Waiting Value and Testing Value: a more general approach to environmental decisions under uncertainty

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Abstract

The aim of this paper is twofold: on one side, we try to generalize the traditional results on environmental option values literature and to incorpotare them into a unique model. This serves the purpose of clarifying some misconceptions on the formulation and on the role of the various kinds of waiting values that emerge in the literature.

On the other side, we introduce a value, namely the *Testing Value*, emerging in all those situations in which the level of information concerning future economic benefit of development (and its future environmental costs) depends on the level of development performed. We show the Testimg Value is always nonnegative and that including it into the analysis could push the decision maker towards a higher level of preservation of the environmental resource. The reason is that the Testing Value could lead the decision maker to develop only a certain amount of the environmental asset (*internal solutions*); on the contary, the Waiting (or Quasi-Option) Value would lead more frequently to *corner solutions*.

1 Introduction

1.1 The history of the Waiting Value

Most of the economic models analyzing environmental choices under uncertainty are characterized by two strong hypotheses regarding the nature of information concerning the irreversible effects of the development of an environmental resource. Decision makers acquire this kind of information without any cost (free information) and only by waiting for time to lapse (exogenous information). Both these assumptions are unrealistic and reductive, at least for the following reasons:

1) Acquisition and appropriation of information usually needs the investment of a certain amount of financial resources and the realization of specific actions in order to learn (for example, research and monitoring activities). Moreover, if the cost of information is higher than its expected value, then, from an economic point of view, it is not optimal to invest resources for acquiring it .

2) It may happen that the irreversible action characterizing the development of an environmental resource produce information about future benefits and costs of the development itself. This is a situation of dependent learning, i.e. characterized by endogenous information: the coming out of new information depends on the decision taken by the policy maker in time period 1.

The issue of irreversibility and uncertainty in environmental decisions has been largely discussed in the last three decades. From the first definition of the QOV, given by Arrow and Fisher (1974), the key concept has been developed in several articles, among others, by Henry (1974), Dasgupta and Heal (1979), Hanemann (1982, 1989), Fisher and Krutilla (1985), Beltratti, Chichilnisky and Heal (1996), Fisher (2000), Pindyck (2000).

Arrow and Fisher (1974), while examining the optimal level of development of a natural resource, identified a concept that they termed "quasi-option" value. The concept emerged from a two-period model of choice (develop or preserve), where development is irreversible [1] and the expected net benefits in future periods are conditional upon the choice in the present period. The quasi-option value takes into account the reduction in the value of expected net benefits of development of a particular area (with respect to the case in which the decision maker knows these net benefits with certainty). This reduction occurs if information about the future consequences of development is not available.

Fisher and Hanemann (1987) [2], under the same context of Arrow and Fisher's model, characterized by *risk neutrality* of the decision maker, *irreversibility* of the action "development", *uncertainty* about the future benefits (of development and preservation) and by independent learning (*exogenous information*): the decision maker can receive *new* information about the environmental asset (about the future benefits of his action) only by letting time to pass; this acquisition of information is independent of the choice made in the first period. [3]

More precisely, in Fisher and Hanemann's model, there are two alternative scenarios for the acquisition of information about the future consequences of development. In one scenario, *exogenous* information is available (and it is known it will be available) in sufficient time to be incorporated in the future development decision; in this scenario, the prospect of future information is fully recognized and incorporated explicitly in the current decision. In the other,

[1] In the sense that development of the environmental resource can take place either "now" or "in the future" but, once undertaken, it is irreversible.

[2] See also Hanemann (1989), Fisher (2000).

[3] In particular, information can emerge with the passage of time (e.g., as the second period approaches, one can make a more accurate assessment of the social value of wilderness preservation in that period) or as the result of a separate research program.

either information is *not available (and it is known that it will be not available in sufficient time to be incorporated into the choice in the future period)* or it is *disregarded* by the decision maker when he sets the current level of development. If we define with $EV_{exo}(c_1)$ the expected net benefits over both periods under the first information scenario and with $EV_0(c_1)$ the same expected net benefits under the second information scenario, and requiring the investment decision be confined to a binary choice between full development ($c_1 = 0$) and no development at all ($c_1 = 1$), we define as option value à la Arrow and Fisher (*quasi-option value*)

$$QOV = [EV_{exo}(1) - EV_{exo}(0)] - [EV_0(1) - EV_0(0)]$$

It is a correction factor, that can be interpreted in the following way: let us rewrite QOV as

$$QOV = [EV_{exo}(1) - EV_0(1)] - [EV_{exo}(0) - EV_0(0)]$$

In the terminology of decision theory, $[EV_{exo}(1) - EV_0(1)]$ is the value of perfect information conditional on having preserved the whole environmental area in $t = 1$ (it is the gain from information with respect to the choice of c_2 conditional on setting $c_1 = 0$). Similarly, $[EV_{exo}(0) - EV_0(0)]$ is the value of perfect information conditional on having destroyed the whole environmental area in $t = 1$. QOV is the difference between these two values. But the irreversibility creates an asymmetry: if one decides to preserve initially, he can always reverse that decision later when he obtains more accurate information about the consequences of development; on the contrary, if he decides to develop (everything) now ($c_1 = 0$), the decision cannot be reversed and any subsequent information has no economic value. Hence, $EV_{exo}(0) = EV_0(0)$ and the expression of the quasi-option value becomes

$$QOV = EV_{exo}(1) - EV_0(1)$$

Thus, a decision to set $c_1 = 1$ preserves flexibility, and QOV is the value of this flexibility. More precisely, it is the gain the decision maker obtains when he can receive information regarding future benefits, *if* he decides to do not develop in the current period (with respect to the case in which he ignores the possibility of receiving this kind of information).

Another interpretation: if the decision maker ignored the possibility of acquiring partial information about the future benefits of development/preservation and myopically based his decision on the maximization of $EV_0(c_1)$, QOV is the shadow tax that would have to be imposed on development in order to steer him to the correct choice on whether or not to develop at all.

This does not mean that one should never develop in the first period; after all, it may happen that $EV_{exo}(1) < EV_{exo}(0)$. Rather, it does mean that the case for preservation is strenghtened when one recognizes the prospect of further information about the future consequences of development: in general, the amount of environmental resource preserved under the first information scenario is higher, i.e. $(c_1^*)_{exo} \geq (c_1^*)_0$. This implies that QOV is always non-negative.

Hanemann (1982) derived also an expression for the waiting value when the assumption that the decision maker acquires *perfect* information about the future benefits (about the preservation/development of the environmental resource) is weakened. He shows the results presented above carry over to the case of *partial* information. In particular, if we call $EV_{pexo}(c_1)$ the expected net benefits over both periods under a scenario of partial learning, we can define the analog of QOV with partial as opposed to perfect information as

$$QOV^{+} = EV_{pexo}(1) - EV_0(1)$$

and, since $EV_{exo}(1) \geq EV_{pexo}(1)$, the inequality $QOV \geq QOV^{+}$ holds (QOV is the upper bound for every QOV^{+}).

However, when there is a continuum of preservation (development) levels, rather than a binary choice between full development and full conservation, some of these conclusions need to be modified. Let $c_1 \in [0,1]$ and $c_2 \in [0, c_1]$, because of irreversibility.

Analyzing this case, Epstein (1980) established that it is not necessarily true that $(c_1^*)_{exo} \geq (c_1^*)_0$. He developed also a set of conditions sufficient for Arrow and Fisher's result to carry over when there is a continuum of preservation (development) levels. [4]

Moreover, Hanemann (1989) derived an expression for the quasi-option value in case of continuous irreversible development: let $(\hat{c}_1)_{exo}$=arg $\underset{c_1 \in [0,1)}{max} EV_{exo}(c_1)$ and similarly $(\hat{c}_1)_0$=arg $\underset{c_1 \in [0,1)}{max} EV_0(c_1)$; the value which would appear to correspond to the QOV in the binary choice case is

$$\begin{aligned} \overline{QOV} &= [EV_{exo}(1) - EV_{exo}\,(\hat{c}_1)_{exo}] - [EV_0(1) - EV_0\,(\hat{c}_1)_0] \\ &= [EV_{exo}(1) - EV_0\,(1)] - [EV_{exo}\,(\hat{c}_1)_{exo} - EV_0\,(\hat{c}_1)_0] \end{aligned}$$

We do not agree with this formulation of the problem: first of all, if we maximize for $c_1 \in [0,1)$, we are not sure that a solution exists (since the choice set is not compact and Weierstrass theorem does not apply); second, even if we would correctly define $(\hat{c}_1)_{exo}$=arg $\underset{c_1 \in [0,1)}{max} EV_{exo}(c_1)$ and similarly $(\hat{c}_1)_0$=arg $\underset{c_1 \in [0,1)}{max} EV_0(c_1)$, it is $EV_{exo}(1) \leq EV_{exo}\,(\hat{c}_1)_{exo}$ and $EV_0(1) \leq EV_0\,(\hat{c}_1)_0$, thus QOV is defined as the difference among two negative quantities; finally, we think that the only advantage the formulation above produces is to reduce the continuous case to a "special binary case" in which the decision maker compares the expected value of the choice of preserving everything with the expected value of the best possible choice when preserving everything is not allowed. This "strange" formulation has to be charged to the need of making the choice $c_1 = 1$ to appear in the formula of the quasi-option value, since $c_1 = 1$ would seem to

[4] Vedi tesi

be crucial for exercising the option in the second period. But, since in Hanemann and Fisher's model, given that information is exogenous, it comes out also when $c_1 \neq 1$, every $c_1 \in (0,1]$ allows the decision maker to make a choice in the second period exploiting the new information received, thus exercising the option that has emerged (obviously the option is higher the higher is the value of c_1 chosen). [5]

Our interpretation of the waiting value (as the money the decision maker is willing to pay in order to shift the decision from now to the future) is nearer to the formulation of Conrad (1980) and of Miller and Lad (1983): Conrad (1980) states that "the quasi-option value, so defined, is identical to the expected net value at time zero of the information in c_1"; Miller and Lad (1983) state that "the existence of a quasi-option value as defined generally, depends only the name on the irreversible character of development. Under conditions of irreversibility, an option value is called a quasi-option value. But any option value, quasi or otherwise, stems from the relative values of flexible and inflexible decisions, not from the existence of irreversibility per se". Thus, the quasi-option value has to be interpreted as the difference between the expected value of the optimal flexible decision and that of the optimal fixed decision: this difference is greater than zero.

Using our notation, we define in general the *waiting value* as

$$WV = EV_{exo}((\hat{c}_1)_{exo}) - EV_0((\hat{c}_1)_0)$$

In the first information scenario, the decision maker can choose c_2 after having received information about the relative benefits of the second period (what Miller and Lad called *flexible decision*); in the second information scenario, where the myopia of the decision maker prevents him from recognizing the possibility of acquiring new information exogenously arriving, the optimal choices of c_1 and c_2 are made simultaneously in the first period (what Miller and Lad called *flexible* or *inflexible decision*). [6]

According to our formulation of the problem the QOV à la Arrow and Fisher becomes a particular case of WV that emerges when the decision maker chooses to preserve the entire environmental resource in the first period: it is the expected value of information conditional on having set $c_1^* = 1$. It has to be intended as the upper bound of the WV, since, by preserving everything in the

[5] If one wants to follow Hanemann and Fisher's idea, a more coherent expression of QOV in the continuous case could be

$$\widetilde{QOV} = \left[EV_{exo}((\hat{c}_1)_{exo}) - EV_{exo}(0)\right] - \left[EV_0((\hat{c}_1)_0) - EV_0(0)\right]$$

with $(\hat{c}_1)_{exo}, (\hat{c}_1)_0 \in [0,1]$; here one compares two different cases: the first one, with $(\hat{c}_1)_{exo}, (\hat{c}_1)_0 \in [0,1]$, in which the option can be exercised in the second period; the second, with $c_1 = 0$, in which is no more possible to exert the option in the second period, despite the new information received.

[6] Hence the decision maker chooses the amount of c_2 without knowing the realizations of second period benefits.

first period, you can choose in the second period an action in the same set of possible actions available in the first period. Hence, $QOV \geq WV$.

1.2 The need for the Testing Value

As for the first objection we moved in the previous subsection to the traditional models of environmental choice under uncertainty, [7] we have to notice that the conclusions that Arrow and Fisher (1974) found on the quasi-option value (QOV) still hold, even if we include the cost of information within the model. They recognize the fact that information about the consequences of development arrive automatically with the passage of time is unrealistic (or, at least, rare): the acquisition of information usually requires the expenditure of resources and occurs only if some agent takes appropriate actions. Indeed, if the cost of information exceeds the expected value of information, i.e. the difference $EV_{exo} - EV_0$, then it is not optimal to invest in acquiring it.

On the other hand, we cannot say the same for our second objection: independently from the fact that information arrives costly or not, Arrow and Fisher's results on the optimality of a complete conservation of environmental resources when their destruction is irreversible are derived in a framework with independent learning, i.e. with exogenous information.

It is commonly accepted in the literature of environmental option values that this result does not hold if information is endogenous. Miller and Lad (1983) and Freeman (1984) showed that *if information concerning future effects of the irreversible depletion of an environmental resource can be obtained only carrying out depletion itself in* $t = 1$*, than it is optimal to develop (only) one portion of the environmental asset in the current period*. In other words, the policy to postpone the choice in order to enable the decision maker to profit of the coming information is sub-optimal when the last one is endogenous: if the uncertainty is primarily about the benefits of development, this strengthens the case for (some) development. On the other hand, even when more information is provided solely by development, *substantial* development [8] may not be indicated.

In fact, Freeman (1984) and Fisher and Hanemann (1987) assumes that full information is provided by *any* amount of development; moreover, no exogenous information arrives. As one can notice, this is a very particular information structure, where only a very particular type of endogenous information is allowed. The resulting expected value function under this scenario, denoted $EV_{endo}(c_1)$, is equivalent to that in the no-information scenario, EV_0, in the event no development is undertaken, and to that in the new (exogenous) in-

[7] The fact that is unreasonable that research is costless and information arrives without conscious planning or effort. Usually, information arrives by mounting some specific research program (for example, research on the medical properties of indigenous plant species). Obviously, these research programs are conducted independently from the development and the preservation itself.

[8] To be intended as the destruction of a high proportion of the environmental area.

formation scenario, EV_{exo}, in the event *any* development is undertaken [9]. In symbols,

$$EV_{endo}(c_1) = \begin{cases} EV_0(1) & \text{if} \quad c_1 = 1 \\ EV_{exo}(c_1) & \text{if} \quad c_1 \in [0, 1) \end{cases}$$

Several results follow form this (particular) formulation of the problem:

1) It can never be optimal to have $(\hat{c}_1)_{endo} = 1$.

2) There is still a corner solution for c_1, in the sense that one either develops fully now $(\hat{c}_1)_{endo} = 0$ or engages in an infinitesimal amount of development $(\hat{c}_1)_{endo} = \varepsilon$.

