the case with the present model which, however, assumes that an agent does not consume or buy the good it produces. More crucially, in the current paper we significantly extend the analysis in [5] by assuming that the agents maximize their utility not only with respect to their consumption, but also with respect to their (costly) production decisions. Likewise crucially, we investigate here the possible impact on the economy of a government that uses money creation to divert exogenously given real quantities of good from the private sector.

2 The model

2.1 Agents, goods, and preferences

In this paper, lower-case letters designate scalars, while capitals designate n-vectors. More specifically, a capital letter indexed by i designate a $n-vector$ with its i-th coordinate equal to 0. Thus, P_i^t is the $n-$vector $(p_i^t(1), p_i^t(2), ..., p_i^t(i-1), 0, p_i(i+1), ..., p_i(n))$, and similarly for all other capital letters, with the exception of $X_i^t \equiv (0, ..., 0,\ x_i^t,\ 0, ..., 0)$. A tilde over a variable is used to designate an expected value.

We consider an economy with n agents i, $i = 1,\ ..., n$, and n goods j, $j =$

1, ..., n, where $n > 2$, and with discrete time $t = 0, 1, 2, ...$. The goods are tradable but perishable and non-durable in the following sense. They can be exchanged among agents an infinite number of times, without alteration or costs. However, they cannot be stored: At any given t, the goods must be either consumed by the agent that holds them, or handed over to another agent. Consumption of a good is the only source of utility, but leads to its physical destruction. We assume that agent i and only agent i produces good i. However, agent i never buys nor consumes good i. Production of a good decreases the utility of the producer.

In each period $t = 0, 1, 2, ...$, each agent i produces a quantity x_i^t of good i and consumes a good basket $C_i^t = (c_i^t(1), c_i^t(2), ..., c_i^t(i-1), 0, c_i^t(i+1), ..., c_i^t(n))$, where the $j-th$ coordinate of C_i^t, that is $c_i^t(j)$, designates the consumption of good j by agent i in period t. The agent chooses x_i^t and C_i^t in order to maximize its utility v_i^t over the two-periods t and $t+1$, where v_i^t is the undiscounted sum of the instantaneous utilities $u_i^t = u_i^t\left(C_i^t, x_i^t\right)$ and $u_i^{t+1} = u_i^{t+1}\left(C_i^{t+1}, x_i^{t+1}\right)$ realized by consuming in t and $t+1$ the good baskets C_i^t and C_i^{t+1} respectively, less the disutility caused by producing good i:

$$v_i^t = u_i^t\left(C_i^t, x_i^t\right) + u_i^{t+1}\left(C_i^{t+1}, x_i^{t+1}\right). \tag{1}$$

The instantaneous utility functions $u_i^t(\cdot)$ are assumed to be time-independent and the same for all agents: $u_i^t(\cdot) = u(\cdot) \ \ \forall i$ and t. We require that the function u be increasing in each $c_i(j),\ j \neq i$, decreasing in x_i, and such that $lim_{c_i^t(j)\to 0}\left[\partial u_i^t\left(C_i^t\right)/\partial c_i^t(j)\right] = \infty \quad \forall j \neq i$. This last condition guarantees that in any period an agent will not concentrate its consumption on a single period or, within a given period, on a subset of goods, but will spread it over time and over all $n-1$ goods it consumes. As a consequence, an agent will never sell its entire current endowment of a good j, $j \neq i$.

2.2 Exchange structure

In our economy, the agents are located on a circle, with (looking towards the outside of the circle) agent i on the right of agent $i-1$ and on the left of agent $i+1$. We assume a wraparound topology. That is, agent n has agent $n-1$ as left and agent 1 as right neighbor, see Figure 1.

Agents interact (that is, exchange goods and money) one with the other sequentially. First, agent 1 interacts with agent 2, then agent 2 with agent 3,..., then agent n with agent 1, etc. The number of interactions between agents is unlimited – the circle is followed clockwise an infinite number of times. We say that there is an unlimited number of exchange cycles of length n. Within an exchange cycle, each agent i interacts with two other agents,

