An Estimated New-Keynesian Business Cycle Model with Equilibrium Unemployment

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Abstract

I estimate an empirical New Keynesian business cycle model with equilibrium search unemployment by matching model and data spectra. This estimation technique allows to handle misspecification by downweighting the frequencies at which the model performs poorly. I study the implications for parameter estimates, impulse responses and the model-data fit of two alternatives weighting schemes. Downweighting very high frequencies helps recovering reasonable estimates and meaningful impulse responses to the various shocks.

Keywords: Inflation, Unemployment, Business Cycles, DSGE models, Frequency Domain Analysis, Misspecification

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1 Introduction

Recently, Smets and Wouters (2003) and Christiano Eichenbaum and Evans (2005) (CEE) have estimated empirical New Keynesian DSGE models to evaluate the role of shocks and frictions for the business cycle. These large scale DSGE models feature enough internal propagation mechanisms to fit the data reasonably well and they have become the workhorse to conduct monetary policy analysis. A robust conclusion of these studies is that labor market frictions are crucial in accounting for the behavior of output and inflation. Yet, with the exception of Trigari (2005) and Walsh (2005b), the current vintage of empirical New Keynesian models does not include the modern theory of unemployment due to Diamond, Mortensen and Pissarides[1]. An empirically grounded labor market block is, however, crucial to uncover the sources of business cycle fluctuations and to understand the dynamics of marginal cost and inflation. Trigari and Walsh both find that labor search reduces the elasticity of marginal cost with respect to output and therefore helps to replicate the sluggish response of inflation to monetary shock without relying on an excessive degree of nominal price stickiness. However, these two authors do not adress the "volatility" and the "persistence" puzzles recently highlighted by Shimer (2005) and Yashiv (2005). Shimer shows that, conditional on productivity shocks of plausible magnitude, the standard Search and Matching model can not replicate the observed high volatility of vacancies and unemployment. He also suggests that the model's wage is too volatile and therefore annihilates most of firms' incentives to hire new workers. Yashiv (2005) establishes the importance of convex hiring cost to match the high persistence of the vacancy and unemployment rates. The first task of this paper is therefore to update the labor market block of Trigari and Walsh's models by following the recent advises of Shimer (2005) and Yashiv (2005). Then, this paper also aims at shedding new lights, using a DSGE ap-

[1] see Pissarides (2000) for a textbook treatment of the Search and Matching model.

proach, on some central empirical macroeconomic relationships that involve the unemployment rate such as the Phillips curve, the Okun's Law and the Beveridge curve.

This paper introduces investment adjustment costs, variable capital utilization, convex hiring costs and real wage rigidity in a New Keynesian business cycle model with labor search and exogenous job destruction[2]. Price stickiness is introduced by assuming that monopolistically competitive firms face quadratic price adjustment costs à la Rotemberg. One advantage of "Rotemberg pricing" over "Calvo pricing" is to avoid price dispersion and, thereby, issues related to firm-specific inputs. The model contains seven shocks. The neutral technology shock is permanent and implies that output, consumption, investment and real wage share a common stochastic trend. The remaining six shocks, investment-specific-technology, preference, cost-push, wage bargaining, exogenous spending and monetary policy are all stationary.

To handle misspecification, I estimate the model with a limited information approach that focuses on the business cycle facts and consists in matching model with data spectra. The spectrum matching estimation technique was proposed by Watson (1993), implemented by Wen (1998) and formalized and extended by Diebold, Ohanian and Berkowitz (1998).

In the model, output is demand driven because of sticky prices, and labor is a predetermined, firm-specific, input because of search and matching frictions. Hence, in the aftermath of a demand shock, the only ways for firms to adjust output and to satisfy demand on impact is by varying the degree of capacity utilization or by affording a costly jump in prices. These features make capital utilization and the rental rate of capital excessively volatile at very high frequencies[3]. The DSGE model being clearly misspec-

[2]See Trigari (2005), Walsh (2005b) and Krause and Lubik (2005) for New Keynesian models with labor search and endogenous job destruction. I abandon endogenous job destruction which delivers a counterfactual positive correlation between vacancy and unemployment.

[3]Introducing a second instantaneous, intensive margin, such as hours per worker, would

ified at high frequencies, I downweight those frequencies in the estimation and aim at maximizing the agreement between the model and the data at low and, especially, at business cycle frequencies. I study the implications for parameter estimates, impulse responses and goodness of fit of two alternatives weighting schemes. Downweighting very high frequencies drastically helps recovering reasonable estimates and meaningful impulse responses to the various shocks.

2 The Model

The model combines the empirical New Keynesian model of Smets and Wouters (2003) with the Search and Matching model of equilibrium unemployment due to Diamond, Mortensen and Pissarides. As in Krause and Lubik (2005), I introduce nominal frictions through Rotemberg's quadratic price adjustments cost rather than through Calvo pricing. Hence, the same firms face both nominal rigidities and search frictions. The economy consists of a representative family, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by $i \in [0, 1]$, and a central bank.

Fluctuations are driven by seven aggregate shocks: a neutral and an investment-specific technology shocks, a monetary policy shock, a preference shock, a markup shock, an exogenous spending shock and a bargaining power shock. The neutral technology shock follows a random walk with drift whereas the other six shocks are all stationary.

Trigari (2005), Walsh (2005) and Krause and Lubik (2005) propose New Keynesian business cycle models with labor search. Beyond the larger set of shocks, my model differs from the existing literature along the following dimensions. First, it features capital accumulation with investment adjustment costs and variable capital utilization as in CEE (2005). This combination

mitigate this problem.

of predetermined extensive margins, i.e. employment and physical capital, with an instantaneous, intensive margin, i.e. capital utilization, is crucial for the definition of real marginal cost and thereby for inflation dynamics. Second, the present model is characterized by a stochastic balanced growth path along which consumption, investment, capital and real wage grow at the same rate whereas unemployment remains stationary. In the empirical analysis, detrending of actual data is model consistent. Third, the labor block includes both convex hiring costs and real wage rigidity as prescribed by Yashiv (2005) and Shimer (2005) respectively.

2.1 The representative family

There is a continuum of identical households of mass one. Each household is a large family, itself made of a continuum of individuals of measure one. Family members are either working or searching for a job[4]. Following Merz (1995) and Andolfatto (1996), I assume that family members pool their income together before the head of the family chooses optimally per capita consumption.

The representative family enters each period $t = 0, 1, 2, ...,$ with B_{t-1} bonds and $\overline{K}_{t-1}$ units of physical capital. At the beginning of each period, bonds mature, providing B_{t-1} units of money. The representative family uses some of this money to purchase B_t new bonds at nominal cost B_t/r_t^B, where r_t^B denotes the gross nominal interest rate between period t and $t+1$.

The representative household owns capital and choose the capital utilization rate, u_t, which transforms physical capital into effective capital according to

$$K_t = u_t \overline{K}_{t-1}.$$

The household rents $K_t(i)$ units of effective capital to intermediate-goods-

[4]The model abstracts from the labor force participation decision. The head of the family forces unemployed family members to search for a job.

producing firm $i \in [0,1]$ at the nominal rate r_t^K. The household's choice of $K_t(i)$ must satisfy

$$K_t = \int_0^1 K_t(i)\, di.$$

The costs of capital utilization is $a(u_t)$ per unit of physical capital. I assume the following functional form for the function a,

$$a(u_t) = \phi_{u1} \times (u_t - 1) + \frac{\phi_{u2}}{2} \times (u_t - 1)^2,$$

and that $u_t = 1$ in steady-state.

Each period, $N_t(i)$ family members are employed at intermediate-goods-producing firm $i \in [0,1]$. Each worker employed at firm $i \in [0,1]$ works a fixed amount of hours and earns nominal wage $W_t(i)$. N_t denotes aggregate employment in period t and is given by

$$N_t = \int_0^1 N_t(i)\, di.$$

The remaining $(1 - N_t)$ family members are searching for a job in period t. Each unemployed member receives nominal unemployment benefits b_t, financed through lump-sum taxes.

During period t, the representative household receives total nominal factor payments $r_t^K K_t + W_t N_t + (1 - N_t) b_t$. In addition, the household also receives nominal profits $D_t(i)$ from each firm $i \in [0,1]$, for a total of

$$D_t = \int_0^1 D_t(i)\, di.$$

In each period $t = 0, 1, 2, ...$,the family uses these resources to purchase finished goods, for both consumption and investment purposes, from the representative finished goods-producing firm at the nominal price P_t. The law of

motion of physical capital is

$$\overline{K}_t \leq (1-\delta)\,\overline{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{1}$$

where δ denotes the depreciation rate. The function S captures the presence of adjustment costs in investment, as in CEE (2005). I assume the following functional form for the function S,

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2$$

where g_I is the steady state growth rate of investment. Hence, along the balanced growth path, $S(g_I) = S'(g_I) = 0$ and $S''(g_I) = \phi_I > 0$. μ_t is an investment-specific technology shock affecting the efficiency with which consumption goods are transformed into capital. The investment-specific shock follows the exogenous stationary autoregressive process

$$\ln(\mu_t) = \rho_\mu \ln\left(\mu_{t-1}\right) + \varepsilon_{\mu t}$$

where $\varepsilon_{\mu t}$ is $i.i.d.N\left(0, \sigma_\mu^2\right)$.