3) For any $c_1 \in (0, 1)$, if a decision-maker were going to develop at all, he would understate the advantage of partial over full development, if he disregarded the possibility of acquiring information through independent learning. In symbols,

$$[EV_{endo}(c_1) - EV_{endo}(0)] > [EV_0(c_1) - EV_0(0)]$$

In particular, the *quasi-option value of the minimum feasible development* (ε *− development*),

$$QOV_\varepsilon = EV_{endo}(\varepsilon) - EV_0(\varepsilon) > 0$$

is positive.

4) Lastly, $(\hat{c}_1)_{endo} \geq (\hat{c}_1)_0$, i.e. if the decision is to develop when one disregards the possibility of (dependent) learning, the correct decision when recognizing this possibility cannot be more development and may be less.

Conscious of the fact that the policy implications depend crucially on the precise manner in which development generates the information (i.e., on the form of the "information production function"), in what follows we try to model endogenous uncertainty in order to be as much "general" as possible.

In the case of oil extraction in one country, for example, there may be uncertainty as to whether and where the land contains oil in commercial quantities. If it is the case, it is likely that the uncertainty can be solved by undertaking some development. But it is without doubts that if you drill in the land (by destroying a part of the natural resource) the deeper you drill the higher the probability of discovering a oil well.

Another example: if you destroy only one or two trees of the Amazon forest, you obtain very little information about the possible extinction of a certain species. If instead you keep on destroying a larger portion of the forest, you can obtain higher information about the pervasive effects of the development activity.

Thus, it seems plausible that, in case information comes out through development of the natural resource, *the level of information coming out depends on*

[9] Note that another (strong) assumption is that the new information coming out when any level of information is undertaken is the same we discover under the exogenous information context. We maintain this hypothesis in our model.

the level of development performed. This is an assumption we will maintain in what follows when we characterize the endogenous information framework (and that differentiates our framework from the one of Freeman (1984) seen above).

Besides, differently from Freeman (1984) and from Fisher and Hanemann (1987) we will define the *Testing value* not as the difference between $EV_{endo}(\varepsilon)$ and $EV_0(\varepsilon)$ but rather as

$$TV = EV(c_1^*) - EV_{exo}((\hat{c}_1)_{exo})$$

where $EV(c_1)$ is the expected value of benefits when there is both exogenous and endogenous information and c_1^* is the optimal preservation level in the current period, under this information structure. Thus, the testing value has to be interpreted as the additional value attached to endogenous information, additional with respect to the information exogenously arriving. In other words, it is the gain the decision maker obtains when he can receive information regarding future benefits, by developing in the current period (with respect to the case in which he ignores the possibility of receiving information through this way). Obviously, if $c_1^* = 0$, $TV = 0$.

The *quasi-option value of the minimum feasible development* defined by Hanemann and Fisher (1987) becomes a particular testing value that emerges when only endogenous information is available (and exogenous information is completely absent) and when the information coming out is the same for every $c_1 \in [0, 1)$.

2 The Model

2.1 Assumptions and notation

Let us consider a two-periods model of choice ($t = 1, 2$), where in $t = 1$ the decision-maker (DM) has to choose the amount of environmental resource he wants to preserve (not develop) until t =2. Assuming the level of the environmental resource be normalized to 1, let us indicate with $c_1 \in [0, 1]$ the amount preserved in period 1. We indicate with b_1 the marginal benefit deriving from the decision of preserving at time $t = 1$ [10]. We assume current benefits from preservation are negative, i.e. $b_1 < 0$; [11] thus, the unique incentive to preserve in $t = 1$ is given by the possibility to receive a positive future benefit in $t = 2$.[12]

In the second period, DM chooses again the amount of the resource to be preserved. Since we assume development is irreversible, it is straightforward that in t =2 it is not possible to preserve more than one has done in t =1: DM's options in 2 are constrained to the decision taken in 1. Thus, if we indicate with c_2 the amount preserved in period 2, by irreversibility, it is $c_2 \in [0, c_1]$: the amount chosen in $t = 2$ cannot be higher than the one chosen in $t = 1$.

In the second period, there are two possible states of the world. With probability π, the true state is revealed before the decision in $t = 2$ is taken by the DM. With probability $1-\pi$, the DM does not know the true state of the world when he chooses the optimal level of c_2 in $t = 2$: this state is revealed after this decision has been taken. We indicate with b_2^i the marginal benefit deriving from preservation in period 2, when the revealed state of the world is s^i, $i = u, f$. The probability distribution over the states of the world is $\left(s^u, p; s^f, 1-p\right)$. [13] We also assume the benefit from preservation is negative if the state of the world is s^u (unfavorable state), and positive if the state of the world is s^f (favorable state), i.e. $b_2^u < 0, b_2^f > 0$. [14]

[10] Benefits (from preservation and from development) derinving from the choice of preserving in $t = 1$ are known with certainty by the decision maker.

[11] Miller and Lad (1983) consider a larger state of nature space, namely

$$\Theta = [b_1(c_1), b_2(c_1, c_2) | (c_1, c_2) \in D]$$

where D is the decision space. The members of Θ are the various pairs of benefits and costs which could possibly accrue during the first period and second period for each possible decision which might be made. Thus, also benefits in period $t = 1$ are not known when choosing the level of preservation c_1.

The component of the pairs in Θ, b_t, can be thought of as four dimensional vectors (pc_t, pb_t, dc_t, db_t) representing preservation costs, preservation benefits, development costs and development benefits associated with the action taken in period t.

[12] We chose to do not contemplate into the analysis the case $b_1 = 0$, since it makes the choice of c_1 irrelevant. The same reasoning holds for b_2.

[13] p is the probability of the state s^u before this state is revealed.

[14] Differently form Beltratti, Chichilnisky and Heal (1996), we do not assume that the expected benefit of preservation in the second period is positive, i.e. $pb_2^u + (1-p)b_2^s > 0$. We let this quantity to be greater, equal or less than zero.

According to our assumptions, we indicate with:

c_2: amount of environmental resource preserved in $t = 2$, when the true state of the world has not been revealed before;

c_2^u: amount of environmental resource preserved in 2, when DM knows the true state of the world is s^u;

c_2^f: amount of environmental resource preserved in 2, when DM knows the true state of the world is s^f.

A more useful way to identify the decision problem is to sum up the sequence of events through four crucial steps:

- Step (a): DM chooses the amount of the environmental resource he wants to preserve in time $t = 1$.

- Step (b): Or the true state of the world is revealed or it is not.

- Step (c): DM chooses the amount of the environmental resource he wants to preserve in time $t = 2$.

- Step (d): If in Step (b) no information has coming out, now the true state of the world is revealed.

The structure of the decision problem is summarized in Figure 1 below.

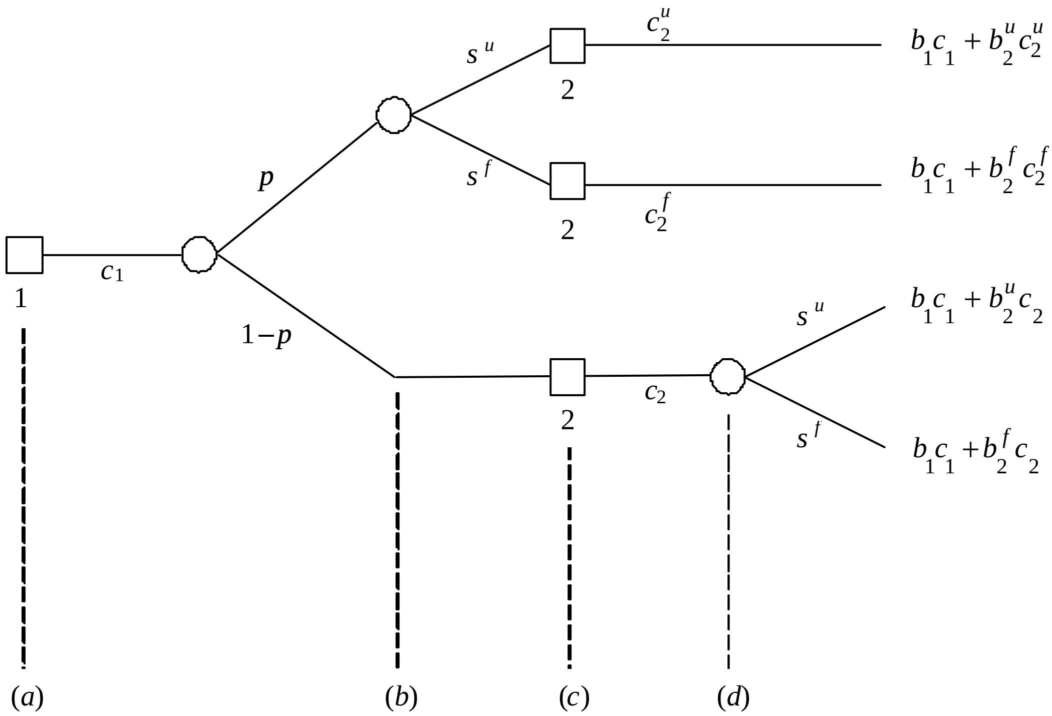


Figure 1.1

It is useful to state a pair of results which simplifies the subsequent analysis. These results holds independently from the kind of information structure we deal with (i.e., independently from the way in which π is defined):

Result 1
If in Step (b) the true state of the world is revealed, then $(c_2^u)^* = 0, (c_2^u)^* = 1$.

Result 2
If in Step (b) the true state of the world is not revealed, then

$$ {c_2}^* = \begin{cases} 0 & \text{if} \quad pb_2^u + (1-p)b_2^s < 0 \\ c_1 & \text{if} \quad pb_2^u + (1-p)b_2^s > 0 \end{cases} $$

2.2 How we model uncertainty

In our model uncertainty characterizes Step (b): DM does not know if the true state of the world will be known or not when he will take his choice c_2 in Step (c). The key parameter is $\pi \in [0,1]$, the probability that the true state is revealed before choosing c_2, i.e. the probability the new (complete) information arrives before Step (c). According to the way in which we define the form and the properties of π (remembering it is a probability), we are able to identify:

- if there is new information coming out or not ($\pi = 0$ or $\pi \in (0,1]$);
- new information arrives with certainty or not ($\pi = 1$ or $\pi \in [0,1)$);
- information can be exogenous, endogenous, or both: in the first case, π does not depend on c_1, the amount of environmental resource preserved in (and, obviously, by $(1-c_1)$, the amount of environmental resource developed in the same Step); in the second case, π does depend on c_1, or, rather, on $(1-c_1)$, the level of development. In particular, we assume (as a particular case of what stated in Section 1.2) *the level of information coming out when information is endogenous is directly proportional to the level of development performed*. In the third case, one part of the information arrives exogenously and the rest comes out accordingly to $(1-c_1)$ hence, $\pi = f(1-c_1)$, with $f'(\cdot) > 0$.

We can summarize every information categories described above in four cases, where Case A, Case B and Case D are derived form Case C respectively with $q = \lambda = 0, \lambda = 0, q = 0$, for $c_1 \in [0,1]$ (see the table below). In particular, Case D is considered separately (even though it is very rare in reality to have only endogenous information) only because it represents the way in which endogenous information in analyzed in literature (i.e. separately from any other kind of information) and thus it allows us to make direct comparisons between the results we find and the results accepted by environmental option values literature.

CASE A: No information

$$\pi = 0 \qquad \text{for} \quad c_1 \in [0,1]$$

CASE B: Exogenous information

$$\pi = q \in (0,1] \qquad \text{for} \quad c_1 \in [0,1]$$

CASE C: Exogenous and Endogenous information

$$\begin{aligned} \pi &= q + \lambda f(1-c_1) \qquad \text{for} \quad c_1 \in [0,1] \\ & \text{with} \\ \lambda &\in (0, 1-q]\,, \\ f(1-c_1) &= 1 - c_1 \qquad \text{for} \quad c_1 \in [0,1] \\ f'(1-c_1) &> 0 \qquad \text{for} \quad c_1 \in [0,1] \end{aligned}$$

CASE D: Endogenous information

$$\begin{aligned} \pi &= \lambda f(1-c_1) \qquad \text{for} \quad c_1 \in [0,1] \\ & \text{with} \\ \lambda &\in (0, 1-q]\,, \\ f(1-c_1) &= 1 - c_1 \qquad \text{for} \quad c_1 \in [0,1] \\ f'(1-c_1) &> 0 \qquad \text{for} \quad c_1 \in [0,1] \end{aligned}$$

The subcase *information arriving with certainty* [15] can be derived in the following ways;

- Case B with $q = 1$;
- Case C with $\lambda = 1 - q$;
- Case D with $\lambda = 1$.

2.2.1 CASE A: No information

The decision problem represented in Figure 1 is reduced to the one in Figure 2.1 below.

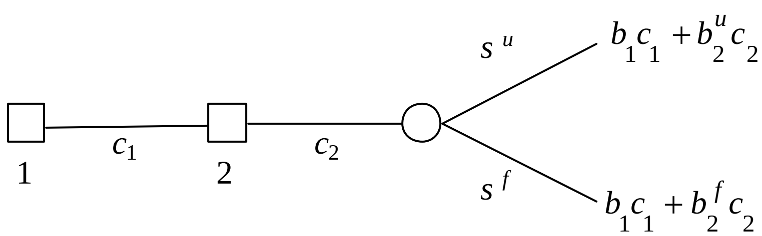


Figure 2.1

[15] Again, we analyze this case separately only because it is the one most frequently analyzed in environmental option values literature: thus, it allows us to make comparisons and to show that our results holds even with the restriction of information arriving with certainty.

The expected value of the lottery is

$$EV(c_1, c_2) = b_1 c_1 + \left[p b_2^u + (1-p) b_2^f \right] c_2$$

Let us write the expected value of preserving as a function of c_1 only, by choosing c_2 optimally in the second period: by this way, we obtain the expected value of preserving in $t = 1$, given that DM's choice in $t = 2$ is optimal.