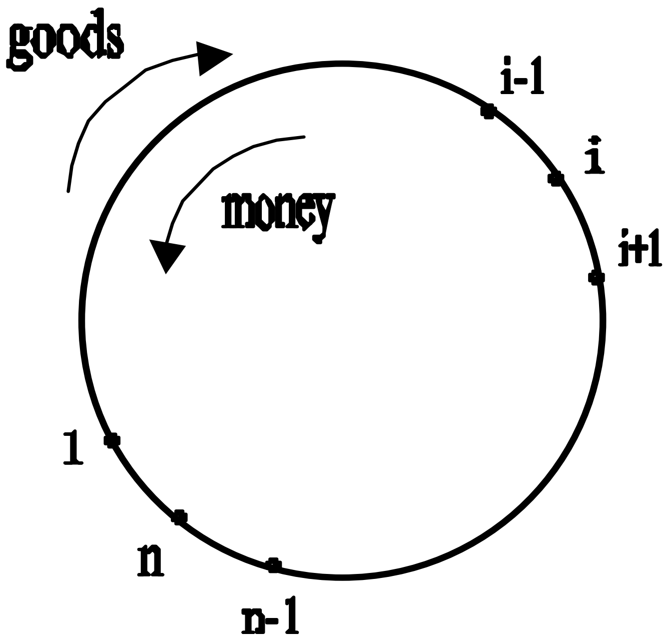

Figure 1: The exchange structure

namely, first with its left neighbor $i-1$, then with its right neighbor $i+1$. This doing, the agents act as price-takers: They assume that the observed prices are not influenced by their own buy/sell decisions. The price-taking hypothesis can be justified by assuming that the agents are not single individuals, but idealizations of (local) competitive markets. The interaction structure is as follows.

When it interacts with agent $i-1$, agent i :

1. Initially has a quantity m_i^{t-1} of money, but no goods.
2. Purchases from agent $i-1$ a vector of goods Y_i^t at price P_i^t with the intention to (a) produce $\tilde{X}_i^t = (0, ..., 0, \tilde{x}_i^t, 0, ..., 0)$; (b) resell part $\tilde{Y}_{i+1}^t$

of the vector $Y_i^t + \tilde{X}_i^t$ to agent $i+1$, at the expected prices ${}^S\tilde{P}_{i+1}^t$, thus realizing an expected monetary income $\tilde{m}_i^t = \tilde{Y}_{i+1}^t {}^S\tilde{P}_{i+1}^t$, where $\tilde{y}_{i+1}^t(i) = x_i^t$; (c) consume the goods $\tilde{C}_i^t$ expected to remain in its possession; and (d) use, one period later, the monetary balance $\tilde{m}_i^t$ to buy from agent $i-1$ a vector of goods $\tilde{Y}_i^{t+1}$ at the expected price ${}^D\tilde{P}_i^{t+1}$, and to consume these goods. Thus, the demand of agent i for goods held by agent $i-1$, ${}^DY_i^t = {}^DY_i^t(m_i^{t-1}, P_i^t, \cdot)$ is the solution of the problem:

$$v_i^t = u_i^t\left(\tilde{C}_i^t, \tilde{x}_i^t\right) + \bar{u}_i^{t+1}\left({}^D\tilde{P}_i^{t+1}, \tilde{m}_i^t, x_i^{t+1}\right)_{|x_i^{t+1}=0} \longrightarrow \max_{Y_i^t, \tilde{x}_i^t} \tag{2}$$

$$\text{s.t.} \quad P_i^t Y_i^t \leq m_i^{t-1}, \;\; Y_i^t \geq 0, \;\; \tilde{x}_i^t \geq 0, \tag{3}$$

$$Y_i^t + \tilde{X}_i^t - \tilde{C}_i^t \geq 0, \;\; \tilde{m}_i^t = {}^S\tilde{P}_{i+1}^t\left(Y_i^t + \tilde{X}_i^t - \tilde{C}_i^t\right), \tag{4}$$

with, according to the price-taking hypothesis, the realized price P_i^t is assumed by the agent to be a parameter independent of its decisions, and where $\bar{u}$ is the indirect utility function associated to u. In (2), the indirect utility function $\bar{u}_i^{t+1}$ is evaluated at $x_i^{t+1} = 0$ since the temporal structure defined by 1.-2. implies that production serves only to generate income that will be used to buy consumption goods one period later. Together with the assumptions of a two-periods revolving planing horizon, this means that, seen from period t, the optimal production in $t+1$ is always $x_i^{t+1} = 0$. Notice that, under the assumption made on u, the first inequality from the left in (3) will always be saturated at the optimum.