The family's budget constraint is given by

$$\begin{aligned} P_t C_t + P_t I_t + B_t / r_t^B \quad \leq \quad & B_{t-1} + W_t N_t + (1-N_t)\, b_t + r_t^K u_t \overline{K}_{t-1} \\ & -P_t a(u_t)\,\overline{K}_{t-1} - T_t + D_t \end{aligned} \tag{2}$$

for all $t = 0, 1, 2, ...$

The family's lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s a_{t+s} \ln\left(C_{t+s} - hC_{t+s-1}\right) \tag{3}$$

where $0 < \beta < 1$. The preference shock a_t follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at} \tag{4}$$

where $0 < \rho_a < 1$, and ε_{at} is $i.i.d.N(0, \sigma_a^2)$. When $h > 0$, the model allows for habit formation in consumption and consumption responds gradually to shocks.

The head of the family chooses C_t, B_t, u_t, I_t, and $\overline{K}_t$ for each $t = 0, 1, 2, ...$ to maximize the expected lifetime utility (3) subject to the constraints (1) and (2).

The first order conditions for this problem are

- C_t :

$$\Lambda_t = \frac{a_t}{C_t - hC_{t-1}} - \beta h E_t \left(\frac{a_{t+1}}{C_{t+1} - hC_t} \right) \tag{5}$$

  where Λ_t denotes the multiplier on (2) and can be interpreted as the utility to the household of an additional unit of wealth at date t.

- B_t :

$$\Lambda_t = \beta r_t^B E_t \left(\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right) \tag{6}$$

- u_t :

$$(\phi_{u1} - \phi_{u2}) + \phi_{u2} u_t = \widetilde{r}_t^K \tag{7}$$

  where $\widetilde{r}_t^K = \frac{r_t^K}{P_t}$ denotes the real rental rate of capital.

- I_t :

$$\begin{aligned} 1 &= \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - g_I \right) \left(\frac{I_t}{I_{t-1}} \right) \right] \\ &\quad + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left(\frac{I_{t+1}}{I_t} - g_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned} \tag{8}$$

  where Υ_t denotes the multiplier on (1) and can be interpreted as the

utility to the household of an additional unit of physical capital at date t. $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$ is the marginal Tobin's Q.

- $\overline{K}_t$:

$$\upsilon_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-\delta)\, \upsilon_{t+1} + \widetilde{r}^K_{t+1} u_{t+1} - a\left(u_{t+1}\right) \right] \right\} \tag{9}$$

2.2 The representative finished goods-producing firm

During each period $t = 0, 1, 2, ...$, the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0,1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t}\, di \right]^{\theta_t/(\theta_t - 1)} \geq Y_t, \tag{10}$$

where θ_t translates into a random shock to the markup of price over marginal cost. This markup shock follows the autoregressive process

$$\ln(\theta_t) = (1-\rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta t}, \tag{11}$$

where $0 < \rho_\theta < 1$, $\theta > 1$, and $\varepsilon_{\theta t}$ is $i.i.d.N\left(0, \sigma^2_\theta\right)$.

Intermediate good i sells at the nominal price $P_t(i)$, while the finished good sells at the nominal price P_t. Given these prices, the finished goods-producing firm chooses Y_t and $Y_t(i)$ for all $i \in [0,1]$ to maximize its profits

$$P_t Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di, \tag{12}$$

subject to the constraint (10) for each $t = 0, 1, 2, ...$ The first-order conditions for this problem are (10) with equality and

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t \tag{13}$$

for all $i \in [0,1]$ and $t = 0, 1, 2, ...$

Competition in the market for the finished good drives the finished goods-producing firm's profits to zero in equilibrium. This zero-profit condition determines P_t as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t} di \right]^{1/(1-\theta_t)}$$

for all $t = 0, 1, 2, ...$

2.3 The representative intermediate goods-producing firm

Each intermediate goods-producing firm $i \in [0,1]$ enters in period t with a stock of $N_{t-1}(i)$ employees carried from the previous period. At the beginning of period t, before production starts, $\rho N_{t-1}(i)$ olds jobs are destroyed, where ρ is the job destruction rate[5]. $N_t(i)$ denotes the pool of employees taking part to production at firm i in period t. The law of motion of the stock of productive workers at firm (i) is

$$N_t(i) = (1-\rho) N_{t-1}(i) + m_{t-1}(i).$$

$m_t(i)$ denotes the flow of new employees hired by firm i in period t, and is given by

$$m_t(i) = q_t V_t(i)$$

where $V_t(i)$ denotes vacancies posted by firm i in period t and q_t is the aggregate probability of filling a vacancy in period t. Notice that workers hired in period t do not take part to period t production. Search frictions in the labor market render the matching process time consuming. As a consequence, a firm-worker pair enjoys a joint surplus that motivates the

[5] The rate of match dissolution is exogenous. This is consistent with Hall (2005) and Shimer (2005)'s finding that recent business cycle fluctuations in the U.S. labor market mostly come from the job creation margin.

existence of a long run relationship between the two parties[6].

The pool of job seekers in period t is denoted by U_t and is given by

$$U_t = 1 - \int_0^1 N_t(i)\, di = 1 - N_t$$

Notice that the model abstracts from the labor force participation decision. The laws of motion of aggregat employment, N_t, and unemployment, u_t, are respectively

$$\begin{aligned} N_t &= (1-\rho) N_{t-1} + m_{t-1}, \\ U_t &= U_{t-1} + \rho N_{t-1} - m_{t-1}. \end{aligned}$$

The matching process is described by the following aggregate CRS function

$$m_t = \zeta U_t^{\sigma} V_t^{1-\sigma}$$

where V_t denotes aggregate vacancies in period t,

$$V_t = \int_0^1 V_t(i)\, di \tag{14}$$

and ζ is a scale parameter that captures the efficiency of the matching technology. The probability, q_t,, to fill a vacancy in period t is given by

$$q_t = \frac{m_t}{V_t}. \tag{15}$$

The probability, s_t, for an unemployed worker to find a job is

$$s_t = \frac{m_t}{U_t}. \tag{16}$$

[6]Search and matching frictions make labor a firm-specific factor of production. To avoid the difficulties arising from the combination of firm-specific inputs and Calvo pricing, I rather introduce nominal rigidities through quadratic price adjustment costs à la Rotemberg.

During each period $t = 0, 1, 2, ...$, the representative intermediate goods-producing firm combines $N_t(i)$ homogeneous employees with $K_t(i)$ units of efficient capital, rented from the households, to produce $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha}$$

A_t is an aggregate labor-augmenting technology shock whose growth rate, $z_t \equiv A_t/A_{t-1}$, follows the exogenous stochastic process

$$\ln(z_t) = (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt},$$

where $z > 1$ and ε_{zt} is $i.i.d.N(0, \sigma_z^2)$, so that the level of technology, A_t, is a unit root process.

Following Yashiv (2005), intermediate goods-producing firms face convex hiring costs, measured in terms of the finished good, and given by

$$\frac{\phi_N}{2}\left[\frac{q_t V_t(i)}{N_t(i)}\right]^2 Y_t,$$

where ϕ_N governs the magnitude of these costs[7].

Intermediate goods substitute imperfectly for one another in the production function of the representative finished goods-producing firm. Hence, each intermediate goods-producing firm $i \in [0, 1]$ sells its output $Y_t(i)$ in a monopolistically competitive market, setting $P_t(i)$, the price of its own product, with the commitment of satisfying the demand for good i at that price. Following Rotemberg (1982), each intermediate goods-producing firm faces costs of adjusting its nominal price between periods, measured in terms of the finished good and given by

$$\frac{\phi_P}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t \tag{17}$$

[7] I use a simplified version of Yashiv (2005)'s hiring cost function.

where ϕ_P governs the magnitude of the price adjustment cost and where $\pi > 1$ denotes the gross steady-state rate of inflation. The labor and price adjustment costs make the intermediate goods-producing firm's problem dynamic. It chooses $K_t(i)$, $N_t(i)$, $V_t(i)$, $Y_t(i)$ and $P_t(i)$ for all $t = 0, 1, 2, ...$to maximize its total market value, given by

$$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[\frac{D_{t+s}(i)}{P_{t+s}} \right] \tag{18}$$

where $\beta^t \Lambda_t / P_t$ measures the marginal utility to the representative household of an additional dollar of profits during period t and where

$$\begin{aligned} D_t(i) &= P_t(i) Y_t(i) - W_t(i) N_t(i) - r_t^K K_t(i) \\ &\quad - \frac{\phi_N}{2} \left(\frac{q_t V_t(i)}{N_t(i)} \right)^2 P_t Y_t - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 P_t Y_t, \end{aligned}$$

subject to the constraints

$$\begin{aligned} Y_t(i) &= \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t, \\ Y_t(i) &\leq K_t(i)^{\alpha} \left[A_t N_t(i) \right]^{1-\alpha}, \\ N_t(i) &= \chi N_{t-1}(i) + q_{t-1} V_{t-1}(i) \end{aligned}$$

where $\chi \equiv 1 - \rho$ is the job survival rate.