By looking at *Result 2*,

$$\begin{array}{llll} (i) & p b_2^u + (1-p) b_2^f < 0 & \Longrightarrow & {c_2}^* = 0 \\ & & \Longrightarrow & EV_0(c_1, {c_2}^* = 0) = b_1 c_1 \\ & & \Longrightarrow & {c_1}^* = 0 \end{array}$$

$$\begin{array}{llll} (ii) & p b_2^u + (1-p) b_2^f > 0 & \Longrightarrow & {c_2}^* = c_1 \\ & & \Longrightarrow & EV_0(c_1, {c_2}^* = c_1) = \left[b_1 + p b_2^u + (1-p) b_2^f \right] c_1 \\ & & \Longrightarrow & {c_2}^* = \begin{cases} 0 & \text{if} \quad b_1 + p b_2^u + (1-p) b_2^f < 0 \\ c_1 & \text{if} \quad b_1 + p b_2^u + (1-p) b_2^f > 0 \end{cases} \end{array}$$

By summarizing, [16]

$$\begin{array}{llll} (i) & p b_2^u + (1-p) b_2^f < 0 & \Longrightarrow & {c_1}^* = {c_2}^* = 0 \\ & & \Longrightarrow & {EV_0}^* = 0 \end{array}$$

$$\begin{array}{llll} (ii)' & \begin{cases} p b_2^u + (1-p) b_2^f > 0 \\ b_1 + p b_2^u + (1-p) b_2^f < 0 \end{cases} & \Longrightarrow & {c_1}^* = {c_2}^* = 0 \\ & & \Longrightarrow & {EV_0}^* = 0 \end{array}$$

$$\begin{array}{llll} (ii)'' & \begin{cases} p b_2^u + (1-p) b_2^f > 0 \\ b_1 + p b_2^u + (1-p) b_2^f > 0 \end{cases} & \Longrightarrow & b_1 + p b_2^u + (1-p) b_2^f > 0 \\ & & \Longrightarrow & {c_1}^* = {c_2}^* = 1 \\ & & \Longrightarrow & {EV_0}^* = b_1 + p b_2^u + (1-p) b_2^f \end{array}$$

Let us represent graphically, for $p = \frac{1}{2}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_0(\cdot)$. We employ a cartesian plane with the relative benefits $-\frac{b_1}{b_2^f}$ on the x-axis and with the relative benefits $-\frac{b_2^u}{b_2^f}$ on the y-axis. [17] Obviously, when these two ratios vary (when the values assumed by the benefits of preserving during

[16] The second condition in $(ii)''$ implies the first one.

[17] The choice of the two ratios as variables for the axes has been supported, not only by its theoric logicality, but also by the willingness of obtaining two positive quantities (since $b_1, b_2^u < 0, b_2^f > 0$), in order to concentrate in the I quadrant the combined analysis of the conditions (inequalities) we found by solving the maximization problem.

the two periods vary), the optimal choice for the first and for the second period change too.

For $p = \frac{1}{2}$, the conditions summarized above become

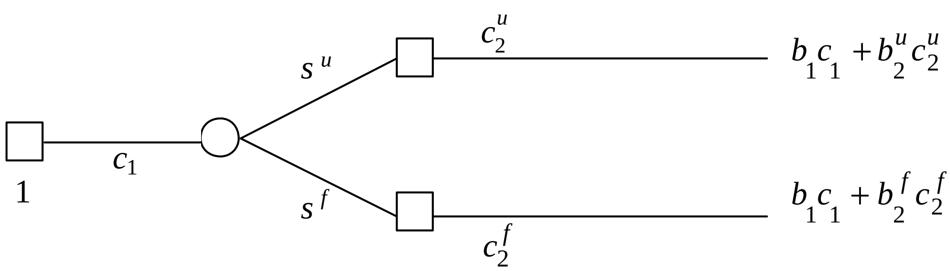
$$
\begin{array}{llcll}
(i) & -\frac{b_2^u}{b_2^f} & > & 1 & \Longrightarrow \quad EV_0{}^* = 0 \\
(ii)' & 1 - 2\left(-\frac{b_1}{b_2^f}\right) & < & -\frac{b_2^u}{b_2^f} < 1 & \Longrightarrow \quad EV_0{}^* = 0 \\
(ii)'' & -\frac{b_2^u}{b_2^f} & < & 1 - 2\left(-\frac{b_1}{b_2^f}\right) & \Longrightarrow EV_0{}^* = b_1 + \frac{1}{2}b_2^u + \frac{1}{2}b_2^f
\end{array}
$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in $Figure$ 2.2 in the Appendix.

2.2.2 CASE B: Exogenous Information

Let us start with case in which the new information arrives exogenously in *Step* (b) with certainty, i.e. DM at *Step* (a) knows that at *Step* (c) with probability 1 he knows the values of the benefit b_2^i (i.e., he knows that the benefit is b_2^u or b_2^f).

A particular case: Perfect (i.e. $\pi = 1$) Exogenous (i.e. $\pi = q$) Information The decision problem represented in Figure 1 is reduced to the one in *Figure* 3.1 below.



$Figure$ 3.1

The expected value of the lottery is

$$EV_{exo|q=1}(c_1, c_2^u, c_2^f) = b_1c_1 + pb_2^u c_2^u + (1-p)b_2^f c_2^f$$

By looking at Re*sult* 1, $(c_2^u)^* = 0$ and that $(c_2^f)^* = c_1$, hence

$$EV_{exo|q=1}(c_1, (c_2^u)^*, (c_2^f)^*) = b_1 c_1 + (1-p) b_2^f c_1$$

For which concerns the optimal level of conservation in the first period, we have

$$\begin{array}{llllllll} (i) & b_1 + (1-p)b_2^f & < & 0 & \Longrightarrow & {c_1}^* = 0 & \Longrightarrow & EV_{exo|q=1}(0) = 0 \\ (ii) & b_1 + (1-p)b_2^f & > & 0 & \Longrightarrow & {c_1}^* = 1 & \Longrightarrow & EV_{exo|q=1}(1) = b_1 + (1-p)b_2^f \end{array}$$

In the same cartesian plane we employ in the previous point, we represent graphically, for $p = \frac{1}{2}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_{exo|q=1}(\cdot)$.

For $p = \frac{1}{2}$, the conditions above become

$$\begin{array}{llllll} (i) & -\frac{b_1}{b_2^f} & > & \frac{1}{2} & \Longrightarrow & {EV_{exo|q=1}}^* = 0 \\ (ii) & -\frac{b_1}{b_2^f} & < & \frac{1}{2} & \Longrightarrow & {EV_{exo|q=1}}^* = b_1 + \frac{1}{2} b_2^f \end{array}$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in *Figure* 3.2 in the Appendix.

The general case When $q \in (0,1)$, *DM* at *Step* (*a*) knows that at *Step* (*b*) with probability q he knows the value realized of the benefit b_2^i. Hence, he is not sure that he will know the true state of the world when he will choose at *Step* (*c*).

Let us solve the *DM*'s utility maximization problem.

Given *Result* 1 and *Result* 2, the realized payoffs are those indicated in *Figure* 3.3 below.

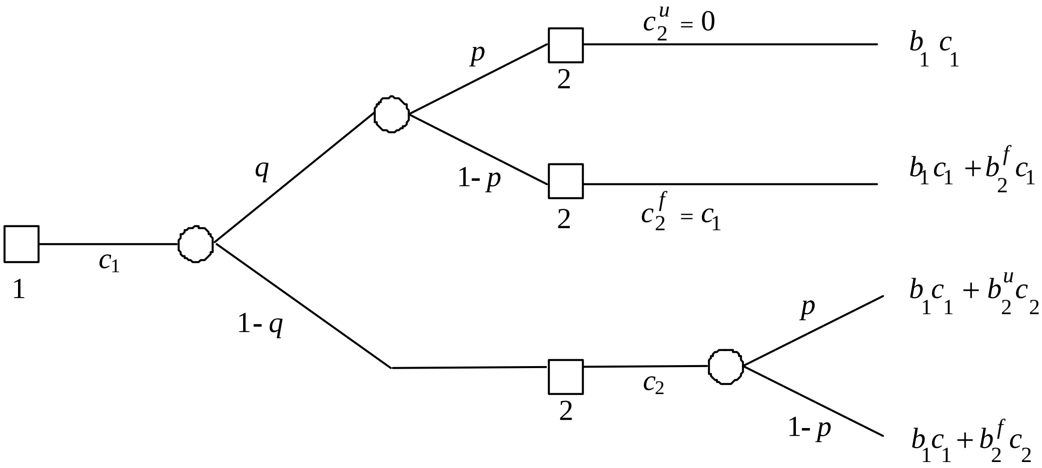


Figure 3.3

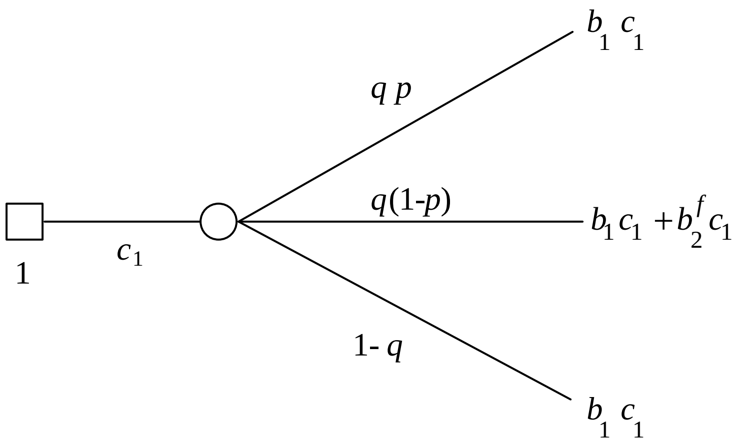
DM's expected payoff is

$$\begin{aligned} EV_{exo}(c_1, (c_2^u)^*, (c_2^f)^*) &= q\left[(pb_1 + (1-p)(b_1 + b_2^f)\right] c_1 + \\ &\quad +(1-q)\left[p(b_1 c_1 + b_2^u c_2) + (1-p)(b_1 c_1 + b_2^f c_2)\right] \\ &= q\left[b_1 + (1-p)b_2^f\right] c_1 + (1-q)\left[b_1 c_1 + (pb_2^u + (1-p)b_2^f)c_2\right] \end{aligned}$$

Let us now analyze the "low part" of the compound lottery represented in *Figure* 3.3.

We use the results found in the previous subsection:

(i) $pb_2^u + (1-p)b_2^f < 0 \implies c_1{}^* = 0 \implies$ The compound lottery in *Figure* 3.3 can be reduced into the one-stage lottery in the figure below.



$Figure$ 3.4

The expected value of the lottery (given that in $Step$ (c) the DM follows an optimal choice strategy independently from the information he has received) is

$$
\begin{aligned}
EV_{exo}\left(c_1, (c_2^u)^*, (c_2^f)^*, {c_2}^* = 0\right) &= q\left[b_1 + (1-p)b_2^f\right] c_1 + (1-q)b_1 c_1 \\
&= \left[b_1 + q(1-p)b_2^f\right] c_1
\end{aligned}
$$

Hence, the optimal level of conservation in the first period is

$$
\begin{array}{llcl}
(i)' & b_1 + q(1-p)b_2^f < 0 & \Longrightarrow & {c_1}^* = 0 \Longrightarrow EV_{exo}(0) = 0 \\
(i)'' & b_1 + q(1-p)b_2^f > 0 & \Longrightarrow & {c_1}^* = 1 \Longrightarrow EV_{exo}(1) = b_1 + q(1-p)b_2^f
\end{array}
$$

(ii) $pb_2^u + (1-p)b_2^f > 0 \Longrightarrow {c_2}^* = c_1 \Longrightarrow$ The compound lottery in $Figure$ 3.4 can be reduced into the one-stage lottery in the figure below.

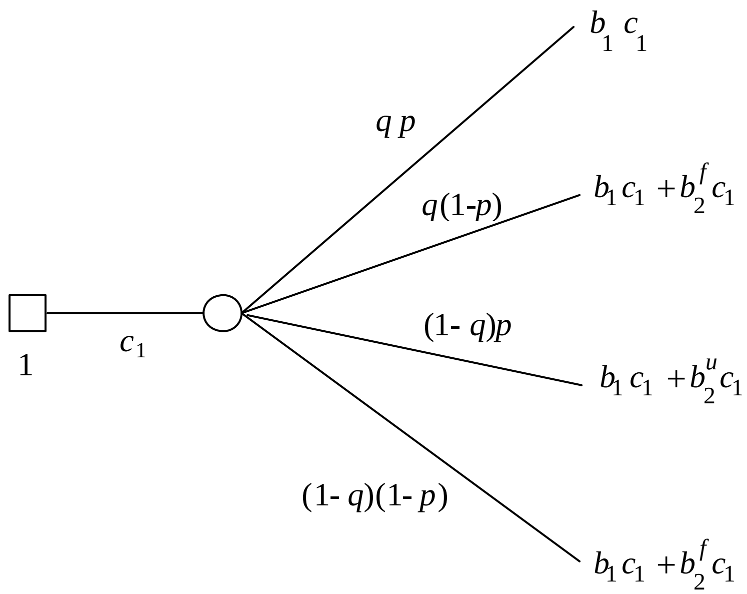


Figure 3.5

that can be still reduced in the lottery

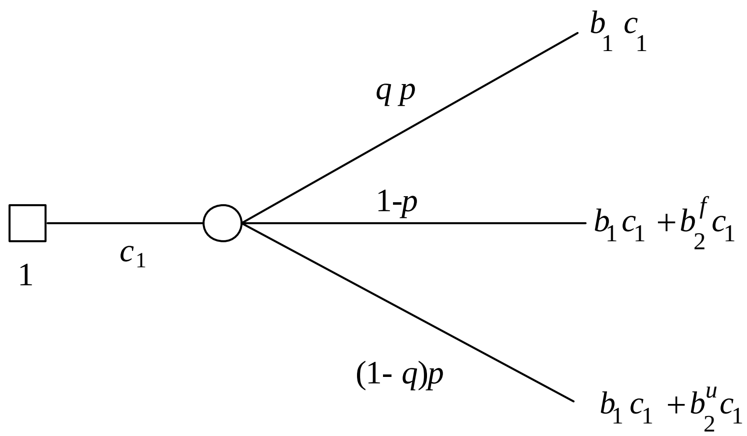


Figure 3.6

The expected value of the lottery (given that in *Step* (c) the *DM* follows an optimal choice strategy independently from the information he has received) is

$$
\begin{aligned}
EV_{exo}\left(c_1,(c_2^u)^*,(c_2^f)^*,=c_1\right) &= q\left[(pb_1+(1-p)(b_1+b_2^f)\right]c_1+ \\
&\quad (1-q)\left[p(b_1c_1+b_2^uc_1)+(1-p)(b_1c_1+b_2^fc_1)\right] \\
&= q\left[(pb_1+(1-p)(b_1+b_2^f)\right]c_1+ \\
&\quad (1-q)\left[p(b_1+b_2^u)+(1-p)(b_1+b_2^f)\right]c_1 \\
&= \left[b_1+(1-q)pb_2^u+(1-p)b_2^f\right]c_1
\end{aligned}
$$

Hence, the optimal level of conservation in the first period is

$$(ii)' \qquad b_1+(1-q)pb_2^u+(1-p)b_2^f < 0 \implies {c_1}^*=0 \implies EV_{exo}(0)=0$$

$$(ii)'' \qquad b_1+(1-q)pb_2^u+(1-p)b_2^f > 0 \implies {c_1}^*=1 \implies EV_{exo}(1)=b_1+(1-q)pb_2^u+(1-p)b_2^f$$

Let us represent graphically, for $p=\frac{1}{2}$ and $q=\frac{1}{3}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_{exo}(\cdot)$.