Following the purchase of Y_i^t from agent $i-1$, agent i interacts with agent $i+1$. Doing so, agent i :

3. Initially has no money, but a basket of goods $Y_i^t \geq 0$;

4. Produces the output X_i^t, increasing its basket to $Y_i^t + X_i^t$, and sells to agent $i+1$ a vector of goods Y_{i+1}^t at price P_{i+1}^t, with the intention to (a) consume immediately the goods remaining in its possession, $C_i^t = Y_i^t + X_i^t - Y_{i+1}^t$; (b) use in $t+1$ the resulting monetary income $m_i^t = P_{i+1}^t Y_{i+1}^t$to buy from agent $i-1$ at the expected price ${}^{DD}\tilde{P}_i^{t+1}$ a basket of goods $\tilde{Y}_i^{t+1} = \tilde{Y}_i^{t+1}({}^{DD}\tilde{P}_i^{t+1}, m_i^t)$; and (c) to consume this basket. Thus, the supply function of agent i when selling to agent $i+1$, ${}^{S}Y_{i+1}^t = {}^{S}Y_{i+1}^t(Y_i^t + X_i, P_{i+1}^t, \cdot)$ is the solution of the problem:

$$v_i^t = u_i^t\left(\left[Y_i^t + X_i^t - Y_{i+1}^t\right], x_i^t\right) + \bar{u}_i^{t+1}\left({}^{DD}\tilde{P}_i^{t+1}, m_i^t, x_i^{t+1}\right)_{|x_i^{t+1}=0} \rightarrow \max_{Y_{i+1}^t} \quad (5)$$

$$\text{s.t.}\quad m_i^t = P_{i+1}^t Y_{i+1}^t, \quad Y_i^t + X_i^t - Y_{i+1}^t \geq 0, \quad Y_{i+1}^t \geq 0. \tag{6}$$

with the realized price P_{i+1}^t assumed by the agent to be independent from its decisions. The function $\bar{u}_i^{t+1}$ is evaluated at $x_i^{t+1} = 0$ in (5) for the same reasons as before.

Note that, since an agent does not consume the good it produces, and since the actual production is done before the exchange takes place, the supply curve for good i is vertical:

$${}^S y_{i+1}^t(i) = x_i^t.$$

Following 5., agent i finds itself in a situation similar to the initial one under 1., that is, with a budget $m_i^{t+1} > 0$, but without goods. Note that we always have $m_i^t = m_{i+1}^{t-1}$. The realized values of P_i^t, Y_i^t, and x_i^t, result from the equilibrium condition:

$${}^S Y_i^t(Y_{i-1}^t + X_{i-1}^t, P_i^t, \cdot) = {}^D Y_i^t(m_i^{t-1}, P_i^t, \cdot).$$

The structure of the interaction between agents $i-1$ and i respectively i and $i+1$ does not depend upon t. Given initial conditions

$$\{m_i^0, Y_i^0, {}^D\tilde{P}_i^1, {}^{DD}\tilde{P}_i^1, {}^S\tilde{P}_{i+1}^0, {}^S\tilde{p}_{i+1}^0(i)\}_{i=1,\ldots,n}$$

with $m_1^0 = 0$, the above assumptions completely define the dynamics of the economy.

Notice that in this model, there are two flows in opposite directions. Money circulates counterclockwise, goods clockwise. There is full conservation of money. The use of money implies accumulation of money balances by the selling, and equal loss of money balances by the buying agent. Thus, at any point of time, some agents always hold a larger monetary balance than others, capturing a central feature of a monetary economy, see [9]. On the other hand, goods are fully consumed within the period when they are produced.

3 Basic properties

To illustrate the basic properties of the model presented above, we assume in the following that the instantaneous utility u is given by:

$$u = \sum_{j \neq 1} c(j)^{1/2} - x_i.$$

Furthermore, we assume that the price anticipations are formed according to:

$$\begin{aligned} {}^{D}\tilde{p}_i^{t+1}(j) &= {}^{DD}\tilde{p}_i^{t+1}(j) = p_i^t(j), \quad \forall i, t, j \neq i, & (7) \\ {}^{S}\tilde{p}_{i+1}^{t}(j) &= p_i^t(j), \quad \forall i, t, j \neq i \;\; i+1, \quad \text{and} & (8) \\ {}^{S}\tilde{p}_{i+1}^{t}(i) &= p_{i+1}^{t-1}(i). & (9) \end{aligned}$$

That is, we assume that at any moment all anticipated purchase and sale prices are equal to the exchange prices currently observed on the market. An exception is ${}^{S}\tilde{p}_{i+1}^{t}(i)$, which is supposed equal to $p_{i+1}^{t-1}(i)$, reflecting the fact that, since good i is not exchanged between agents $i-1$ and i, $p_i^t(i)$ is not defined.