The first order conditions for this problem are

- $K_t(i)$:

$$\widetilde{r}_t^K K_t(i) = \xi_t(i) \left(\alpha A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha} \right) \tag{19}$$

- $N_t(i)$:

$$\begin{aligned}\frac{\Psi_t(i)}{\Lambda_t} &= \xi_t(i)\left[(1-\alpha)A_t^{1-\alpha}K_t(i)^{\alpha}N_t(i)^{-\alpha}\right]-\widetilde{W}_t(i) \\ &\quad +\phi_N\left(\frac{Y_t}{N_t(i)}\right)x_t(i)^2+\beta\chi E_t\frac{\Psi_{t+1}(i)}{\Lambda_{t+1}}\frac{\Lambda_{t+1}}{\Lambda_t}\end{aligned} \tag{20}$$

where $\widetilde{W}_t(i)=\frac{W_t(i)}{P_t}$ denotes the real wage, $\Xi_t(i)$ measures the value to firm i, expressed in utils, of an additional unit of output in period t. Hence, $\xi_t(i)=\Xi_t(i)/\Lambda_t$ represents firm i's real marginal cost in period t. $\Psi_t(i)$ measures the value to firm i, expressed in utils, of an additional job in period t, and $x_t(i)\equiv\frac{q_tV_t(i)}{N_t(i)}$ is the hiring rate. This condition tells that the costs and benefits of hiring an additional worker must be equal.

- $V_t(i)$:

$$\phi_N\left(\frac{Y_t}{N_t(i)}\right)x_t(i)=\beta E_t\frac{\Psi_{t+1}(i)}{\Lambda_{t+1}}\frac{\Lambda_{t+1}}{\Lambda_t} \tag{21}$$

- Vacancy posting condition

$$\begin{aligned}&\phi_N\left(\frac{Y_t}{N_t(i)}\right)x_t(i) \\ =\ &\beta E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left\{\begin{array}{c}\xi_{t+1}(i)\left[(1-\alpha)A_{t+1}^{1-\alpha}K_{t+1}(i)^{\alpha}N_{t+1}(i)^{-\alpha}\right] \\ -\widetilde{W}_{t+1}(i)+\phi_N\left(\frac{Y_{t+1}}{N_{t+1}(i)}\right)\left(x_{t+1}(i)^2+\chi x_{t+1}(i)\right)\end{array}\right\}\end{aligned} \tag{22}$$

- $P_t(i)$:

$$\begin{aligned}0 &= (1-\theta_t)\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t}\left(\frac{Y_t}{P_t}\right)+\theta_t\xi_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t-1}\left(\frac{Y_t}{P_t}\right) \\ &\quad -\phi_P\left(\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right)\left(\frac{Y_t}{\pi P_{t-1}(i)}\right) \\ &\quad +\beta\phi_P E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{t+1}(i)}{\pi P_t(i)}-1\right)\left(\frac{P_{t+1}(i)}{\pi P_t(i)}\right)\left(\frac{Y_{t+1}}{P_t(i)}\right)\right]\end{aligned} \tag{23}$$

2.4 Wage setting

Unemployment benefits in nominal terms, b_t, are proportional to the value of the nominal wage along the balanced growth path, $W_{ss,t}$,

$$b_t = \tau W_{ss,t},$$

where τ is the replacement ratio. The fact that hiring costs and unemployments benefits both share the common stochastic trend ensures that the unemployment rate is stationary.

Jobs and workers at a given intermediate goods-producing firm are homogeneous. $W_t(i)$ denotes the nominal wage paid for any job at firm i in period t. Following Krause and Lubik (2005), $W_t(i)$ is a convex combination of the standard Nash wage $W_t^{NB}(i)$ and a wage norm $\overline{W}_t$,

$$W_t(i) = \gamma W_t^{NB}(i) + (1-\gamma)\overline{W}_t.$$

I set the wage norm equal to the balanced growth path value of the Nash wage,

$$\overline{W}_t = W_{ss,t}^{NB}.$$

The derivation of the Nash wage, $W_t^{NB}(i)$, is described in appendix 1. The final expression for the Nash wage is

$$\begin{aligned} W_t^{NB}(i) &= \eta_t \left\{ \begin{array}{c} \left[(1-\alpha) A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{-\alpha}\right] P_t \xi_t(i) \\ + \left(\frac{\phi_N P_t Y_t}{N_t(i)}\right)\left(x_t(i)^2 + s_t x_t(i)\right) \end{array} \right\} \\ &\quad - (1-\eta_t) b_t \end{aligned}$$

where η_t denotes the worker's bargaining power at time t. η_t evolves exogenously over time according to

$$\ln \eta_t = \left(1-\rho_\eta\right)\ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t},$$

where $0 < \eta < 1$ and $\varepsilon_{\eta t}$ is $i.i.d.N\left(0, \sigma_{\eta}^{2}\right)$.

2.5 Government

The central bank adjusts the short-term nominal interest rate r_t^B following a Taylor type rule

$$\ln\left(\frac{r_t^B}{r^B}\right) = \rho_r \ln\left(\frac{r_{t-1}^B}{r^B}\right) + \rho_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \rho_y \ln\left(\frac{Y_t/A_t}{Y_{ss,t}/A_{ss,t}}\right) + \rho_\Theta \ln\left(\frac{\Theta_t}{\Theta}\right) + \varepsilon_{rt}, \tag{24}$$

where $\Theta_t \equiv \frac{V_t}{U_t}$ denotes labor market's tightness, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate and ε_{rt} is an $i.i.d.N\left(0, \sigma_r^2\right)$ monetary policy shock.

Exogenous spending is a time-varying fraction of GDP

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$

where g_t evolves exogenously over time according to

$$\ln g_t = \left(1 - \rho_g\right) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{gt}.$$

The exogenous spending shock ε_{gt} is $i.i.d.N\left(0, \sigma_g^2\right)$.

2.6 Symmetric equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $P_t(i) = P_t$, $N_t(i) = N_t$, $V_t(i) = V_t$, $K_t(i) = K_t$ for all $i \in [0,1]$ and $t = 0, 1, 2, \ldots..$ Moreover, workers are homogeneous and all workers at a given firm i receive the same nominal wage $W_t(i)$, so that in a symmetric equilibrium $W_t(i) = W_t$ for all $i \in [0,1]$ and $t = 0, 1, 2, \ldots.$ In addition, $B_t = B_{t-1} = 0$ must hold for all $t = 0, 1, 2, \ldots.$

2.7 The stationary system

Output, consumption, investment, capital and real wage share the stochastic trend induced by the unit root process of neutral technological progress. I first rewrite the model in terms of stationary variables, and then loglinearize the transformed model-economy around its steady-state. This approximate model can then be solved using the AIM solution method. The following variables are stationary and need not to be transformed: u_t, r_t^B, $\widetilde{r}_t^K$, υ_t, ξ_t, N_t, U_t, V_t, q_t, s_t, x_t, $\pi_t = \frac{P_t}{P_{t-1}}$, μ_t, a_t, z_t, θ_t, η_t, g_t. I define the transformed variables $y_t = Y_t/A_t$, $k_t = K_t/A_{t-1}$, $\overline{k}_t = \overline{K}_t/A_t$, $c_t = C_t/A_t$, $\lambda_t = A_t\Lambda_t$, $i_t = I_t/A_t$, $\widetilde{w}_t = \widetilde{W}_t/A_t$, $\widetilde{w}_t^{NB} = \widetilde{W}_t^{NB}/A_t$, and $\widetilde{bar}_t = \widetilde{b}_t/A_t$,.

2.8 The steady-state

In the absence of shocks, the economy converges to a steady-state growth path in which all stationary variables are constant: for all t, $y_t = y$, $k_t = k$, $\overline{k}_t = \overline{k}$, $u_t = u = 1$, $\lambda_t = \lambda$, $\upsilon_t = \upsilon$, $\xi_t = \xi$, $c_t = c$, $\widetilde{r}_t^K = \widetilde{r}^K$, $i_t = i$, $N_t = N$, $U_t = U$, $V_t = V$, $x_t = x$, $q_t = q$, $s_t = s$, $\widetilde{w}_t = \widetilde{w}$, $\widetilde{w}_t^{NB} = \widetilde{w}^{NB}$, $\widetilde{bar}_t = \widetilde{bar}$, $r_t^B = r^B$, $\pi_t = \pi$, $\mu_t = \mu = 1$, $a_t = a = 1$, $z_t = z$, $\theta_t = \theta$, $\eta_t = \eta$, $g_t = g$, $g_{yt} = g_{ct} = g_{It} = g_{At} = z$, $cyr_t = cyr$, $iyr_t = iyr$, $\Theta_t = \Theta$, $\widetilde{r}_t^B = \widetilde{r}^B$.