For $p=\frac{1}{2}$and $q=\frac{1}{3}$, the conditions (inequalities) written above become

$$(i)' \quad \begin{cases} -\frac{b_2^u}{b_2^f}>1 \\ -\frac{b_1}{b_2^f}>\frac{1}{6} \end{cases} \implies c_1^*=0, c_2^*=0 \implies {EV_{exo}}^*=0$$

$$(i)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f}>1 \\ -\frac{b_1}{b_2^f}>\frac{1}{6} \end{cases} \implies c_1^*=1, c_2^*=0 \implies {EV_{exo}}^*=b_1+\frac{1}{6}b_2^f$$

$$(ii)' \quad \begin{cases} -\frac{b_2^u}{b_2^f}<1 \\ -\frac{b_2^u}{b_2^f}>\frac{3}{2}-3\left(-\frac{b_1}{b_2^f}\right) \end{cases} \implies c_1^*=c_2^*=0 \implies {EV_{exo}}^*=0$$

$$(ii)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f}<1 \\ -\frac{b_2^u}{b_2^f}<\frac{3}{2}-3\left(-\frac{b_1}{b_2^f}\right) \end{cases} \implies c_1^*=c_2^*=1 \implies {EV_{exo}}^*=b_1+\frac{1}{3}b_2^u+\frac{1}{2}b_2^f$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in *Figure* 3.7 in the Appendix.

2.3 The calculus of the Waiting Value

Let us calculate the Waiting Value using the expression we introduce in Section 1.1, i.e.

$$WV = EV_{exo}{}^{*} - EV_{0}{}^{*}$$

Since $EV_{exo}{}^{*}$ and $EV_0{}^{*}$ vary according to the values of b_1, b_2^u and b_2^f, we have to calculate the difference between the two optimal expected values by looking at the different regions we identify in the cartesian plane $\left(-\frac{b_1}{b_2^f}, -\frac{b_2^u}{b_2^f}\right)$: we do it by comparing *Figure* 3.7 with *Figure* 2.2.

For $-\frac{b_2^u}{b_2^f} > 1$ and $-\frac{b_1}{b_2^f} > 1, \qquad WV = 0.$

For $-\frac{b_2^u}{b_2^f} > 1$ and $-\frac{b_2^u}{b_2^f} > \frac{1-p}{(1-q)p} - \frac{1}{(1-q)p}\left(-\frac{b_1}{b_2^f}\right), \qquad WV = 0.$

For $-\frac{b_2^u}{b_2^f} < 1$ and $-\frac{b_2^u}{b_2^f} < q(1-p), \qquad WV = b_1 + q(1-p)b_2^f > 0.$

For $\frac{p}{1-p} - \frac{1}{1-p}\left(-\frac{b_1}{b_2^f}\right) < -\frac{b_2^u}{b_2^f} < 1$ and $-\frac{b_2^u}{b_2^f} < \frac{1-p}{(1-q)p} - \frac{1}{(1-q)p}\left(-\frac{b_1}{b_2^f}\right)$,
$WV = b_1 + (1-q)pb_2^u + (1-p)b_2^f > 0.$

For $-\frac{b_2^u}{b_2^f} < \frac{p}{1-p} - \frac{1}{1-p}\left(-\frac{b_1}{b_2^f}\right), \qquad WV = -qpb_2^u > 0.$

Thus, the waiting value is increasing in the probability of receiving new information (exogenously), in the probability of the state s^u (unfavorable state) before this state is revealed and in the level of benefits in the favorable state. It is decreasing in the level of benefits in the current period (taken in absolute value and given that they are negative by assumption) and in the level of benefits in the unfavorable state (taken in absolute value and given that they are negative by assumption). Briefly,

$$WV = WV(\underset{+,-}{p}, \underset{+}{q}, \underset{+}{b_1}, \underset{+}{b_2^u}, \underset{+}{b_2^f})$$

Obviously, if, following the literature on quasi-option values, we would calculate the Waiting value as the difference between the expected benefits in case of *certain* exogenous information and the expected benefits in the no information case, i.e.

$$WV_{|q=1} = EV_{exo|q=1}{}^{*} - EV_0{}^{*} \geq 0$$

we would find an upper bound for the Waiting value we have calculated above, i.e. $WV_{|q=1} \geq WV$, since WV is increasing in q.

Our result is more robust: we find that *in absence of endogenous information, even though exogenous information does not arrive with certainty (but with a given probability* $q \in (0,1)$*), the Waiting Value is always non-negative, thus* *forcing the DM towards a higher level of preservation of the environmental area during the first and the second period of choice.*

In fact, looking at the results shown in *Figure* 2.2 and 3.7, it is not difficult to see that it is always

$$\begin{array}{rcll} (c_1{}^*)_{exo} & \geq & (c_1{}^*)_0 & \quad (a) \\ (c_2{}^*)_{exo} & \geq & (c_2{}^*)_0 & \quad (b) \end{array}$$

Result (*b*) is derived following this reasoning: because of irreversibility, $(c_2{}^*)_{exo} \leq (c_1{}^*)_{exo}, (c_2{}^*)_0 \leq (c_1{}^*)_{exo}$, but since $(c_1{}^*)_{exo} \geq (c_1{}^*)_0$ in Step (*c*) *DM* has a larger choice set from which choosing $(c_2{}^*)_{exo}$; since the choice $(c_2{}^*)_0$ is possible also in case of exogenous information (because it is surely $(c_1{}^*)_{exo} \geq (c_2{}^*)_0$), then $(c_2{}^*)_{exo}$ is at least equal to $(c_2{}^*)_{exo}$. Besides, $(c_1{}^*)_{exo}, (c_1{}^*)_0, (c_2{}^*)_{exo}, (c_2{}^*)_0 \in \{0, 1\}$ thus the proof is even easier.

2.3.1 CASE C: Endogenous and Exogenous Information

A particular case: Perfect (i.e. $\pi = q + \lambda = 1$) Exogenous and Endogenous Information Given *Result* 1 and *Result* 2, the realized payoffs are those indicated in *Figure* 3.3.

Moreover, we can reason as in Case B: given that $(c_2^u)^* = 0$ and that $(c_2^f)^* = c_1$, the expected value of the lottery is

$$\begin{aligned} EV_{q+\lambda=1}(c_1, (c_2^u)^*, (c_2^f)^*) &= [q + (1-q)(1-c_1)] \left[(pb_1 + (1-p)(b_1 + b_2^f) \right] c_1 + \\ &\quad \{1 - [q + (1-q)(1-c_1)]\} \left[p(b_1 c_1 + b_2^u c_2) + (1-p)(b_1 c_1 + b_2^f c_2) \right] \end{aligned}$$

Let us now analyze the "low part" of the compound lottery represented in *Figure* 3.2, after having replaced q with $q + (1-q)(1-c_1)$ (value of π in the case we are analyzing) [18].

We use the results found in Case A.

(*i*) $\quad pb_2^u + (1-p)b_2^f < 0 \quad \Longrightarrow \quad c_2{}^* = 0 \quad \Longrightarrow$ We reduce the compound lottery into a one-stage lottery similar to the one in *Figure* 3.4 (after having replaced q with $q + (1-q)(1-c_1)$).

The expected value of the lottery (given that in *Step* (*c*) the *DM* follows an optimal choice strategy independently from the information he has received) is

$$\begin{aligned} EV_{q+\lambda=1}\left(c_1, (c_2^u)^*, (c_2^f)^*, c_2{}^* = 0\right) &= \{1 - (1-p)\,[q + (1-q)(1-c_1)]\}\, b_1 c_1 + \\ &\quad + (1-p)\,[q + (1-q)(1-c_1)]\,(b_1 + b_2^f) c_1 \\ &= b_1 c_1 + (1-p)\,[1 - (1-q)c_1]\, b_2^f c_1 \end{aligned}$$

[18] Notice that for $c_1 = 1$ (preservation of the whole environmental resource) Case C reduces to Case B.

By the First Order Condition, we obtain

$$\frac{dEV(c_1)}{dc_1} = b_1 + (1-p)b_2^f - 2(1-p)(1-q)b_2^f c_1 = 0$$

$$\Longrightarrow \qquad {c_1}^* = \frac{b_1 + (1-p)b_2^f}{2(1-p)(1-q)b_2^f}$$

Hence, given that the quantity $\frac{b_1+(1-p)b_2^f}{2(1-p)(1-q)b_2^f}$ could be lower than 0 or greater than 1, the optimal level of conservation in the first period is

$${c_1}^* = \begin{cases} 0 & \text{if} \quad b_1 < -(1-p)b_2^f \qquad (i)' \\ \frac{b_1+(1-p)b_2^f}{2(1-p)(1-q)b_2^f} & \text{if} \quad -(1-p)b_2^f < b_1 < (1-p)(1-2q)b_2^f \qquad (i)'' \\ 1 & \text{if} \quad b_1 > (1-p)(1-2q)b_2^f \qquad (i)''' \end{cases}$$

The optimal expected benefit function $EV_{q+\lambda=1}(\cdot)$ is

$$EV^*_{q+\lambda=1}(b_1, b_2^u, b_2^f) = \begin{cases} 0 & \text{if} \quad b_1 < -(1-p)b_2^f \qquad (i)' \\ \frac{\left[b_1+(1-p)b_2^f\right]^2}{4(1-p)(1-q)b_2^f} & \text{if} \quad b_1 < -(1-p)b_2^f \qquad (i)'' \\ b_1 + (1-p)b_2^f & \text{if} \quad b_1 < -(1-p)b_2^f \qquad (i)''' \end{cases}$$

(ii) $\quad pb_2^u + (1-p)b_2^f > 0 \quad \Longrightarrow \quad {c_2}^* = c_1 \quad \Longrightarrow$ The compound lottery can be reduced into a one-stage lottery similar to the one in $Figure$ 3.6 (after having replaced q with $q + \lambda(1-c_1)$).

The expected value of the lottery (given that in $Step$ (c) the DM follows an optimal choice strategy independently from the information he has received) is

$$\begin{aligned} EV_{compl}(c_1, (c_2^u)^*, (c_2^f)^*, {c_2}^* &= c_1) = b_1c_1 + (1-p)b_2^f c_1 + \{1 - [q + (1-q)(1-c_1)]\}\, pb_2^u c_1 \\ &= b_1c_1 + (1-p)b_2^f c_1 + (1-q)pb_2^u (c_1)^2 \end{aligned}$$

By the First Order Condition, we obtain

$$\frac{dEV_{q+\lambda=1}(c_1)}{dc_1} = b_1 + (1-p)b_2^f + 2(1-q)pb_2^u c_1 = 0$$

$$\Longrightarrow \qquad {c_1}^* = -\frac{b_1 + (1-p)b_2^f}{2(1-q)pb_2^u}$$

Hence, given that the quantity $-\frac{b_1+(1-p)b_2^f}{2(1-q)pb_2^u}$ could be lower than 0 or greater than 1, the optimal level of preservation in the first period is

$$c_1^* = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \qquad (ii)' \\ -\frac{b_1+(1-p)b_2^f}{2(1-q)pb_2^u} & \text{if} \quad 0 < b_1 + (1-p)b_2^f < -2(1-q)pb_2^u \qquad (ii)'' \\ 1 & \text{if} \quad b_1 + (1-p)b_2^f > -2(1-q)pb_2^u \qquad (ii)''' \end{cases}$$

The optimal expected benefit function $EV^*{}_{q+\lambda=1}(\cdot)$ is

$$EV^*_{q+\lambda=1}(b_1, b_2^u, b_2^f) = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \qquad (ii)' \\ -\frac{\left[b_1+(1-p)b_2^f\right]^2}{4(1-q)pb_2^u} & \text{if} \quad 0 < b_1 + (1-p)b_2^f < -2(1-q)pb_2^u \qquad (ii)'' \\ b_1 + (1-q)pb_2^u + (1-p)b_2^f & \text{if} \quad b_1 + (1-p)b_2^f > -2(1-q)pb_2^u \qquad (ii)''' \end{cases}$$

· Let us represent graphically, for $p = \frac{1}{2}$ and $q = \frac{1}{3}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_{q+\lambda=1}(\cdot)$.

For $p = \frac{1}{2}$and $q = \frac{1}{3}$, the conditions (inequalities) written above become [19]

$$(i)' \qquad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} > \frac{1}{2} \end{cases} \Longrightarrow \quad c_1^* = 0, c_2^* = 0$$

$$(i)'' \qquad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{2} \end{cases} \Longrightarrow \quad c_1^* = \frac{3}{2}\frac{b_1}{b_2^f} + \frac{3}{4}, c_2^* = 0$$

$$(ii)' \qquad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} > \frac{1}{2} \end{cases} \Longrightarrow \quad c_1^* = c_2^* = 0$$

$$(ii)'' \qquad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ 0 < \frac{b_1}{b_2^f} + \frac{1}{2} < -\frac{2}{3}\frac{b_2^u}{b_2^f} \end{cases} \Longrightarrow \quad c_1^* = c_2^* = -\frac{3}{2}\frac{b_1}{b_2^u} - \frac{3}{4}\frac{b_2^f}{b_2^u}$$

$$=$$

$$(ii)''' \qquad -\frac{b_2^u}{b_2^f} \quad < \quad -\frac{2}{3}\left(-\frac{b_1}{b_2^f}\right) + \frac{3}{4} \Longrightarrow c_1^* = c_2^* = 1$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in *Figure* 4.1 in the Appendix.

· Let us represent graphically, for $p = \frac{1}{2}$ and $q = \frac{2}{3}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_{q+\lambda=1}(\cdot)$.

[19] For which concerns case $(i)'''$, it has not been considered since for $q < \frac{1}{2}$ the condition $b_1 + (1-p)b_2^f > -2(1-q)pb_2^u$ is never verified. In other words, for $q < \frac{1}{2}$, the condition $b_1 + (1-p)b_2^f < -2(1-q)pb_2^u$ holds for every $b_1, b_2^u < 0, b_2^f > 0$ and for every $p \in (0,1)$.

For $p = \frac{1}{2}$ and $q = \frac{1}{3}$, the conditions (inequalities) written above become [20]

$$(i)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} > \frac{1}{2} \end{cases} \implies c_1^* = 0, c_2^* = 0$$

$$(i)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ \frac{1}{6} < -\frac{b_1}{b_2^f} < \frac{1}{2} \end{cases} \implies c_1^* = 3\frac{b_1}{b_2^f} + \frac{3}{2}, c_2^* = 0$$

$$(i)''' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{6} \end{cases} \implies c_1^* = 1, c_2^* = 0$$

$$(ii)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} > \frac{1}{2} \end{cases} \implies c_1^* = c_2^* = 0$$

$$(ii)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ 0 < \frac{b_1}{b_2^f} + \frac{1}{2} < -\frac{1}{3}\frac{b_2^u}{b_2^f} \end{cases} \implies c_1^* = c_2^* = -3\frac{b_1}{b_2^u} - \frac{3}{2}\frac{b_2^f}{b_2^u}$$

$$(ii)''' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} < -3\left(-\frac{b_1}{b_2^f}\right) + \frac{3}{2} \end{cases} \implies c_1^* = c_2^* = 1$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in *Figure* 4.2 in the Appendix.