While the assumption ${}^{D}\tilde{p}_i^{t+1}(j) = {}^{DD}\tilde{p}_i^{t+1}(j) = p_i^t(j)$ can fairly easily be relaxed, seemingly without great impact on the results, the assumption ${}^{S}\tilde{p}^{t}_{i+1}(j) = p^{t}_{i}(j)$ implies that the agents believe that they will be able to sell to their right neighbor at a price equal (or, at least, closely related) to the price at which they buy from the left neighbor. This is false – under this hypothesis, the model will generate a marked asymmetry between the buy and sell prices. In other words, the hypothesis introduces a new and important market friction, this time at the level of the agents' behavior. Hypothesis (8) has the great advantage, however, to insure that the non-negativity constraints on the production and exchanged quantities are always satisfied, making the results amenable to presentation in a short paper.

3.1 Point equilibrium under uniform monetary balances

As a benchmark, let us first consider what happens on the long-run if we start with a situation where (a) all agents initially have the same monetary balance m but no goods, with the exception of (say) agent 1 that (b) has no money but an arbitrary weakly positive basket of goods. We call "point equilibrium" a situation such that, for any i the prices and variables $Y_i, {}^{D}\tilde{P}_i, {}^{DD}\tilde{P}_i, {}^{S}\tilde{P}_{i+1}, {}^{S}\tilde{p}_{i+1}(i)$ and thus C_i, x_i and $u(C_i, x_i)$ are time invariant. Numerical evidence[1] shows that under the conditions (a)-(b) above, the economy has a unique, stable point equilibrium such that for $k = 1, 2, ..., n,\ k \neq i, i-1,$, the relative prices $p_i(i-k-1)/p_i(i-k)$ are constant and equal to:

$$p_i\left[\mathrm{mod}\left\{\mathrm{mod}(i-k)-1\right\}\right]/p_i\left[\mathrm{mod}(i-k)\right] = \sqrt{2}w(n), \tag{10}$$

where the modulo mod is taken $n+1$ and where $w(n)$ is a multiplicative factor that depends only on n, with $dw/dn > 0$, $d^2w/dw^2 < 0$. More precisely, $w(3) = 1.06065$, $\lim_{n\longrightarrow\infty} w(n) \simeq 1.2$.Similarly, the relative quantities exchanged are given by:

$$y_i\left[\mathrm{mod}(i-k)\right]/y_i\left[\mathrm{mod}\left\{\mathrm{mod}(i-k)-1\right\}\right] = 2w(n). \tag{11}$$

Since:

[1] We were unable to obtain analytical solutions for $n > 3$.

$$c_i(j) = \begin{cases} 0, \quad i = j; \text{ and} \\ y_i(j) - y_{i+1}(j), \quad i \neq j, \end{cases}$$

one has also:

$$c_i\left[\mathrm{mod}(i-k)\right]/c_i\left[\mathrm{mod}\left\{\mathrm{mod}(i-k)-1\right\}\right] = 2w(n),\ k \neq i, i-1. \quad (12)$$

The produced quantities x_i, finally, are the same or all agents. They are decreasing in n with $d^2x/dx^2 < 0$ – specifically, for $n = 3$, $x = 1/5$; for $n = 4$, $x \simeq 0.1795$; for $n = 5$, $x \simeq 0.1689$; etc.

Note that, at the point equilibrium, the individual vectors of consumptions and produced and traded goods are trivial translations one from the other. All agents realize the same instantaneous utility u. The produced, exchanged, and consumed quantities do not depend upon m, nor the relative prices. On the long run, the only impact of a once-and-for-all change in the balance m is a proportional change in the absolute prices. That is, money is neutral in this configuration.