2.9 The linearized system

1. y_t

$$\left[g^{-1} - \left(\frac{\phi_N}{2}x^2\right)\right]\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \phi_{u1}\frac{\overline{k}}{zy}\widehat{u}_t + g^{-1}\widehat{g}_t + \phi_N x^2\widehat{x}_t \quad (25)$$

2. k_t

$$\widehat{k}_t = \widehat{u}_t + \widehat{\overline{k}}_{t-1} \quad (26)$$

3. $\overline{k}_t$

$$z\widehat{\overline{k}}_t + (1-\delta-z)\,\widehat{\mu}_t + (1-\delta)\,\widehat{z}_t = (1-\delta)\,\widehat{\overline{k}}_{t-1} + (z-1+\delta)\,\widehat{i}_t \quad (27)$$

4. λ_t

$$\widehat{\lambda}_t = \widehat{r}_t^B + E_t\left(\widehat{\lambda}_{t+1} - \widehat{z}_{t+1} - \widehat{\pi}_{t+1}\right) \quad (28)$$

5. c_t

$$\begin{aligned}(z-\beta h)\,(z-h)\,\widehat{\lambda}_t &= zh\widehat{c}_{t-1} - \left[z^2+\beta h^2\right]\widehat{c}_t - hz\widehat{z}_t \\ &\quad + (z-h)\,z\widehat{a}_t - (z-h)\,\beta h E_t\widehat{a}_{t+1} \\ &\quad + \beta hz E_t\widehat{z}_{t+1} + \beta hz E_t\widehat{c}_{t+1}\end{aligned} \quad (29)$$

6. $\widetilde{r}_t^K$

$$\widehat{u}_t = \frac{\widetilde{r}^K}{\phi_{u2}}\widehat{\widetilde{r}}_t^K \quad (30)$$

7. i_t

$$\widehat{i}_t - \widehat{i}_{t-1} + \widehat{z}_t - \frac{1}{\phi_I z^2}\left(\widehat{\upsilon}_t + \widehat{\mu}_t\right) = \beta\left(\widehat{i}_{t+1} - \widehat{i}_t + \widehat{z}_{t+1}\right) \quad (31)$$

8. υ_t

$$\frac{z}{\beta}\left(\widehat{\lambda}_t - \widehat{\lambda}_{t+1} + \widehat{z}_{t+1} + \widehat{\upsilon}_t\right) = (1-\delta)\,\widehat{\upsilon}_{t+1} + \widetilde{r}^K\widehat{\widetilde{r}}_{t+1}^K \quad (32)$$

9. N_t

$$\widehat{N}_{t+1} = \chi\widehat{N}_t + (1-\chi)\left(\widehat{q}_t + \widehat{V}_t\right) \quad (33)$$

10. U_t

$$\widehat{U}_t = -\frac{N}{U}\widehat{N}_t \quad (34)$$

11. q_t

$$\widehat{q}_t = -\sigma\widehat{\Theta}_t \quad (35)$$

12. s_t

$$\widehat{s}_t = (1-\sigma)\,\widehat{\Theta}_t \tag{36}$$

13. V_t

$$\widehat{x}_t \equiv \widehat{q}_t + \widehat{V}_t - \widehat{N}_t \tag{37}$$

14. Θ_t

$$\widehat{\Theta}_t = \widehat{V}_t - \widehat{U}_t \tag{38}$$

15. x_t

$$\begin{aligned} & \left(\frac{\phi_N x}{\beta}\right)\left(\widehat{y}_t - \widehat{N}_t\right) + \left(\frac{\phi_N x}{\beta}\right)\widehat{x}_t \qquad (39) \\ = \; & \left[(1-\alpha)\,\xi - \frac{N\widetilde{w}}{y} + \phi_N x^2 + \chi\phi_N x\right]\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\right) \\ & + \left[(1-\alpha)\,\xi + \phi_N x^2 + \chi\phi_N x\right]\left(\widehat{y}_{t+1} - \widehat{N}_{t+1}\right) \\ & - \frac{N\widetilde{w}}{y}\widetilde{w}_{t+1} + \left(2\phi_N x^2 + \chi\phi_N x\right)\widehat{x}_{t+1} + (1-\alpha)\,\xi\widehat{\xi}_{t+1} \end{aligned}$$

16. u_t

$$\widehat{y}_t = \alpha\widehat{k}_t - \alpha\widehat{z}_t + (1-\alpha)\,\widehat{N}_t \tag{40}$$

17. ξ_t

$$\widehat{\widetilde{r}}_t^{\,K} = \widehat{\xi}_t + \widehat{y}_t + \widehat{z}_t - \widehat{k}_t \tag{41}$$

18. π_t

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \left(\frac{\theta-1}{\phi_P}\right)\widehat{\xi}_t - \frac{1}{\phi_P}\widehat{\theta}_t \tag{42}$$

19. $\widetilde{w}_t$

$$\widehat{\widetilde{w}}_t = \gamma\widehat{\widetilde{w}_t^{NB}} \tag{43}$$

20. $\widetilde{w}_t^{NB}$

$$\left(\frac{\widetilde{w}^{NB}N}{\eta y}\right)\widehat{\widetilde{w}}_t^{NB} = [(1-\alpha)\xi + \phi_N x(x+s)]\left(\widehat{\eta}_t + \widehat{y}_t - \widehat{N}_t\right) + [(1-\alpha)\xi]\widehat{\xi}_t + [\phi_N x(2x+s)]\widehat{x}_t + (\phi_N sx)\widehat{s}_t + \left(\frac{\widetilde{barN}}{y}\right)\widehat{\eta}_t \quad (44)$$

21. r_t^B

$$\widehat{r}_t^B = \rho_r \widehat{r}_{t-1}^B + \rho_\pi \widehat{\pi}_t + \rho_y \widehat{y}_t + \rho_\Theta \widehat{\Theta}_t + \varepsilon_{rt} \quad (45)$$

22. μ_t

$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \varepsilon_{\mu t} \quad (46)$$

23. a_t

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_{at} \quad (47)$$

24. z_t

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_{zt} \quad (48)$$

25. θ_t

$$\widehat{\theta}_t = \rho_\theta \widehat{\theta}_{t-1} + \varepsilon_{\theta t} \quad (49)$$

26. η_t

$$\widehat{\eta}_t = \rho_\eta \widehat{\eta}_{t-1} + \varepsilon_{\eta t} \quad (50)$$

27. g_t

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \varepsilon_{gt} \quad (51)$$

3 Econometric Methodology and Results

I estimate the DSGE model's parameters by minimizing a well-defined distance between model's and data's multivariate spectra. This estimation method has been suggested by Watson (1993), implemented by Wen (1998) and formalized and further extended by Diebold, Ohanian and Berkowitz (1998) (henceforth DOB). It is a limited information procedure which concentrates on the information contained in all second moments (variance, persistence, covariances) of the (simulated and actual) data. As noted by DOB, "all traditional business cycle statistics have spectral analogues: univariate persistence is reflected in typical spectral shape, multivariate issues of comovements are summarized by the coherence, lead-lag relationships are captured by phase shifts". Hence, this procedure selects the parameters values that maximize the agreement between model and data along the "second moment dimension" which is clearly the dimension that business cycle models are designed to capture. It disregards other characteristics of the data such as Skewness and Kurtosis[8]. Evaluating a DSGE model along the spectrum dimension is however a much more demanding exercise than the informal comparison of some model's and data's second moments. Indeed, Watson (1993) recommends to "present both model and data spectra as a convenient way of comparing their complete set of second moments."

I now briefly describes the estimation methodology. I consider the following measure of goodness-of-fit,

$$G_W(\theta) = \sum_j W(\omega_j) \operatorname{tr} \left[D'(\omega_j;\theta) D(\omega_j;\theta) \right]$$

where

$$D(\omega_j;\theta) = F_m(\omega_j;\theta) - F^*(\omega_j), \ \omega_j \in (0,\pi).$$

[8]The danger of a limited information approach is that the selected information may not be sufficient to identify some of the DSGE parameters. This problem is stressed by Canova and Sala (2006) for impulse response matching.