The general case When $\lambda \in [0, 1-q)$, *DM* at *Step* (a) knows that, even if he decides to destroy the entire environmental resource ($c_1 = 0$), at *Step* (c) he will know the value realized of the benefit b_2^i with probability $q + \lambda < 1$. Hence, even if he destroy everything, he is not sure that he will know the true state of the world when he will choose at *Step* (c).

Given *Result* 1 and *Result* 2, the realized payoffs are those indicated in *Figure* 3.3.

Reasoning as in Case B, given that $(c_2^u)^* = 0$ and that $(c_2^f)^* = c_1$,

$$\begin{aligned} EV(c_1, (c_2^u)^*, (c_2^f)^*) &= [q + \lambda(1 - c_1)] \left[(pb_1 + (1-p)(b_1 + b_2^f) \right] c_1 + \\ & \{1 - [q + \lambda(1 - c_1)]\} \left[p(b_1 c_1 + b_2^u c_2) + (1-p)(b_1 c_1 + b_2^f c_2) \right] \end{aligned}$$

Let us now analyze the "low part" of the compound lottery represented in *Figure* 3.3, after having replaced q with $q + \lambda(1 - c_1)$ (value of π in the case we are analyzing) [21].

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[20] As one can notice, only conditions $(i)'$ and $(i)''$ remain the same as in the case where $q < \frac{1}{2}$. In particular, for $q > \frac{1}{2}$, we find valus of b_1, b_2^u and b_2^f for which condition $(i)'''$ (and the related area in the graph) takes place.

[21] Notice that for $c_1 = 1$ (preservation of the whole environmental resource) Case C reduces to Case B.

We use the results found in the previous case. We refer again to *Figure* 3.4, *Figure* 3.5 and *Figure* 3.6, by changing the probability assigned to the branches of the trees (with $q + \lambda(1 - c_1)$ in place of q):

(i) $\quad pb_2^u + (1-p)b_2^f < 0 \quad \Longrightarrow \quad {c_2}^* = 0 \quad \Longrightarrow$ The expected value of the lottery (given that in *Step* (c) the DM follows an optimal choice strategy independently from the information he has received) is

$$\begin{aligned} EV\left(c_1, (c_2^u)^*, (c_2^f)^*, {c_2}^* = 0\right) &= \{1 - (1-p)\left[q + \lambda(1-c_1)\right]\} b_1 c_1 + \\ &\quad + (1-p)\left[q + \lambda(1-c_1)\right](b_1 + b_2^f)c_1 \\ &= b_1 c_1 + (1-p)\left[q + \lambda(1-c_1)\right] b_2^f c_1 \end{aligned}$$

By the First Order Condition, we obtain

$$\begin{aligned} \frac{dEV(c_1)}{dc_1} &= b_1 + (1-p)(q+\lambda)b_2^f - 2(1-p)\lambda b_2^f c_1 = 0 \\ \Longrightarrow \quad & {c_1}^* = \frac{b_1 + (1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f} \end{aligned}$$

Hence, given that the quantity $\frac{b_1+(1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f}$ could be lower than 0 or greater than 1, the optimal level of conservation in the first period is

$$ {c_1}^* = \begin{cases} 0 & \text{if} \quad b_1 < -(1-p)(q+\lambda)b_2^f \qquad (i)' \\ \frac{b_1+(1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f} & \text{if} \quad -(1-p)(q+\lambda)b_2^f < b_1 < (1-p)(\lambda - q)b_2^f \qquad (i)'' \\ 1 & \text{if} \quad b_1 > (1-p)(\lambda - q)b_2^f \qquad (i)''' \end{cases} $$

The optimal expected benefit function $EV^*(\cdot)$ is

$$ EV^*(b_1, b_2^u, b_2^f) = \begin{cases} 0 & \text{if} \quad b_1 < -(1-p)(q+\lambda)b_2^f \qquad (i)' \\ \frac{\left[b_1+(1-p)(q+\lambda)b_2^f\right]^2}{4(1-p)\lambda b_2^f} & \text{if} \quad -(1-p)(q+\lambda)b_2^f < b_1 < (1-p)(\lambda - q)b_2^f \qquad (i)'' \\ b_1 + (1-p)(q+\lambda)b_2^f & \text{if} \quad b_1 > (1-p)(\lambda - q)b_2^f \qquad (i)''' \end{cases} $$

(ii) $\quad pb_2^u + (1-p)b_2^f > 0 \quad \Longrightarrow \quad {c_2}^* = c_1 \quad \Longrightarrow$ The expected value of the lottery (given that in *Step* (c) the DM follows an optimal choice strategy independently from the information he has received) is

$$\begin{aligned} EV\left(c_1, (c_2^u)^*, (c_2^f)^*, {c_2}^* = c_1\right) &= b_1 c_1 + (1-p)b_2^f c_1 + \{1 - \left[q + \lambda(1-c_1)\right]\} p b_2^u c_1 \\ &= b_1 c_1 + (1-p)b_2^f c_1 + p(1 - q - \lambda + \lambda c_1)b_2^u c_1 \end{aligned}$$

By the First Order Condition, we obtain

$$\frac{dEV(c_1)}{dc_1} = b_1 + (1-p)b_2^f + p(1-q-\lambda)b_2^u + 2\lambda p b_2^u c_1 = 0$$
$$\Longrightarrow \qquad {c_2}^* = -\frac{b_1 + (1-p)b_2^f + p(1-q-\lambda)b_2^u}{2p\lambda b_2^u}$$

Hence, given that the quantity $-\frac{b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u}{2p\lambda b_2^u}$ could be lower than 0 or greater than 1, the optimal level of conservation in the first period is

$$ {c_1}^* = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < -(1-q-\lambda)b_2^u \qquad (ii)' \\ -\frac{b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u}{2p\lambda b_2^u} & \text{if} \quad -(1-q-\lambda)b_2^u < b_1 + (1-p)b_2^f < -(1-q+\lambda)b_2^u \qquad (ii) \\ 1 & \text{if} \quad b_1 + (1-p)b_2^f > -(1-q+\lambda)b_2^u \qquad (ii)''' \end{cases}$$

The optimal expected benefit function $EV^*(\cdot)$ is

$$EV^*(b_1, b_2^u, b_2^f) = \begin{cases} 0 \\ \quad \text{if} \quad b_1 + (1-p)b_2^f < -(1-q-\lambda)b_2^u \qquad (ii)' \\ -\frac{\left[b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u\right]^2}{4p\lambda b_2^u} \\ \quad \text{if} \quad -(1-q-\lambda)b_2^u < b_1 + (1-p)b_2^f < -(1-q+\lambda)b_2^u \qquad (ii)'' \\ b_1 + (1-p)b_2^f + p(1-q)b_2^u \\ \quad \text{if} \quad b_1 + (1-p)b_2^f > -(1-q+\lambda)b_2^u \qquad (ii)''' \end{cases}$$

· Let us represent graphically, for $p = \frac{1}{2}, q = \frac{1}{3}$ and $\lambda = \frac{1}{3}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV(\cdot)$.

For $p = \frac{1}{2}, q = \frac{1}{3}$ and $\lambda = \frac{1}{3}$, the conditions (inequalities) written above become [22]

$$(i)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} > \frac{1}{3} \end{cases} \Longrightarrow c_1^* = 0, c_2^* = 0$$

$$(i)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{3} \end{cases} \Longrightarrow c_1^* = \frac{b_1}{b_2^f} + 3, c_2^* = 0$$

$$(ii)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} > 6\frac{b_1}{b_2^f} + 3 \end{cases} \Longrightarrow c_1^* = c_2^* = 0$$

$$(ii)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{1}{6}\frac{b_2^u}{b_2^f} < \frac{b_1}{b_2^f} + \frac{1}{2} < -\frac{1}{2}\frac{b_2^u}{b_2^f} \end{cases} \Longrightarrow c_1^* = c_2^* = -\frac{1}{2} - 3\frac{b_1}{b_2^u} - \frac{3}{2}\frac{b_2^f}{b_2^u}$$

$$(ii)''' \quad -\frac{b_2^u}{b_2^f} < -2\left(-\frac{b_1}{b_2^f}\right) + 1 \Longrightarrow c_1^* = c_2^* = 1$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in *Figure* 4.3 in the Appendix.

· Let us represent graphically, for $p = \frac{1}{2}, q = \frac{1}{3}$ and $\lambda = \frac{1}{4}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV(\cdot)$.

For $p = \frac{1}{2}, q = \frac{1}{3}$ and $\lambda = \frac{1}{4}$, the conditions (inequalities) written above

[22] For which concerns case $(i)'''$, it has not been considered since for $q \geq \lambda$, the condition $b_1 > (1-p)(\lambda - q)b_2^f$ is never verified. In other words, for $q \geq \lambda$, the condition $b_1 > (1-p)(\lambda - q)b_2^f$ holds for every $b_1, b_2^u < 0, b_2^f > 0$ and for every $p \in (0,1)$.

become

$$(i)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} > \frac{7}{24} \end{cases} \Longrightarrow c_1^* = 0, c_2^* = 0$$

$$(i)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ \frac{1}{24} < -\frac{b_1}{b_2^f} < \frac{7}{24} \end{cases} \Longrightarrow c_1^* = 4\frac{b_1}{b_2^f} + \frac{7}{3}, c_2^* = 0$$

$$(i)''' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{24} \end{cases} \Longrightarrow c_1^* = 1, c_2^* = 0$$

$$(ii)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} > \frac{24}{5}\frac{b_1}{b_2^f} + \frac{6}{5} \end{cases} \Longrightarrow c_1^* = c_2^* = 0$$

$$(ii)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{5}{24}\frac{b_2^u}{b_2^f} < \frac{b_1}{b_2^f} + \frac{1}{2} < -\frac{11}{24}\frac{b_2^u}{b_2^f} \end{cases} \Longrightarrow c_1^* = c_2^* = -\frac{5}{6} - 4\frac{b_1}{b_2^u} - 2\frac{b_2^f}{b_2^u}$$

$$(ii)''' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ \frac{b_2^u}{b_2^f} < \frac{24}{11}\frac{b_1}{b_2^f} + \frac{12}{11} \end{cases} \Longrightarrow c_1^* = c_2^* = 1$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in $Figure$ 4.4 in the Appendix.

· Let us represent graphically, for $p = \frac{1}{2}, q = \frac{1}{2}$ and $\lambda = \frac{1}{4}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_{compl}(\cdot)$.

For $p = \frac{1}{2}, q = \frac{1}{2}$ and $\lambda = \frac{1}{4}$, the conditions (inequalities) written above become

$$(i)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} > \frac{3}{8} \end{cases} \Longrightarrow \quad c_1^* = 0, c_2^* = 0$$

$$(i)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ \frac{1}{8} < -\frac{b_1}{b_2^f} < \frac{3}{8} \end{cases} \Longrightarrow \quad c_1^* = 4\frac{b_1}{b_2^f} + \frac{3}{2}, c_2^* = 0$$

$$(i)''' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{8} \end{cases} \Longrightarrow \quad c_1^* = 1, c_2^* = 0$$

$$(ii)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} > 8\frac{b_1}{b_2^f} + 4 \end{cases} \Longrightarrow \quad c_1^* = c_2^* = 0$$

$$(ii)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{1}{8}\frac{b_2^u}{b_2^f} < \frac{b_1}{b_2^f} + \frac{1}{2} < -\frac{3}{8}\frac{b_2^u}{b_2^f} \end{cases} \Longrightarrow \quad c_1^* = c_2^* = -\frac{1}{2} - 4\frac{b_1}{b_2^u} - 2\frac{b_2^f}{b_2^u}$$

$$(ii)''' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ \frac{b_2^u}{b_2^f} < \frac{8}{3}\frac{b_1}{b_2^f} + \frac{4}{3} \end{cases} \Longrightarrow \quad c_1^* = c_2^* = 1$$

This conditions, together with the optimal values of c_1 and c_2 are represented graphically in $Figure$ 4.5 in the Appendix.

2.3.2 CASE D: Endogenous Information only

We choose to analyze the case $\lambda = 1$, since it serves better the aim of this subsection: comparing our results with those common in environmental option values literature, in which the most representative models of environmental decisions under uncertainty under uncertainty and irreversibility are those of Freeman (1984) and of Fisher and Hanemann (1987): in these models, information is completely endogenous ($q = 0$) and it arrives with certainty ($\lambda = 1$).

Let us solve again DM's utility maximization problem.

Reasoning as in Case B, given that $(c_2^u)^* = 0$ and that $(c_2^f)^* = c_1$,

$$\begin{aligned} EV_{endo}(c_1, (c_2^u)^*, (c_2^f)^*) &= (1 - c_1)\left[(pb_1 + (1-p)(b_1 + b_2^f)\right] c_1 + \\ &\quad + c_1\left[p(b_1 c_1 + b_2^u c_2) + (1-p)(b_1 c_1 + b_2^f c_2)\right] \\ &= (1 - c_1)\left[b_1 + (1-p)b_2^f\right] c_1 + c_1\left[b_1 c_1 + (pb_2^u + (1-p)b_2^f)c_2\right] \end{aligned}$$

Let us now analyze the "low part" of the compound lottery represented in $Figure$ 3.3, after having replaced q with $(1 - c_1)$ (value of π in the case we are analyzing) [23].

We use the results found in the previous case. We refer again to *Figure* 3.4, *Figure* 3.5 and *Figure* 3.6, by changing the probability assigned to the branches of the trees (with $(1-c_1)$ in place of q):

(i) $pb_2^u + (1-p)b_2^f < 0 \implies c_2{}^* = 0 \implies$ The expected value of the lottery (given that in *Step* (c) the DM follows an optimal choice strategy independently from the information he has received) is

$$\begin{aligned} EV_{endo}(c_1, (c_2^u)^*, (c_2^f)^*, c_2{}^* &= 0) = (1-c_1)\left[b_1 + (1-p)b_2^f\right] c_1 + b_1(c_1)^2 \\ &= \left[b_1 + (1-c_1)(1-p)b_2^f\right] c_1 \end{aligned}$$

By the First Order Condition, we obtain

$$\begin{aligned} \frac{dEV_{endo}(c_1)}{dc_1} &= b_1 + (1-p)b_2^f - c_1 = 0 \\ &\implies \quad c_1{}^* = \frac{b_1 + (1-p)b_2^f}{2(1-p)b_2^f} \end{aligned}$$

Since the quantity $\frac{b_1+(1-p)b_2^f}{2(1-p)b_2^f}$ is always less than 1 (for every $b_1 < 0$ and $b_2^f > 0$) , it will never be $c_1{}^* = 1$ in the subcase (i). Hence, the optimal level of conservation in the first period is

$$c_1{}^* = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \quad (i)' \\ \frac{b_1+(1-p)b_2^f}{2(1-p)b_2^f} & \text{if} \quad b_1 + (1-p)b_2^f > 0 \quad (i)'' \end{cases}$$

The optimal expected benefit function $EV_{endo}^*(\cdot)$ is

$$EV_{endo}^*(b_1, b_2^u, b_2^f) = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \quad (i)' \\ \frac{\left[b_1+(1-p)b_2^f\right]^2}{4(1-p)b_2^f} & \text{if} \quad b_1 + (1-p)b_2^f > 0 \quad (i)'' \end{cases}$$

(ii) $pb_2^u + (1-p)b_2^f > 0 \implies c_2{}^* = c_1 \implies$ The expected value of the lottery (given that in *Step* (c) the DM follows an optimal choice strategy independently from the information he has received) is

$$EV_{endo}(c_1, (c_2^u)^*, (c_2^f)^*, c_2{}^* = c_1) = pb_1 + pb_2^u(c_1)^2 + (1-p)b_2^f c_1$$

By the First Order Condition, we obtain

$$\begin{aligned} \frac{dEV_{endo}(c_1)}{dc_1} &= b_1 + 2pb_2^u c_1 + (1-p)b_2^f = 0 \\ &\implies \quad c_1{}^* = -\frac{b_1 + (1-p)b_2^f}{2pb_2^u} \end{aligned}$$

[23] Notice that for $c_1 = 1$ (preservation of the whole environmental resource) Case C reduces to Case A.