Equation (10) show that at the equilibrium, when buying from agent $i-1$, agent i pays the lowest price for the good produced by $i-1$. The second lowest price is for the good produced by $i-2$, etc. The highest price is paid for agent's $i+1$ good. Conversely, agents buy and consume larger quantities of those goods that are produced close to home than of those that are produced further away – distance being measured in terms of the physical exchange, that is, counterclockwise. See (11) and (12). The local price of a good increases with the distance from production. The monetary expenditures on any given good j, $p_i(j)y_i(j)$, decrease with the production distance and tend towards 0 for goods that are far away.

The point equilibrium is not efficient. In particular, the utility of all agents is higher at the (efficient) symmetric competitive equilibrium, where:

$$p(i) = \frac{4m}{n-1}, x_i = \frac{n-1}{4}, c_i(j) = \frac{1}{4},\ j \neq i \tag{13}$$

Thus, compared to the competitive equilibrium, our agents produce too little. Also, they consume relatively too much of neighboring goods, too little of distant ones. The inefficiency increases with the size n of the economy.

3.2 Periodic equilibrium under asymmetric monetary balances

A simple experiment, illustrated in Figure 2 for an economy with 4 agents, $n = 4$, contributes to better understanding the mechanisms at work in the model. Assume that all agents have initially the same monetary balances m and that the economy is at its point equilibrium until, say, period 5. In this period, just before it interacts with agent 1, the monetary balance of agent 2 is increased to $m + \Delta m$. In the figure, m is initially equal to 10, and $\Delta m = 2$. From the onset, let us emphasize that the resulting values of the real variables and, thus, of utility, do not depend upon the absolute values of m and Δm, but only on the ratio $\Delta m/m$.

Following the injection of money Δm, agent 2 buys from agent 1 higher quantities of all traded goods than previously, at higher prices. The instantaneous utility of agent 1 falls, both because it produces more and because it is left with fewer goods to consume. As a counterpart, its monetary balance increases to $m + \Delta m$ instead of m at the point equilibrium. The exchange of higher produced and traded quantities against a higher monetary balance is considered beneficial by both agents 1 and 2 because they do not fully anticipate the global impact of the local injection of money Δm. In particular, P_1^5 is initially the only price affected by the money injection. This may in turn locally affect the anticipations by agents 1 and 2, but no other variables. Agent 2, that bought more goods, sells more to agent 3 at lower prices and consumes more, realizing a higher utility, etc.

The economy converges towards a new equilibrium almost instantaneously. Driven by the circulation of Δm around the circle, this equilibrium is now periodic, with period $n-1$: The realized and anticipated prices and quantities experienced by the individual agents repeat themselves every $n-1$ exchange cycles. At the equilibrium, all agents realize the same mean utility and sell and consume (up a trivial translation) the same quantities over each $n-1$ exchange cycles. The time average utility, however, is lower than the one realized at the initial point equilibrium, reflecting the fact that there are now additional prediction errors due to the fluctuating nominal demand. These prediction errors lead to less efficient trading and to a loss of utility – a point already noted in [5] for the case of fixed production. However, a completely new phenomenon appears in the present model. The prediction errors also lead to an increased production by an agent every time it trades with a right neighbor that holds the higher balance $m+\Delta m$, that is, every $n-1$ periods. Since the balance $m+\Delta m$ circulates from agent to agent, the time average production of all goods is increased. The augmented production translates into higher consumption of all goods (save one's own) by all agents, increasing their utility. That is, although the quantity of money in the economy is neutral, its distribution is not.