The function $W(\omega_j)$ weights the trace of the squared divergence between the model spectrum, $F_m(\omega_j;\theta)$, and a consistent estimate of the data spectrum, $F^*(\omega_j)$, across frequencies $\omega_j \in [0,\pi]$. The spectrum-matching estimator, $\widehat{\theta}_{GW}$, is then given by

$$\widehat{\theta}_W = \arg\min_{\theta} \left[G_W(\theta)\right].$$

Before taking the model to the data, I slightly relaxe the aggregate resource constraint. The reason is that capital utilization is the unique instantaneous margin in the model. Since the model economy is hitted by demand shocks every period, capital utilization is bounded to be very volatile to allow aggregate supply to meet aggregate demand in equilibrium. The coefficient in front of capital utilization in the log-linearized aggregate resource constraint is $\left(\phi_{u1}\frac{\overline{k}}{zy}\right)$. The value of this coefficient is picked by the following four steady state conditions

$$\begin{aligned}
\phi_{u1} &= \widetilde{r}^K, \\
\widetilde{r}^K &= \frac{z}{\beta} - 1 + \delta, \\
\beta &= \frac{z\pi}{r^B}, \\
\frac{k}{zy} &= \frac{\xi\alpha}{\widetilde{r}^K}.
\end{aligned}$$

Combining them, we obtain

$$\phi_{u1}\frac{k}{zy} = \xi\alpha.$$

Hence the value of $\phi_{u1}\frac{k}{zy}$ is determined by $\xi = \frac{\theta-1}{\theta}$ and α. The problem is that any reasonable calibration of θ and α delivers a value of $\phi_{u1}\frac{k}{zy}$ greater than 0.15. Such a lower bound for $\phi_{u1}\frac{\overline{k}}{zy}$, combined with the high volatility of $\widehat{u}_t$ implies that, in equilibrium, the costs of deviating capacity utilization from its steady-state value are of the same magnitude than the investment share of output, $\phi_{u1}\frac{\overline{k}}{zy}\widehat{u}_t \approx \frac{i}{y}\widehat{i}_t$. In other words, the value of the coefficient

in front of $\widehat{u}_t$ implied by the steady-state conditions is about one order of magnitude too large. One way to mitigate this problem would be to include another instantaneous margin in the model. However, in order to take the present DSGE model to the data, I take a shortcut by introducing an extra parameter, κ, to relaxe the steady state condition, $\phi_{u1}\frac{k}{zy} = \xi\alpha$. Hence, the empirical DSGE model to be estimated features the following relaxed resource constraint

$$\left[g^{-1} - \left(\frac{\phi_N}{2}x^2\right)\right]\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \kappa\left(\phi_{u1}\frac{\overline{k}}{zy}\right)\widehat{u}_t + g^{-1}\widehat{g}_t + \phi_N x^2\widehat{x}_t,$$

where the relaxation parameter, κ, is going to be estimated.

As pointed out by DOB, differential weighting by frequency may be helpful to handle model misspecification. Following the tradition in macroeconomics, I consider a partition of the frequency domain $[0, \pi]$ into three intervals. The first interval, $\left[0, \frac{2\pi}{32}\right)$ is labelled the "Trend Region" and corresponds to the very low frequencies that are associated with cycles of periods infinity to 32 quarters. The second interval, $\left[\frac{2\pi}{32}, \frac{2\pi}{6}\right]$, corresponds to the "business cycles", usually defined as cycles with periods running from 32 to 6 quarters. Finally, the last interval, $\left(\frac{2\pi}{6}, \pi\right]$, captures the high frequencies, associated with seasonalities and measurement errors. In the model, the feature of output being demand-driven combined with the presence of a unique instantaneous margin, implies that this margin is going to absorb most of the instantaneous effect of demand shocks. In other words, on impact, a monetary policy shock or a preference shock are going to be accomodated entirely and exclusively by the instantaneous responses of the price level and capital utilization. This restriction implied by the DSGE model is likely to be a source of misspecification at the frequencies that are mostly affected by the model's variables *impact* responses to shocks, namely the very high frequencies. Hence, in addition to the relaxation parameter, κ, introduced above, I will treat model misspecification by considering two alternatives

weighting functions for the discrepancy between model and data spectra at various frequencies. Figure 3 plots the two weighting functions. The first weighting function, labeled "Flat weighting" is a step function that assigns equal weight, $\frac{1}{3}$, to each of the three regions. The height of the step function over a given region is determined by the number of equally-spaced points, at which the distance function is evaluated, falling into that region. Because I am working with quarterly data, most of the frequency domain $[0, \pi]$ is the support of cycles of relatively short periods/high frequencies. The second weighting function is derived from a normal density truncated at frequency zero, centered at frequency $\frac{2\pi}{20}$, i.e. the middle point of the business cycle region, and whose standard deviation is determined so that the "High Frequency" region, $\left(\frac{2\pi}{6}, \pi\right]$, weights 10 percent. The first weighting scheme is similar to the implementation of a flat prior. Instead, the second weighting function gives more weight to business cycle frequencies, which are the most relevant ones for the model at hand, and downweights drastically the very high frequencies, suspected to be the most contaminated by misspecification. The validity of the restrictions implied the DSGE model at frequency zero, namely the balanced growth hypothesis, which predicts that output, consumption, investment and real wage share a common stochastic trend is uncertain in light of the recent evidence provided by Karl Whelan and Jonas Fisher. Accordingly, the hump-shaped weighting function also reduces slightly the weight of the "Trend" region. Both weighting functions integrate to unity.

The following teen parameters, $\delta, \rho, \tau, q, U, G/Y, (\phi_N x^2/2), z, \pi, r^B$ are calibrated prior to estimation. Table 1 summarizes the calibration whereas Table 2 reports the parameters and steady-state values implied by this calibration. The quarterly depreciation rate δ is set equal to 0.025, a value commonly used in the literature. The calibration of the labor market block's parameters U, q, and ρ follows exactly Yashiv (2005). The size of the labor force is normalized to one so that $N + U = 1$ so that the unemployment rate

$U/(N+U)$ is equal to unemployment U. The steady-state unemployment rate U is set equal to 0.1. The steady-state separation rate ρ is set equal to 0.085. This value is consistent with the evidence provided by Davis, Haltiwanger and Schuh (1996) who compute a quarterly worker separation rate of about 8 percent and by Hall (1995) who reports this rate to be between 8 and 10 percent. Yashiv (2005) sets the steady-state probability of filling a vacancy, $q = 0.90$, based on estimates obtained by Burdett and Cunningham (1998). The replacement rate τ is set equal to 0.35. I set the steady state value of hiring cost, $(\phi_N x^2/2)$, equal to one percent, a value commonly used in the search and matching literature. I set $G/Y = 0.22$, the value used by Primiceri, Schaumburg and Tambalotti (2007). Finally, the steady-state rate of inflation π, the steady-state nominal interest rate, r^B, and the the growth rate of the economy along the balanced growth path, z, are respectively set equal to their sample averages, 1.0062, 1.012, 1.0045.

The seven following variables are used in estimation: per capita output, consumption and investment in real terms, real wages, labor market tightness (i.e. the ratio of vacancy over unemployment), inflation and the nominal short term interest rate. I use quarterly U.S. data. The dataset spans a sample from 1984:QI to 2006:QI. Following Del Negro, Schorfheide, Smets and Wouters (2004) and Primiceri, Schaumburg and Tambalotti (2007), I measure nominal consumption using readings on nominal personal consumption expenditures of non-durables and services. Nominal investment corresponds to the sum of personal consumption expenditures of durables and gross private domestic investment. Nominal output is measured by nominal GDP. Per capita real GDP, consumption and investment are obtained by dividing the nominal series by the GDP deflator and population. Real wages corresponds to nominal compensation per hour in the non-farm business sector, divided by the GDP deflator. Consistently with the model, I measure population by the labor force which is itself defined as the sum of official unemployment and official employment. The vacancy rate is measured by the index of Help

wanted advertising in newspapers divided by labor force. The unemployment rate is the ratio of official unemployment over labor force. The labor market tightness indicator is measured by the ratio of the vacancy rate over the unemployment rate. Inflation corresponds to the quarterly log difference in the GDP deflator measures, while for nominal interest rate, I use the effective Federal Funds rate.

Figure 1 plots the data in levels while figure 2 plots log deviation from sample mean of the stationary, model-consistent, transformations of the data. These later are used to estimate the DSGE model. Unemployment and vacancy are both very persistent and strongly negatively correlated. This negative correlation, which has been very stable over time, is known as the Beveridge curve. By considering the vacancy/unemployment ratio, I use the Beveridge curve to remove the very persistent trend shared by unemployment and vacancy. Using the vacancy/unemployment ratio to estimate the model is also consistent with theory since market tightness is a sufficient statistic, or state variable, in the search and matching model.

An estimate of data spectrum is computed by fitting the seven data time series with a VAR with four lags. The relatively short sample size prevents me to estimate data spectrum non parametrically. Finally, when estimating the DSGE model, the parameter space is restricted to the determinacy region.

Tables 3 and 4 report estimates of the structural parameters and shock processes respectively. The hump-shaped weighting function delivers estimates in line the values commonly found in the literature. Instead, the flat weighting function drives several estimates against their upper or lower bounds. This finding confirms my prior that misspecification is particularly severe at very high frequencies. Disregarding high frequencies and focusing on the frequency region of interest during the estimation is a worthy device that helps recovering meaningful estimates. Table 5 reports the parameters and steady-state values implied by combining estimates with steady state conditions. Not surprinsigly, the hump-shaped weighting produces much

more reasonable results than the equal-weighting scheme.