Hence, the optimal level of conservation in the first period is

$$c_1{}^* = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \quad (ii)' \\ -\frac{b_1+(1-p)b_2^f}{2pb_2^u} & \text{if} \quad 0 < b_1 + (1-p)b_2^f < -2pb_2^u \quad (ii)'' \\ 1 & \text{if} \quad b_1 + (1-p)b_2^f > -2pb_2^u \quad (ii)''' \end{cases}$$

The optimal expected benefit function $EV_{endo}^*(\cdot)$ is

$$EV_{endo}^*(b_1, b_2^u, b_2^f) = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \quad (ii)' \\ pb_1 + \frac{[b_1]^2 - \left[(1-p)b_2^f\right]^2}{4pb_2^u} & \text{if} \quad 0 < b_1 + (1-p)b_2^f < -2pb_2^u \quad (ii)'' \\ pb_1 + pb_2^u + (1-p)b_2^f & \text{if} \quad b_1 + (1-p)b_2^f > -2pb_2^u \quad (ii)''' \end{cases}$$

· Let us represent graphically, for $p = \frac{1}{2}$ and $q = \frac{1}{3}$, the results we have obtained on the optimal level of c_1 and c_2 and on the behavior of the expected benefit function $EV_{endo}(\cdot)$.

For $p = \frac{1}{2}$and $q = \frac{1}{3}$, the conditions (inequalities) written above become [24]

$$(i)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} > \frac{1}{2} \end{cases} \Longrightarrow \quad c_1^* = 0, c_2^* = 0$$

$$(i)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{2} \end{cases} \Longrightarrow \quad c_1^* = \frac{b_1}{b_2^f} + \frac{1}{2}, c_2^* = 0$$

$$(ii)' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ -\frac{b_2^u}{b_2^f} > \frac{1}{2} \end{cases} \Longrightarrow \quad c_1^* = c_2^* = 0$$

$$(ii)'' \quad \begin{cases} -\frac{b_2^u}{b_2^f} < 1 \\ 0 < \frac{b_1}{b_2^f} + \frac{1}{2} < -\frac{b_2^u}{b_2^f} \end{cases} \Longrightarrow \quad c_1^* = c_2^* = -\frac{b_1 + \frac{1}{2}b_2^f}{2pb_2^u}$$

$$(ii)''' \quad -\frac{b_2^u}{b_2^f} \quad < \quad \frac{1}{2} - \left(-\frac{b_1}{b_2^f}\right) \Longrightarrow c_1^* = c_2^* = 1$$

This conditions, together with the optimal values of c_1 and c_2 and with EV_{endo} are represented graphically in $Figure$ 5.1 in the Appendix.

[24]For which concerns case $(ii)'''$, since the condition $b_1 + (1-p)b_2^f > -2pb_2^u$ implies the condition $b_1 + (1-p)b_2^f > 0$, only the first one has to be considered.

2.4 The calculus of the Testing Value

According to the definition we introduced in Section 1.2, we define the Testing value as

$$TV = EV^* - EV^*_{exo}$$

Hence,

$$EV^* - EV^*_0 = WV + TV$$

We could calculate the testing value not as an *additional* value of endogenous to exogenous information (as we did above), but as a value emerging in the particular information context in which *only* exogenous information is available, i.e.

$$TV' = EV^*_{endo} - EV^*_0$$

In case the utility function would be linear (and in our case it is), one could verify that $TV = TV'$.

In this paper, we calculate

$$TV' = EV^*_{endo|\lambda=1} - EV^*_0$$

only to the scope of replicating the usual way of calculating the Testing value one can find in the few models (in environmental literature) in which it appears (or it seems to appear), to have a base of comparison between our results and those common in literature, that have been exposed in Section 1.2.

Now,let's turn to the first definition of TV, the one we are agree with. Since ${EV_{endo}}^*$ and ${EV_0}^*$ vary according to the values of b_1, b_2^u and b_2^f, we have to calculate the difference between the two optimal expected values by looking at the different regions we identify in the cartesian plane $\left(-\frac{b_1}{b_2^f}, -\frac{b_2^u}{b_2^f}\right)$; moreover, we calculate the TV for different values of λ and q , for to reasons:

- to show that our results hold independently from the values one can assign to the probabilities λ and q; [25]

- to analyze the behavior of the TV as a function of λ and q.

We do it by comparing *Figure* 4.3, 4.4 and 4.5 with *Figure* 2.2.

Let us first look at *Figure* 4.3 and calculate the $TV(b_1, b_2^u, b_2^f)$ in case $p = \frac{1}{2}, q = \frac{1}{3}$ and $\lambda = \frac{1}{3}$:

For $-\frac{b_2^u}{b_2^f} > 1$ and $-\frac{b_1}{b_2^f} > \frac{1}{3}$, $\qquad TV = 0.$

For $-\frac{b_2^u}{b_2^f} > 1$ and $\frac{1}{6} < -\frac{b_1}{b_2^f} < \frac{1}{3}$, $\qquad TV = \frac{3}{2}\frac{(b_1 + \frac{1}{3}b_2^f)^2}{b_2^f} > 0.$

[25] We will show also that our results hold for every $p \in [0,1]$; but, since this is immediate (you have only to olok at the formulas we write down and you will find it evident), we don't need to do any comparative statics analises concentrated on p.

For $-\frac{b_2^u}{b_2^f} > 1$ and $0 < -\frac{b_1}{b_2^f} < \frac{1}{6}$, $\qquad TV = \frac{3}{2}\frac{(b_1)^2}{b_2^f} > 0.$

For $-\frac{b_2^u}{b_2^f} < 1$ and $-\frac{b_2^u}{b_2^f} > 6\frac{b_1}{b_2^f} + 3$, $\qquad TV = 0.$

For $-\frac{b_2^u}{b_2^f} < 1, -\frac{b_2^u}{b_2^f} < 6\frac{b_1}{b_2^f} + 3$ and $-\frac{b_2^u}{b_2^f} > 3\frac{b_1}{b_2^f} + \frac{3}{2}$, $\qquad TV = -\frac{3\left(b_1 + \frac{1}{2}b_2^f + \frac{1}{6}b_2^u\right)^2}{2b_2^u} > 0.$

For $-\frac{b_2^u}{b_2^f} < 1, -\frac{b_2^u}{b_2^f} < 3\frac{b_1}{b_2^f} + \frac{3}{2}$ and $-\frac{b_2^u}{b_2^f} > 2\frac{b_1}{b_2^f} + 1$, $\qquad TV = -\frac{3}{2}\frac{\left(b_1 + \frac{1}{2}b_2^f - \frac{1}{2}b_2^u\right)^2}{b_2^u} > 0.$

For $-\frac{b_2^u}{b_2^f} < 2\frac{b_1}{b_2^f} + 1$, $\qquad TV = 0.$

Let us first look at *Figure* 4.4, in drawn $p = \frac{1}{2}$ and $\lambda = \frac{1}{4} < q = \frac{1}{3}$: the only "relevant" difference with the case in which $\lambda \geq q$ is on the north-west of the cartesian plane, where we can find a region s.t. even with endogenous information (additional to the exogenous one) DM preserves everything in the first period ($c_1^* = 1$) and nothing in the second ($c_2^* = 0$).

Let us calculate the Testing value for values of benefits belonging to this region, defined by the inequalities

$$\begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{24} \end{cases}$$

$$TV = (1-p)\lambda b_2^f = \frac{1}{8}b_2^f > 0$$

3 Conclusions

When a decision maker ignores the possibility of acquiring exogenously further information about the consequences of an irreversible development action, he inevitably underestimates the net advantage of preservation over development and prejudices the decision somewhat in favor of immediate development (even though this does not mean that immediate development is necessarily the wrong decision). The quasi-option value (QOV) takes into account the reduction in the value of expected net benefits of development of a particular area (with respect to the case in which the decision maker knows these net benefits with certainty). This reduction occurs if information about the future consequences of development is not available.

Starting from the analysis of the meaning of the QOV, we have defined a more general *Waiting Value* (WV) as the value attached to the increase in expected utility [26] (of preservation and development net benefits) due to the possibility of acquiring new information exogenously. We have shown the Waiting Value is always non-negative, thus forcing the DM towards a higher level of preservation of the environmental area during the first and the second period of choice.

Miller and Lad (1983), Freeman (1984), Hanemann and Fisher (1987) and again Fisher (2000) has shown that if the information about the consequences of an irreversible development action can be obtained only by undertaking development, this strengthens the case for some development. By generalizing their analyses, we have introduced throughout the paper a $Testing\ Value$ (TV),defined as the additional value attached to endogenous information (additional with respect to the information exogenously arriving); in other words, it is the gain the decision maker obtains when he can receive information regarding future benefits, by developing in the current period (with respect to the case in which he ignores the possibility of receiving information through this way). The need for a testing value emerges in all those situations in which uncertainty concerning the future economic benefit of development (and its future environmental costs) generates itself information.

We have shown the testing value pushes DM in the same direction of the waiting value (i.e. towards a higher level of preservation of environmental resources). Moreover, in many cases the testing value pushes DM towards preservation of environmental resources more than waiting value (alone) does. With regard to the level of preservation in the "exogenous and endogenous" information scenario, we find that, with respect to the case in which only exogenous information is available, in many cases (depending on the values of b_1, b_2^u and b_2^f), c_1^* and c_2^* are higher (see $Figure$ 4.3, 4.4 and 4.5 as compared to $Figure$ 2.2). This means that in all these cases, accounting for the testing value pushes DM towards a higher level of preservation of the environmental resource. The reason is that the testing value could lead DM to develop only a certain amount of the environmental asset (*internal solutions*); on the contrary, the waiting (or quasi-option) value would lead more frequently to *corner solutions*.

[26] In our case, increase in *expected value* of benefits, since the individual utility function is linear.supposed to be linear (*risk neutral decision maker*).