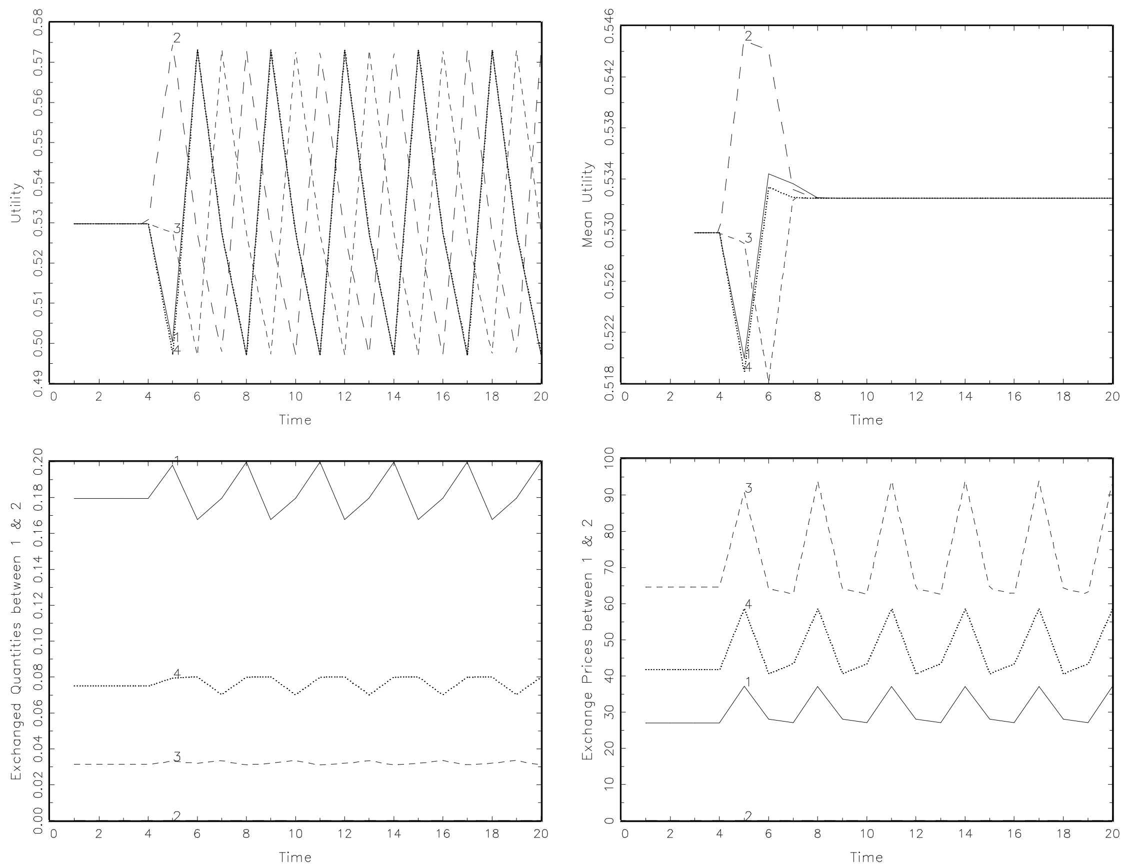

Figure 2: Equilibrium under asymmetric monetary balances

In figure 1, the utility gain due to increased consumption dominates the loss caused by higher prediction errors. The average utility is higher after the injection Δm than before. One might conjecture that an asymmetric distribution of money will always increase the utility when the asymmetry in money distribution is small, but that too high an asymmetry will cause a decrease in average utility – due to higher prediction errors together with a lower marginal utility of consumption. Figure 3, that plots the time averages at equilibrium of utility, production, traded quantities and exchange prices against the additional balance Δm held by one agent, shows that this is indeed the case. Although the level of output keeps increasing monotonically with Δm, average utility first increases, then decreases. Notice that the decrease in utility starts well before the average production exceeds the competitive optimum $x_i = \frac{n-1}{4}$, that is here, $x_i = \frac{3}{4}$. Notice also that the increase in exchanged quantities and in prices with Δm is asymmetric. For neighboring goods, quantities increase strongly and prices moderately. For remote goods, prices increase markedly while quantities are little affected. Thus, a less symmetric distribution of money leads to an increase in consumption that is stronger for neighboring goods than for remote ones.

3.3 Real taxation

In this section, we consider an economy with uniform monetary balances as presented earlier, but introduce now an additional agent, the government. The government is assumed to buy from each agent i, in every period, a fixed quantity g of the good produced by this agent. That is, at each transaction between agents $i-1$ and i, the demand faced by agent $i-1$ for good $i-1$ is assumed to be no longer given by ${}^{D}y_i^t(i-1)$, but by ${}^{D}y_i^t(i-1)+g(i-1)$,