The fit of the model can be assessed by looking at figures 4 and 5. Figure 4 plots univariate spectral densities of the data and corresponding model's variables. The area below the spectral density is equal to the variance, whereas the shape of the spectral density reveals the persistence of a variable. The mode of the spectral density indicates the most relevant cycle in accounting for fluctuations in a variable. The model estimated with the hump-shaped weighting function does a better job than the one estimated with the flat weighting function for output growth, consumption growth and investment growth. Output and consumption are slightly too volatile in the model, whereas model's investment is too smooth. Both models are able to replicate the observed volatility and persistence of the vacancy/unemployment ratio. However, the model has hard time in explaining the behavior of inflation, short term interest rate and wage growth. Despite the presence of wage rigidity, wage growth is much too volatile in the model at all frequencies but zero (the trend). The model's inflation and short term interest rates are much too volatile at low frequencies. The estimated model fits quantities reasonably well but can not account for the behavior of prices. This failure might be attributed to the lack of a second instantaneous margin, such as hours per worker. The modeling of real wage is clearly unsatisfactory. Figure 5 plots the model's and data's phase and coherence for output growth, inflation and tightness. The phase of output and inflation at frequency ω_j measures the lead of output over inflation at this frequency. The coherence of output and inflation at frequency ω_j measures the squared correlation between these two variables at that frequency. In the data, output growth and tightness are strongly correlated at business cycle frequencies wherease in the model they are not. Instead, inflation and tightness in the model are perfectly correlated at very low frequencies whereas in the data the correlation between these two variables is close to zero at very low frequencies. The shape of the coherence of growth and tightness over the frequency domain indicates that the

Okun's law is particularly strong in the data for cycles with a period of eight years. The coherence between inflation and tightness shows that the Philips curve is a medium to short run relationship. The model estimated with the hump-shaped weighting function reveals itself unable to capture these two facts.

Figure 6 to 12 plot impulse responses for the two estimated models. The model estimated with the hump-shaped weighting function exhibits strong internal propagation mechanisms. Remember that leisure does not enter into the representative household's utility function and that labor supply is inelastic. Therefore there is no intratemporal substitution effect. However, the utility of the family is concave and implies a desire for consumption smoothing. Hence, in that model, two important propagation channels of shocks work through the family's intertemporal budget constraint, by affecting the wealth of the family. Family's resources are made of labor and capital incomes which are themselves determined by employment, the wage rate, the capital stock and the rental rate of capital. The first wealth effect-transmission mechanism is triggered by variations in the number of family members working. Persistent fluctuations in employment will affect the household's human wealth. The relevance of this transmission channel is due the income pooling hypothesis and to the calibration of the replacement rate $\tau = 0.35$, which implies that the contribution of an unemployed worker to the family real labor income is only about one third the one of an employed worker. This effect is most clearly seen in the impulse responses to a wage bargaining shock. The wage increases, despite the absence of any productivity improvement. This drastically reduces the firms incentives to hire new workers. Vacancy falls considerably on impact and does not return to steady-state before twenty quarters. This, in turn, triggers a persistent fall in employment which affects the family's lifetime labor income negatively. As the family feels poorer, it reduces consumption despite the positive jump in wage. Due to the high wage, firms prefer to rely more on capital and less on labor. Together with

the fall in investment, the firm's behavior explains the rise of the rental rate of capital.

The second transmission mechanism working through the wealth of the family and playing an important role in the model comes from variations in the the rental rate of capital. Since both employment and physical capital are predetermined, any instantaneous variation in output comes from a change in capital utilization and thereby in the rental rate of capital.

Regarding firms, because varying the degree of capital utilization is costly, they try to avoid relying exclusively on that intensive margin. Firms hiring behavior is forward looking. They start posting vacancies as soon as they foresee a persistent increase in aggregate demand.

The wage rate mainly mimics the behavior of the vacancy unemployment ratio. The rental rate is mainly affected by the degree of capital utilization in the short run and by the supply of capital in the long run.

The model estimated with the hump-shaped weighting function yields more volatile and more persistent impulse responses, thereby allowing the model to produce fluctuations at business cycle frequencies.

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Table 1: Calibrated Parameters

Capital depreciation rate	δ	0.025
Job destruction rate	ρ	0.085
Replacement rate	τ	0.35
Probability to fill a vacancy within a quarter	q	0.90
Unemployment rate	U	0.10
Exogenous spending/output ratio	G/Y	0.22
Hiring cost/output ratio	$\frac{\phi_N}{2}x^2$	0.01
Quarterly growth rate of output	z	1.0045
Quarterly gross inflation rate	π	1.0062
Quarterly nominal gross interest rate	r^B	1.012

Table 2: Parameters Implied by Calibration

Discount factor	β	0.9978
Employment rate	N	0.90
Job survival rate	χ	0.915
Hiring rate	x	0.085
Employment adjustment costs parameter	ϕ_N	2.768
Mean of exogenous spending shock	g	1.282
Real rental rate for capital	$\tilde{r}^K$	0.032
Capital utilization cost first parameter	ϕ_{u1}	0.032
Vacancy	V	0.085
Market tightness	Θ	0.85
Job finding rate	s	0.765
Marginal Tobin's q	υ	1
Investment/capital ratio	I/K	0.0294

Table 3: Structural Parameters Estimates

		Equal Frequency Weighting $\omega_{Trend} = \omega_{BC} = \omega_{HF} = 1/3$	Hump-shaped Frequency Weigthing mode $= 2\pi/20$ and $\omega_{HF} = 0.10$
Capital share	α	0.4000	0.3259
Elasticity of substitution across intermediate goods	θ	14.9842	8.3903
Habit persistence in comsumption	h	0.5000	0.8281
Elasticity of matches to unemployment	σ	0.8000	0.5757
Investment adjustment costs parameter	ϕ_I	150.0000	11.0409
Capital utilization cost second parameter	ϕ_{u2}	0.0005	0.0010
Price adjustment costs parameter	ϕ_P	155.8847	71.07
Wage rigidity parameter	γ	0.1689	0.4637
Monetary policy inertia	ρ_r	0.9950	0.8661
Response of monetary policy to inflation	ρ_π	7.000	0.6593
Response of monetary policy to output	ρ_y	0.0001	0.2190
Response of monetary policy to labor market tightness	ρ_Θ	0.0001	0.0289
Parameter relaxing elast. of output to capacity utilization	κ	0.2874	0.0976
Minimized distance function		0.0011	0.0032

Table 4: Shock Processes Parameters Estimates

		Equal Frequency Weighting $\omega_{Trend} = \omega_{BC} = \omega_{HF} = 1/3$	Hump-shaped Frequency Weighting mode $= 2\pi/20$ and $\omega_{HF} = 0.10$
Persistence of preference shock	ρ_a	0.4327	0.7457
Standard deviation of preference shock	σ_a	0.0001	0.0151
Persistence of investment specific shock	ρ_μ	0.8275	0.9000
Standard deviation of investment specific shock	σ_μ	0.0013	0.0020
Persistence of technology growth shock	ρ_z	0.0500	0.2018
Standard deviation of technology shock	σ_z	0.0249	0.0075
Persistence of markup shock	ρ_θ	0.2000	0.7571
Standard deviation of markup shock	σ_θ	0.0001	0.0081
Persistence of bargaining power shock	ρ_η	0.7523	0.8234
Standard deviation of bargaining power shock	σ_η	0.0289	0.0690
Persistence of exogenous spending shock	ρ_g	0.1000	0.8929
Standard deviation of exogenous spending shock	σ_g	0.0001	0.0072
Standard deviation of monetary policy shock	σ_r	0.0019	0.0016

Table 5: Parameters and Steady-State Values Implied by Estimation

		Equal Frequency Weighting $\omega_{Trend} = \omega_{BC} = \omega_{HF} = 1/3$	Hump-shaped Frequency Weighting mode $= 2\pi/20$ and $\omega_{HF} = 0.10$
Bargaining power parameter	η	0.7902	0.7998
Matching efficiency	ζ	0.7903	0.8196
Real marginal cost	ξ	0.933	0.8808
Consumption/output ratio	C/Y	0.4224	0.5027
Investment/output ratio	I/Y	0.3476	0.2673
Labor share	$\widetilde{W}N/Y$	0.5594	0.5933
Capital/output ratio	K/Y	11.8305	9.0968