Appendix

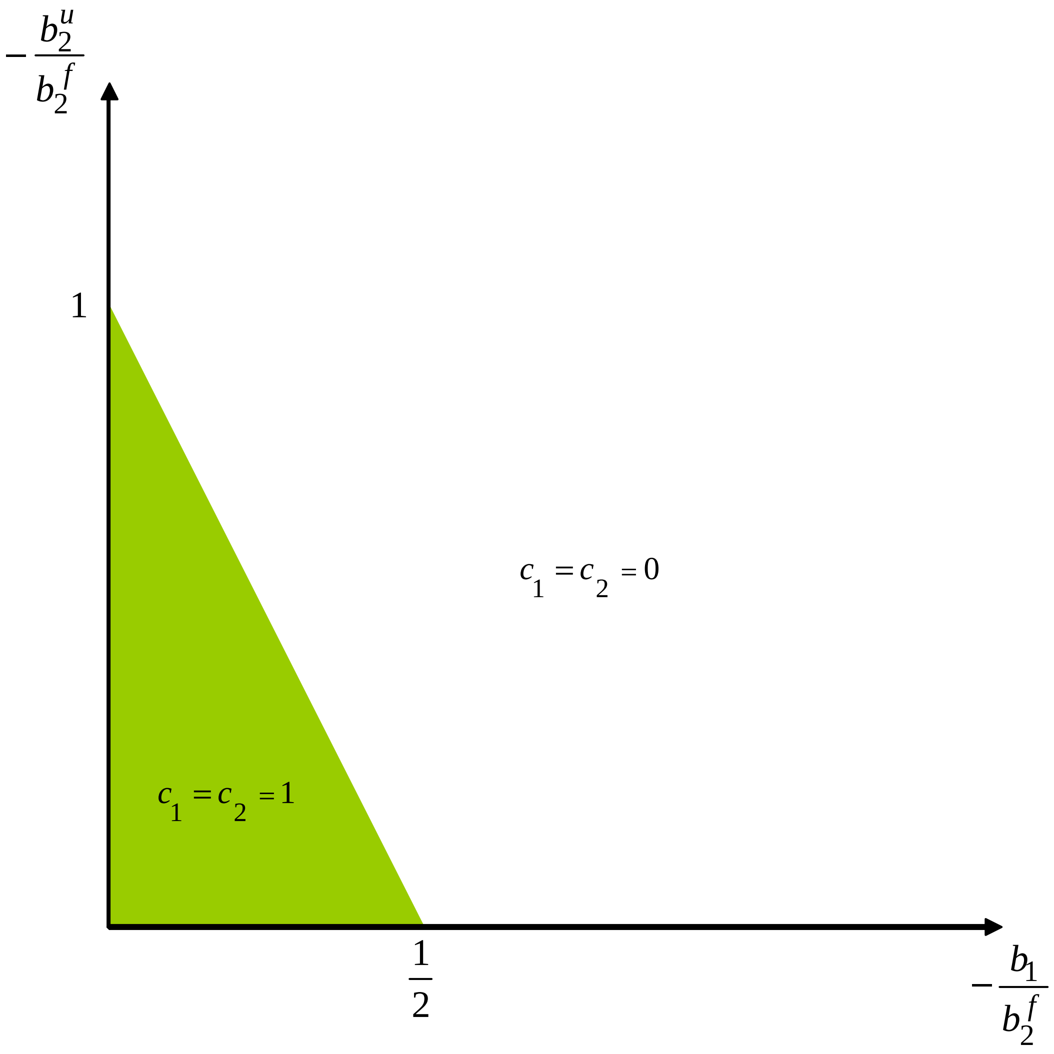


Figure 2.2

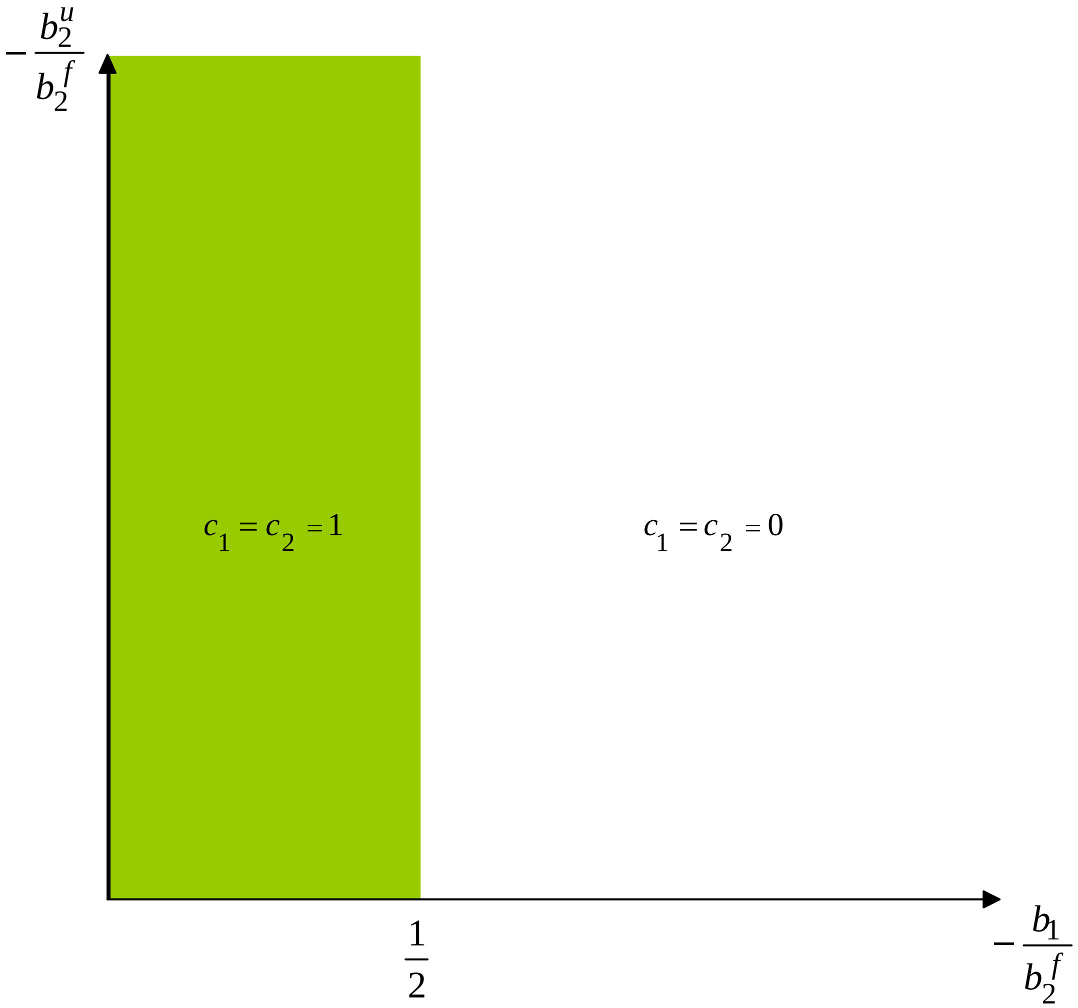


Figure 3.2

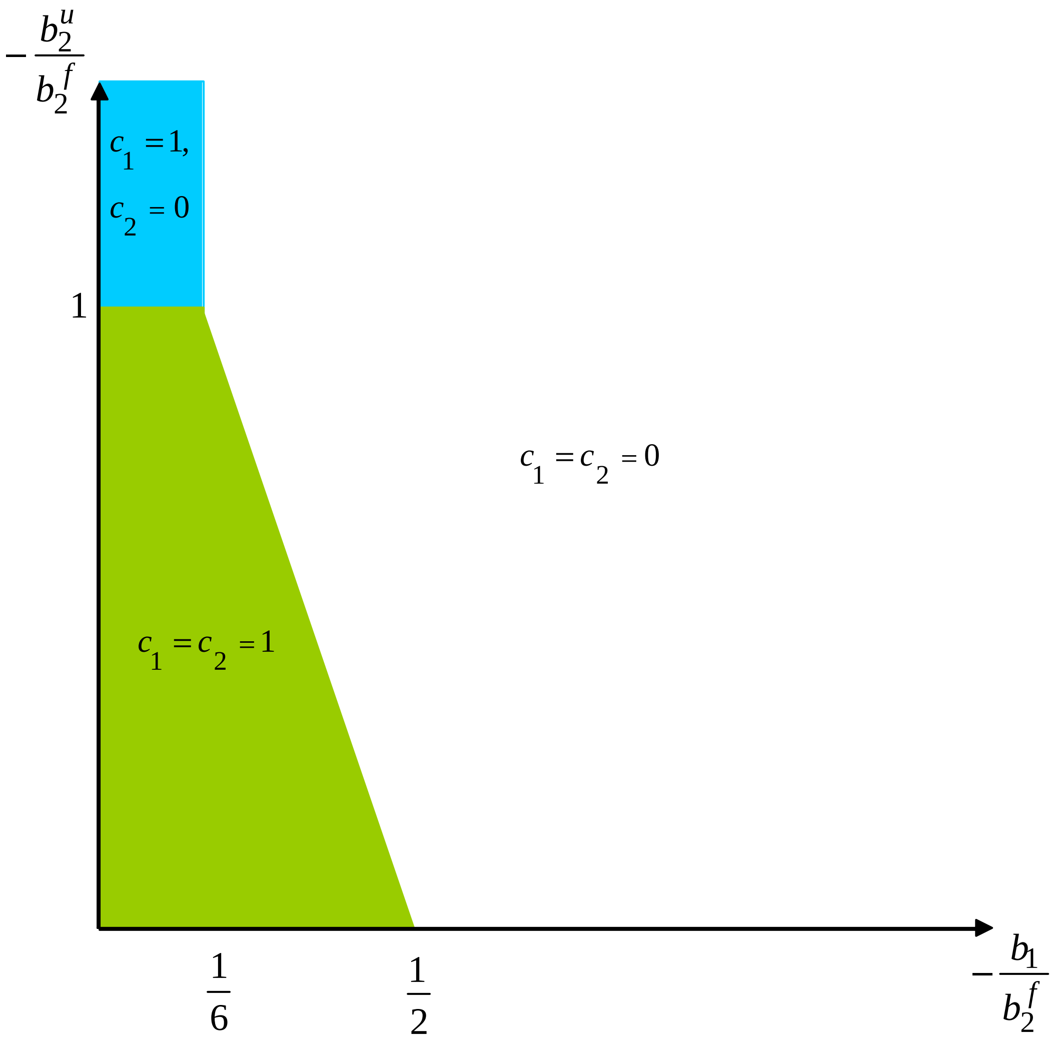
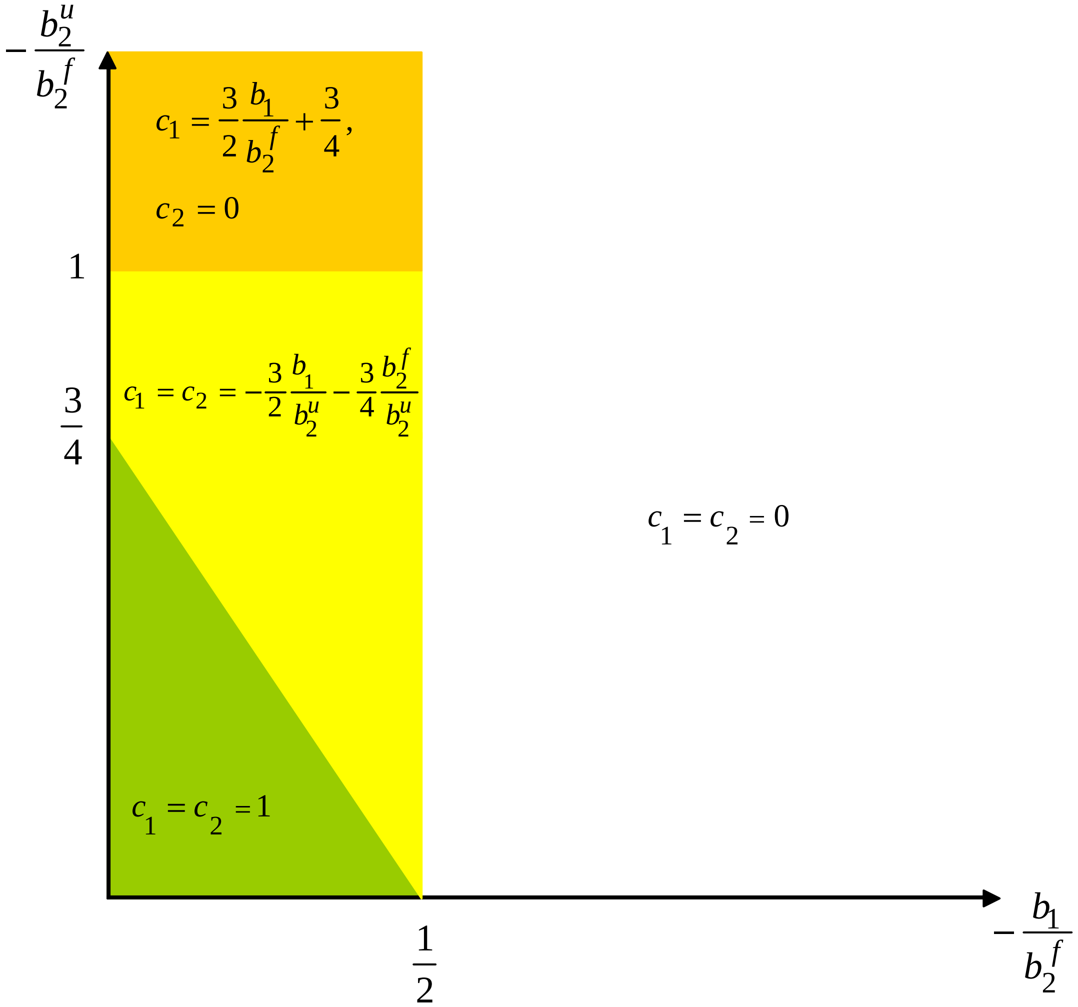


Figure 3.7



$-\frac{b_2^u}{b_2^f}$
$c_1 = \frac{3}{2}\frac{b_1}{b_2^f} + \frac{3}{4},$
$c_2 = 0$
1
$c_1 = c_2 = -\frac{3}{2}\frac{b_1}{b_2^u} - \frac{3}{4}\frac{b_2^f}{b_2^u}$
$\frac{3}{4}$
$c_1 = c_2 = 0$
$c_1 = c_2 = 1$
$\frac{1}{2}$
$-\frac{b_1}{b_2^f}$