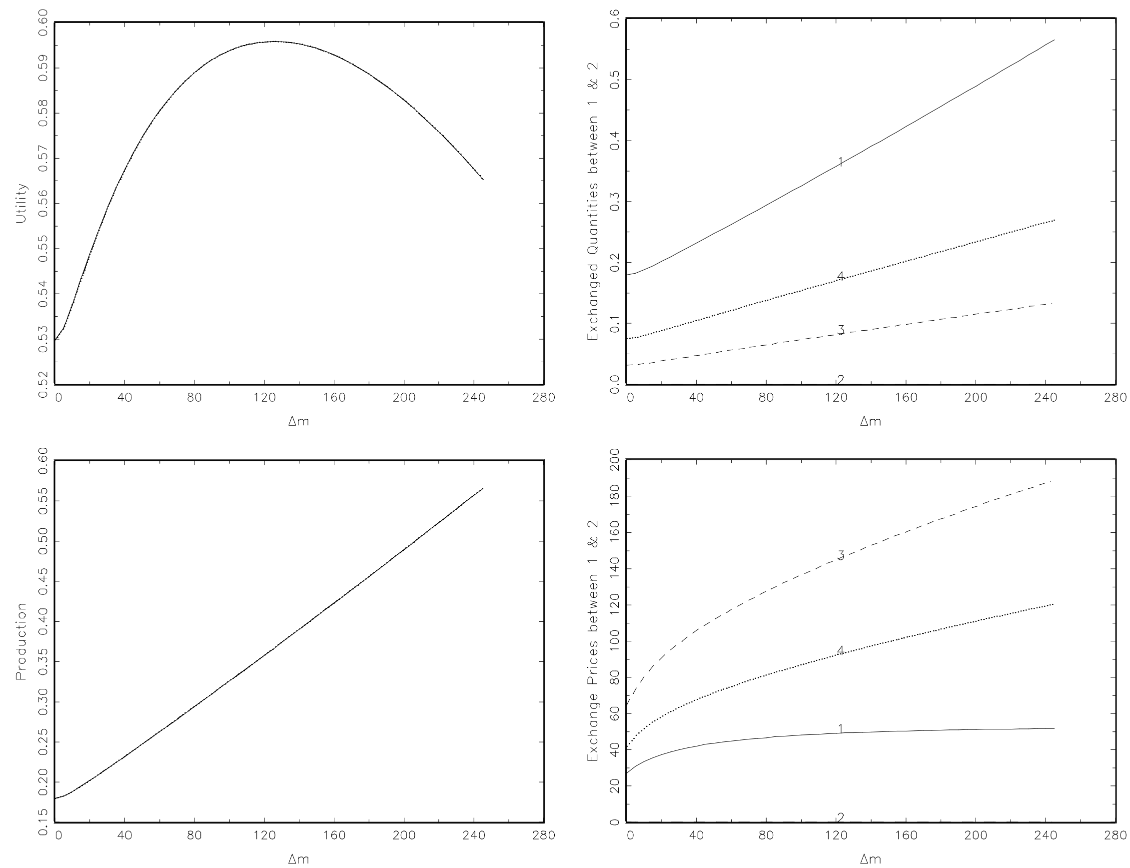

Figure 3: Impact of a skewed distribution of money at the periodic equilibrium

where $g(i-1) = g \ \forall i$. Like all other agents, the government is assumed to be a price taker. Thus, at each transaction, the government and the agent i both pay the same market price for their respective purchases of good $i-1$.

The government is assumed to use money creation to finance its purchases. That is, when it makes a purchase from agent $i-1$ in period t, it injects the additional amount money $\Delta m_{i-1}^t = p_i^t(i-1)g$ in the economy.

This injection raises the realized and anticipated price. Thus, at the next purchase, the government will have to pay more to buy the same quantity of goods. In other words, in order to buy a constant quantity of each good, the government has to inject into the economy ever increasing amounts of nominal money.

Compared with the case without governmental demand, the seller $i-1$, confronted with an increased demand for good $i-1$, produces and sells a larger quantity of this good, wrongly expecting to be able to use the thus obtained monetary balance for future purchases at the current prices. This lowers its instantaneous utility. On the other hand, since the marginal utility of money m_{i-1}^{t} diminishes, $i-1$ sells less and consumes more of the other goods, raising its instantaneous utility. The combined impact of these two contradictory effects is always a net decrease of the agents utility, as shown on the North-West graph of figure 4, where the relationship between the agents' time average utility at the equilibrium and the governmental demand g is shown. The North-East graph of the same figure, on the other hand, shows that the social utility u^S obtained by adding to the sum of private utilities the analogously defined utility of governmental consumption:

$$u^S = \sum_{i=1}^{n} u_i(C_{i)} + \sum_{i=1}^{n} \sqrt{g(i)},$$

is first increasing, then decreasing in g, reflecting again a conflict between the increased production caused by governmental demand and the disruption in exchanges due to accrued anticipations errors. This suggests that there may be, in the model, an optimal level of real taxation plus redistribution. The government may increase the agents utilities by taxing, up to a certain point, the producers, and redistribute the so obtained good among consumers – thus abating the existing efficiencies in exchange.