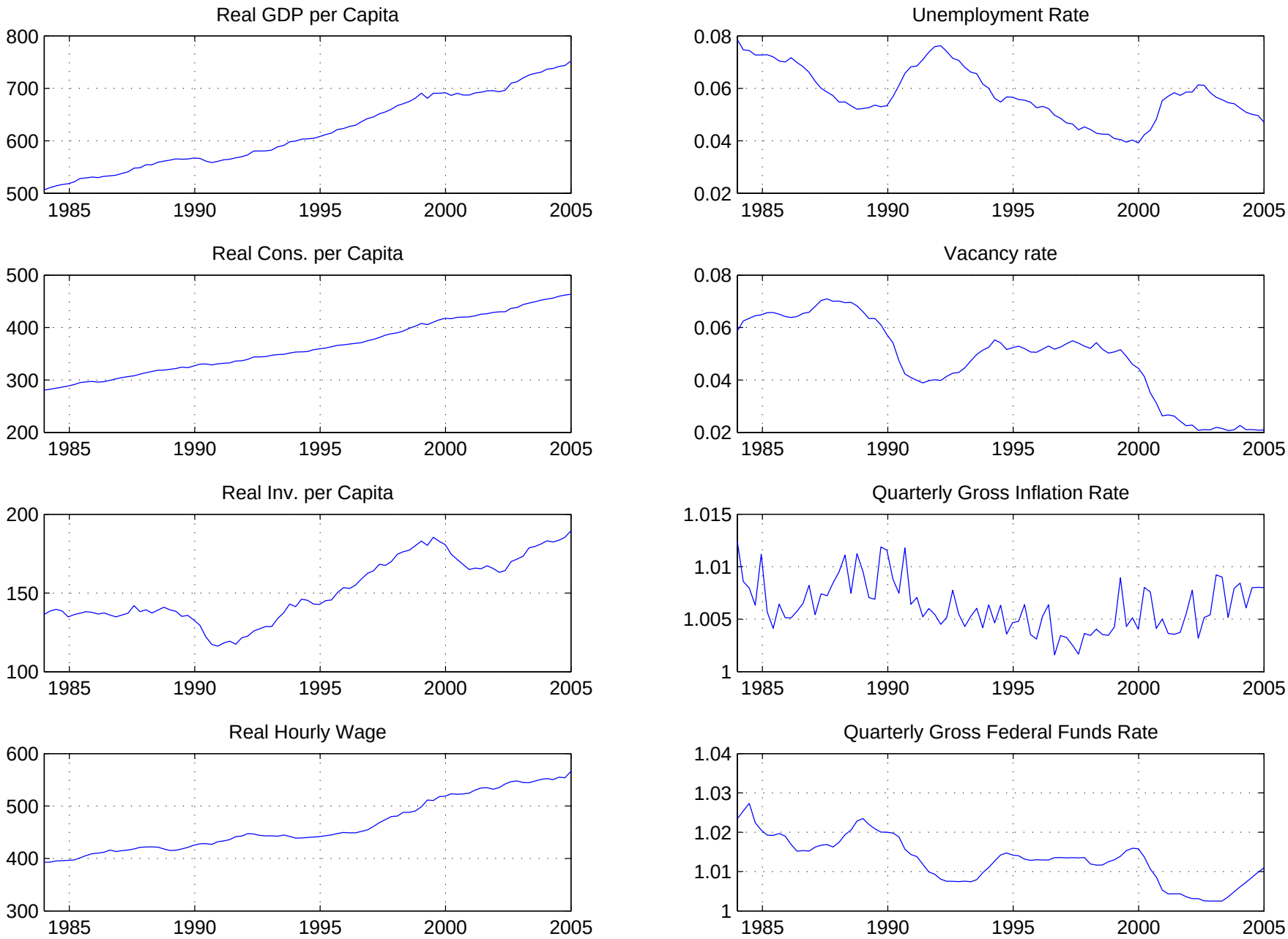

Figure 1. Data, United States.

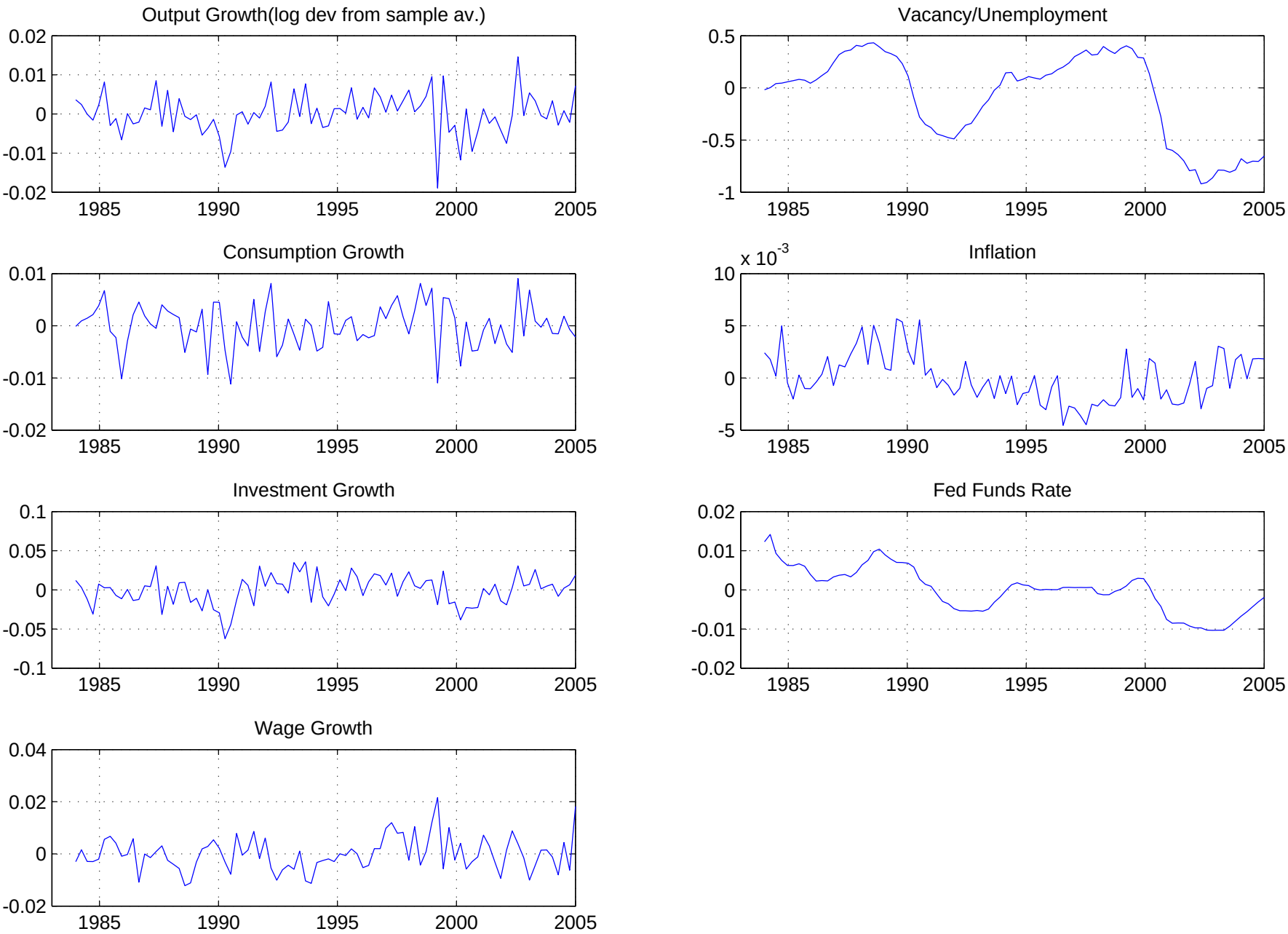

Figure 2. Data used in estimation. Log deviation from sample mean.

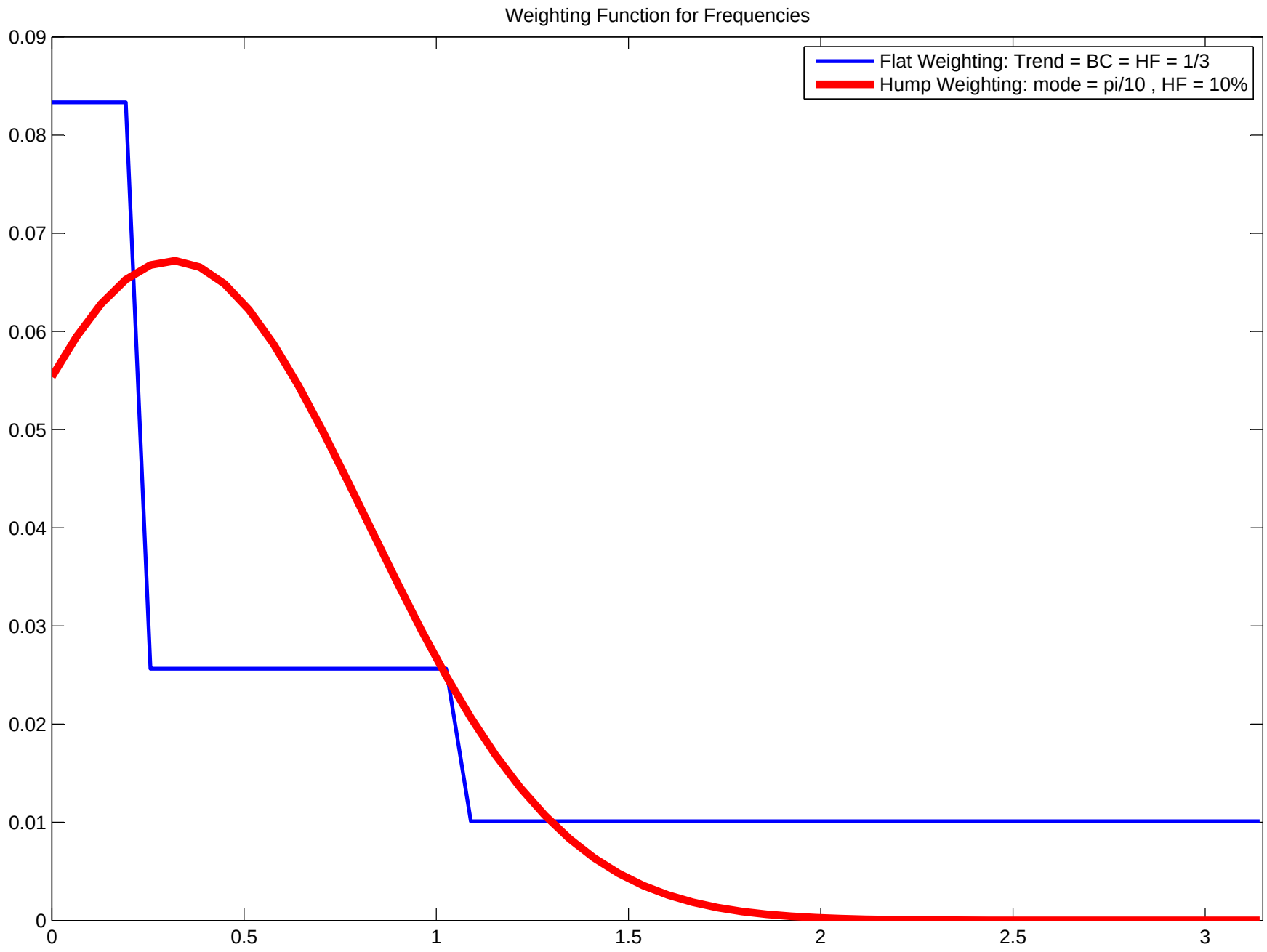
Weighting Function for Frequencies
Flat Weighting: Trend = BC = HF = 1/3
Hump Weighting: mode = pi/10 , HF = 10%
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.5
1
1.5
2
2.5
3