Figure 4.1

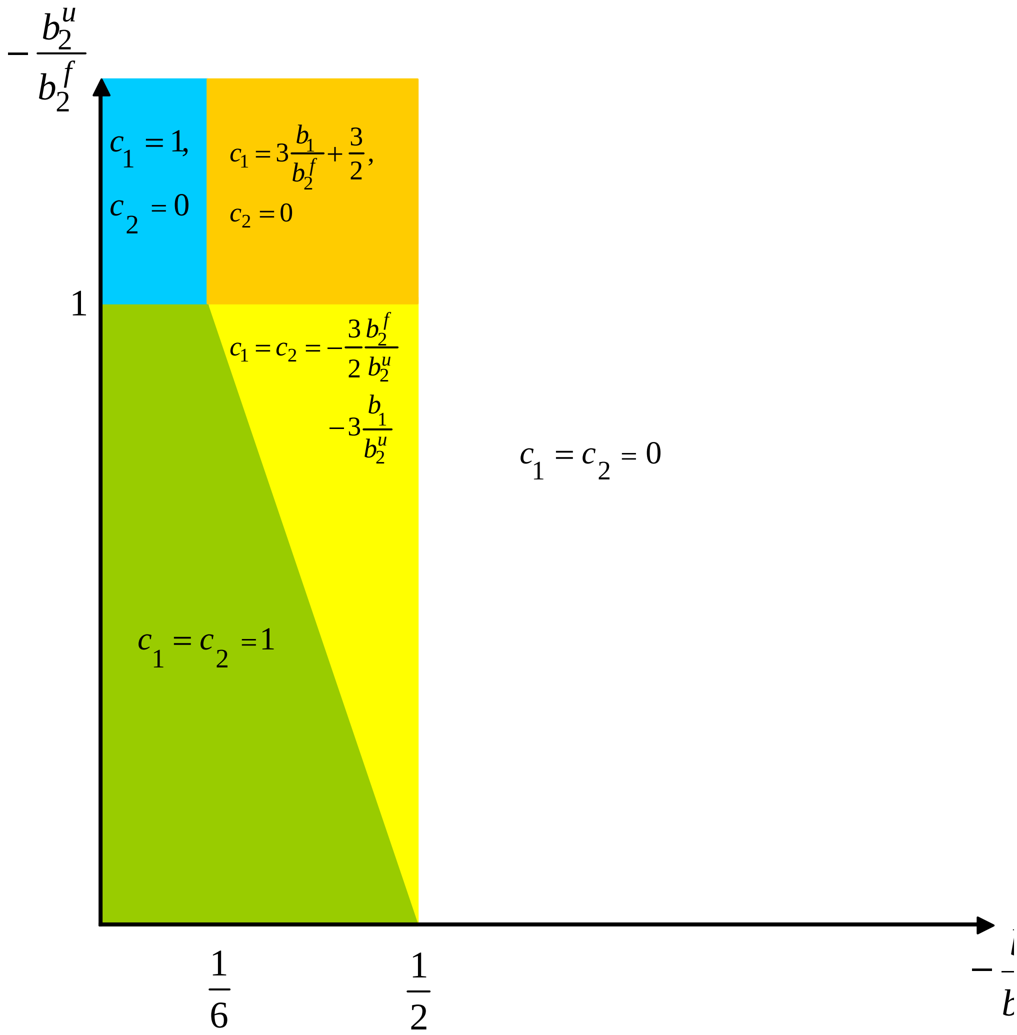
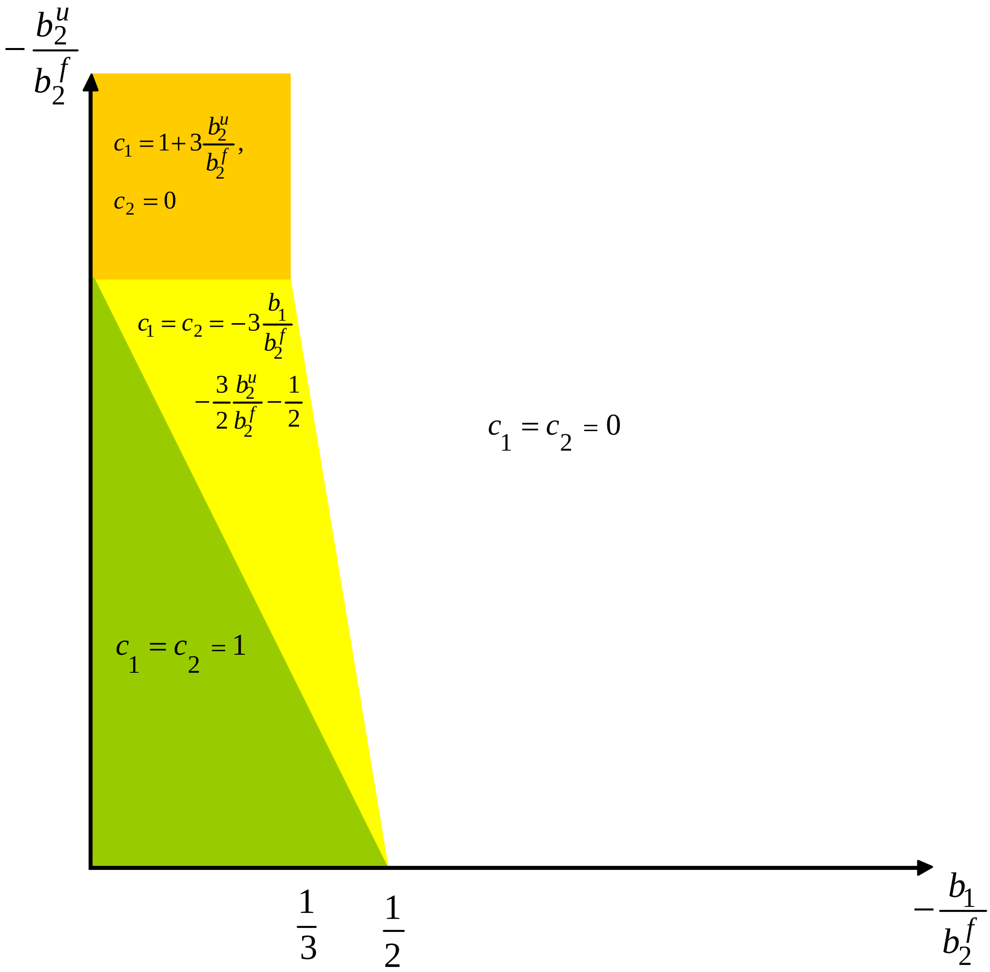
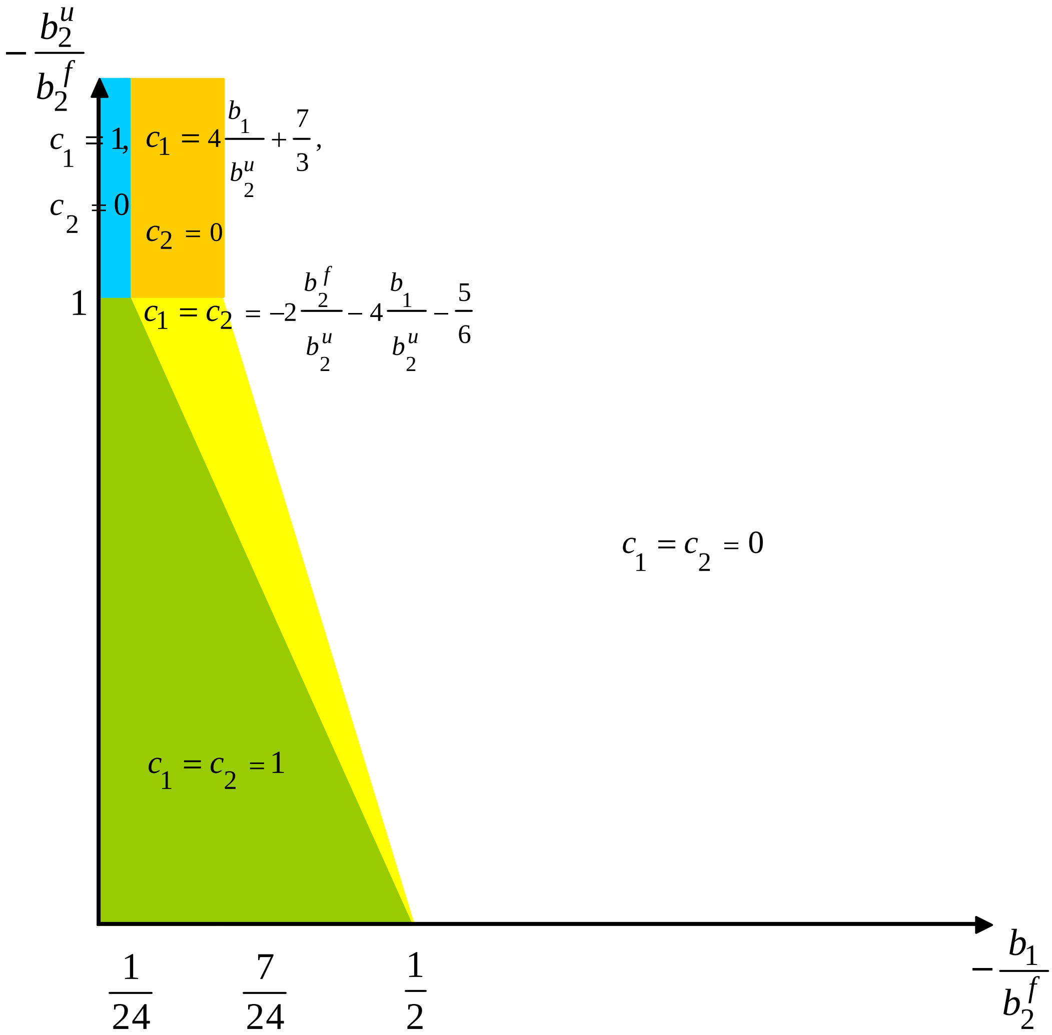

$-\frac{b_2^u}{b_2^f}$
$c_1 = 1,$
$c_2 = 0$
$c_1 = 3\frac{b_1}{b_2^f} + \frac{3}{2},$
$c_2 = 0$
1
$c_1 = c_2 = -\frac{3}{2}\frac{b_2^f}{b_2^u}$
$-3\frac{b_1}{b_2^u}$
$c_1 = c_2 = 0$
$c_1 = c_2 = 1$
$\frac{1}{6}$
$\frac{1}{2}$

Figure 4.2



$-\frac{b_2^u}{b_2^f}$
$c_1 = 1 + 3\frac{b_2^u}{b_2^f}$,
$c_2 = 0$
$c_1 = c_2 = -3\frac{b_1}{b_2^f}$
$-\frac{3}{2}\frac{b_2^u}{b_2^f} - \frac{1}{2}$
$c_1 = c_2 = 0$
$c_1 = c_2 = 1$
$\frac{1}{3}$
$\frac{1}{2}$
$-\frac{b_1}{b_2^f}$

Figure 4.3



$-\frac{b_2^u}{b_2^f}$
$c_1 = 1,$
$c_2 = 0$
$c_1 = 4\frac{b_1}{b_2^u} + \frac{7}{3},$
$c_2 = 0$
1
$c_1 = c_2 = -2\frac{b_2^f}{b_2^u} - 4\frac{b_1}{b_2^u} - \frac{5}{6}$
$c_1 = c_2 = 0$
$c_1 = c_2 = 1$
$\frac{1}{24}$
$\frac{7}{24}$
$\frac{1}{2}$
$-\frac{b_1}{b_2^f}$

Figure 4.4

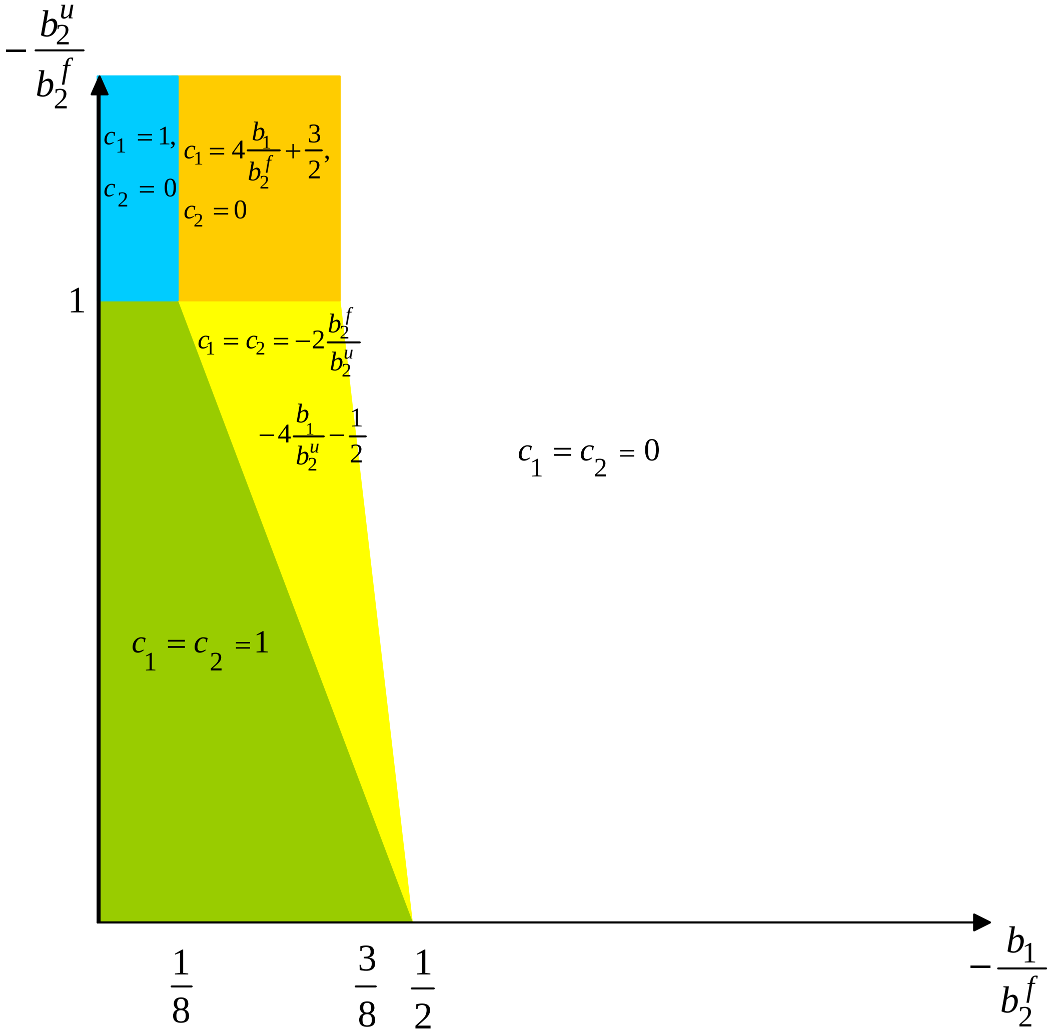
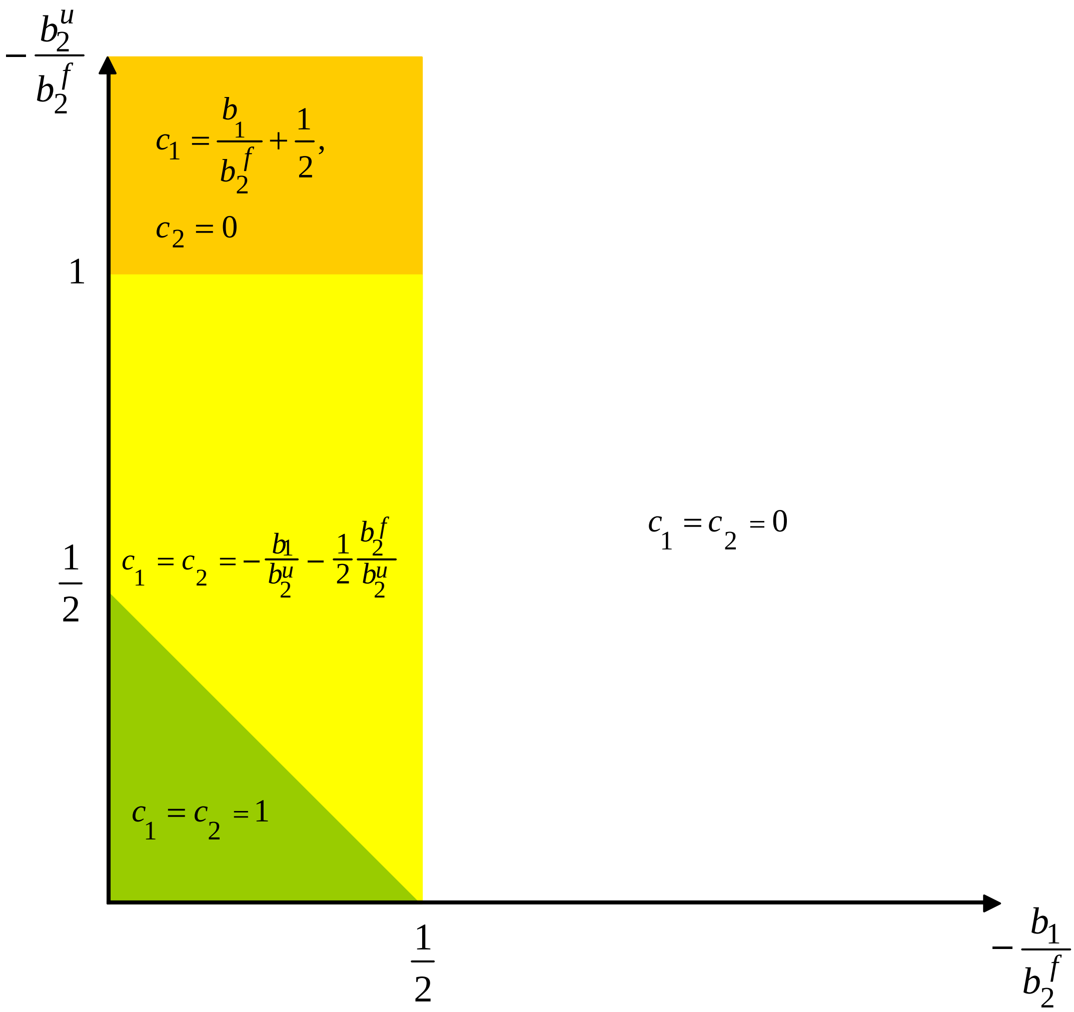

$-\frac{b_2^u}{b_2^f}$
$c_1 = 1,$
$c_2 = 0$
$c_1 = 4\frac{b_1}{b_2^f} + \frac{3}{2},$
$c_2 = 0$
1
$c_1 = c_2 = -2\frac{b_2^f}{b_2^u}$
$-4\frac{b_1}{b_2^u} - \frac{1}{2}$
$c_1 = c_2 = 0$
$c_1 = c_2 = 1$
$\frac{1}{8}$
$\frac{3}{8}$
$\frac{1}{2}$
$-\frac{b_1}{b_2^f}$

Figure 4.5



$-\frac{b_2^u}{b_2^f}$
$c_1 = \frac{b_1}{b_2^f} + \frac{1}{2}$,
$c_2 = 0$
1
$\frac{1}{2}$
$c_1 = c_2 = -\frac{b_1}{b_2^u} - \frac{1}{2}\frac{b_2^f}{b_2^u}$
$c_1 = c_2 = 0$
$c_1 = c_2 = 1$
$\frac{1}{2}$
$-\frac{b_1}{b_2^f}$

Figure 5.1

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