The South-West and South-East graphs of figure 4 plot the governmental demand g against the percentage of net output x_i exchanged between the agents 4 and 1 respectively against the percentage of total output taxed by the government. One recognizes that taxation indeed reduces the exchanges between agents in the economy, and that governmental demand is very costly in real terms since it generates a roughly five time higher fall in disposable real income.

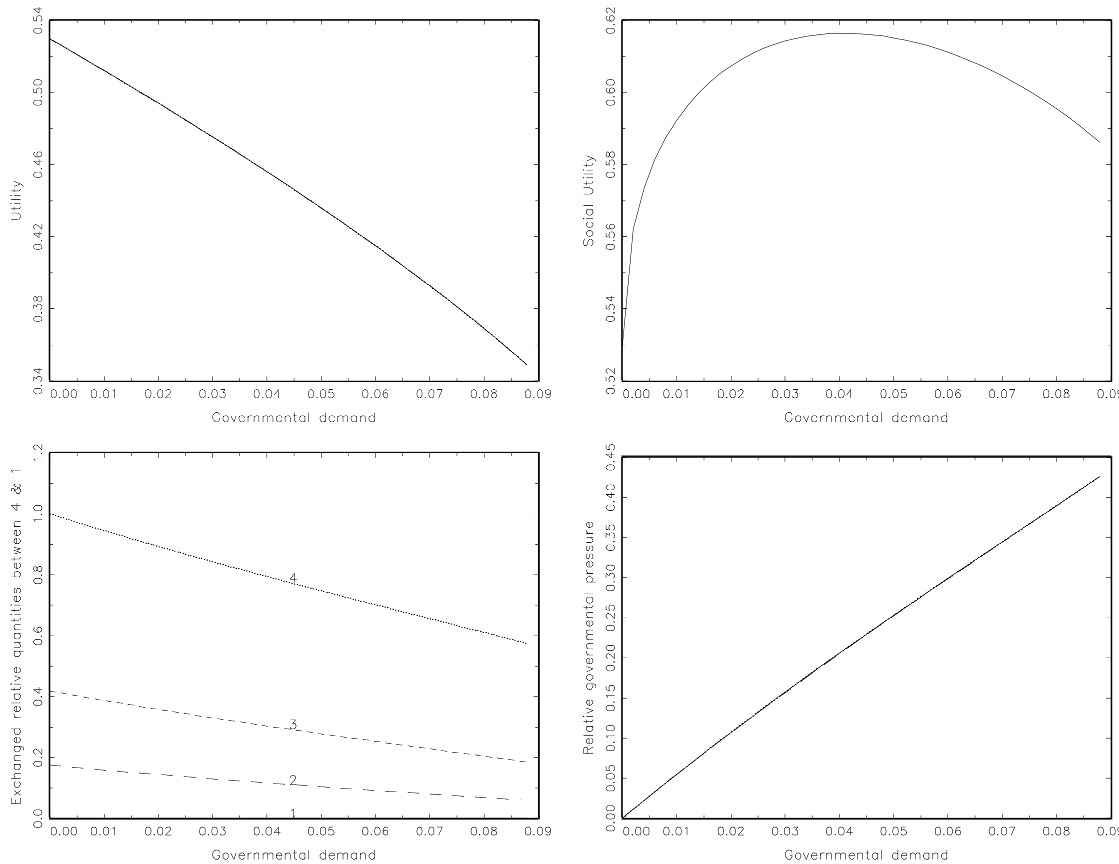

Figure 4: Impact of real taxation at equilibrium

4 Conclusion

Already in its most simple version, the model suggested here generates non-trivial insights on the role of fiat money such as the real impacts of balances distribution and of real taxation through money creation. The model can be extended in numerous directions. In particular, more sophisticated anticipation formation mechanisms should be taken into account. More complex spatial networks could be considered, leading to spin glas-type models, e.g. Altogether, the model appears to offer a flexible and comprehensive platform for investigating the real impacts of money in an economy with locally interacting agents.

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