Figure 3

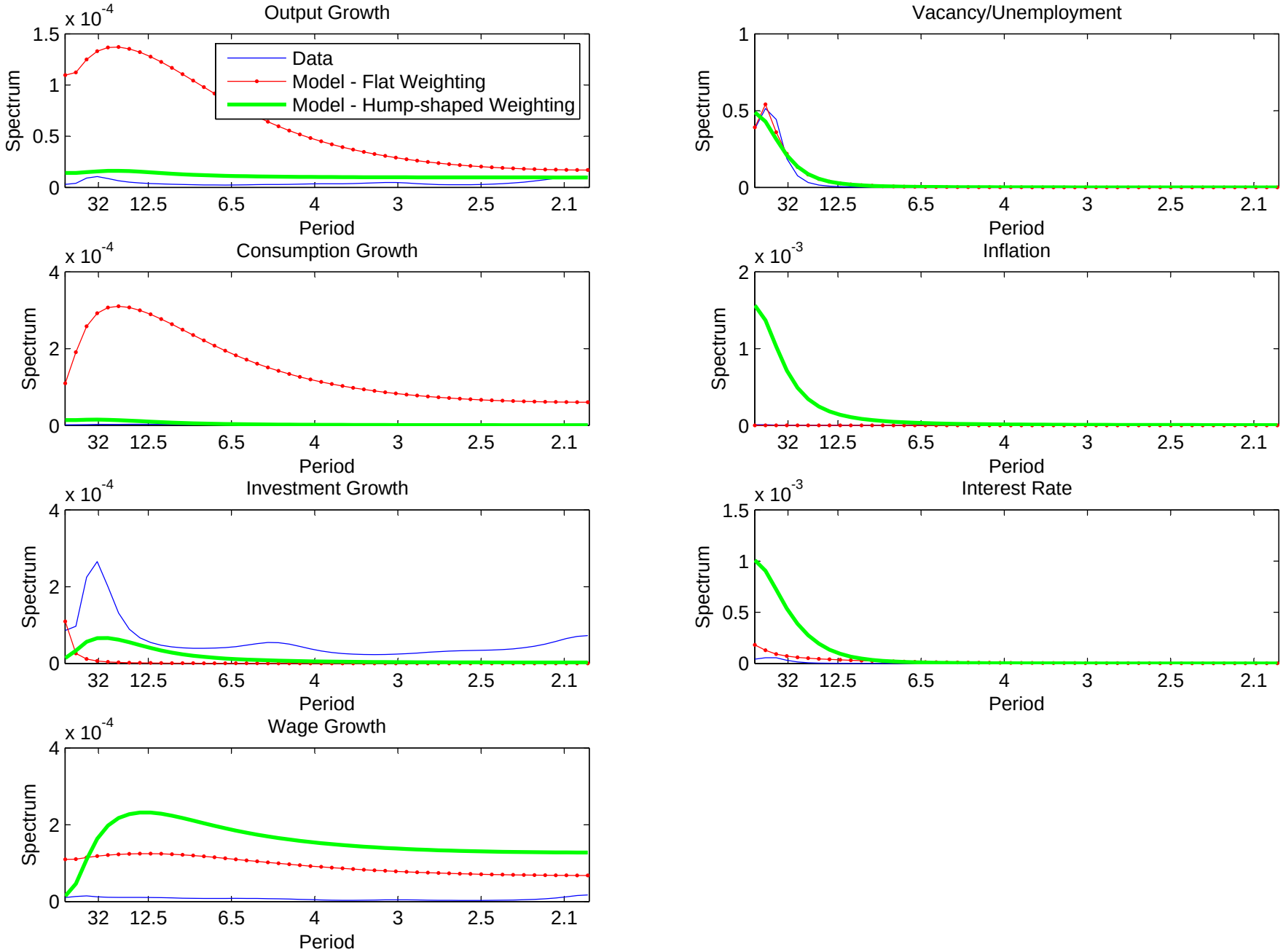

Figure 4. Spectrum. Estimated Models vs Data

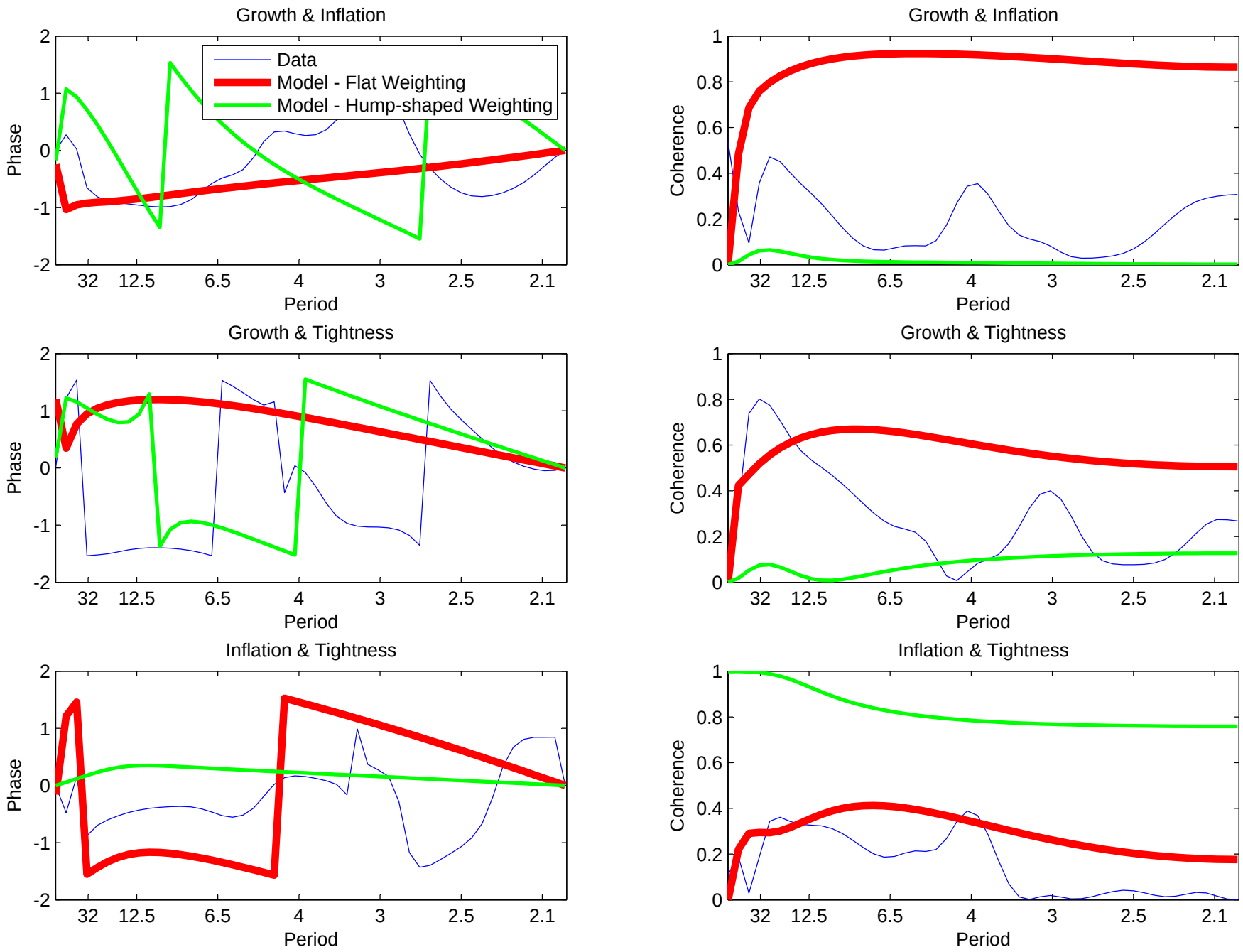

Figure 5. Phase and Coherence. Estimated Models vs Data

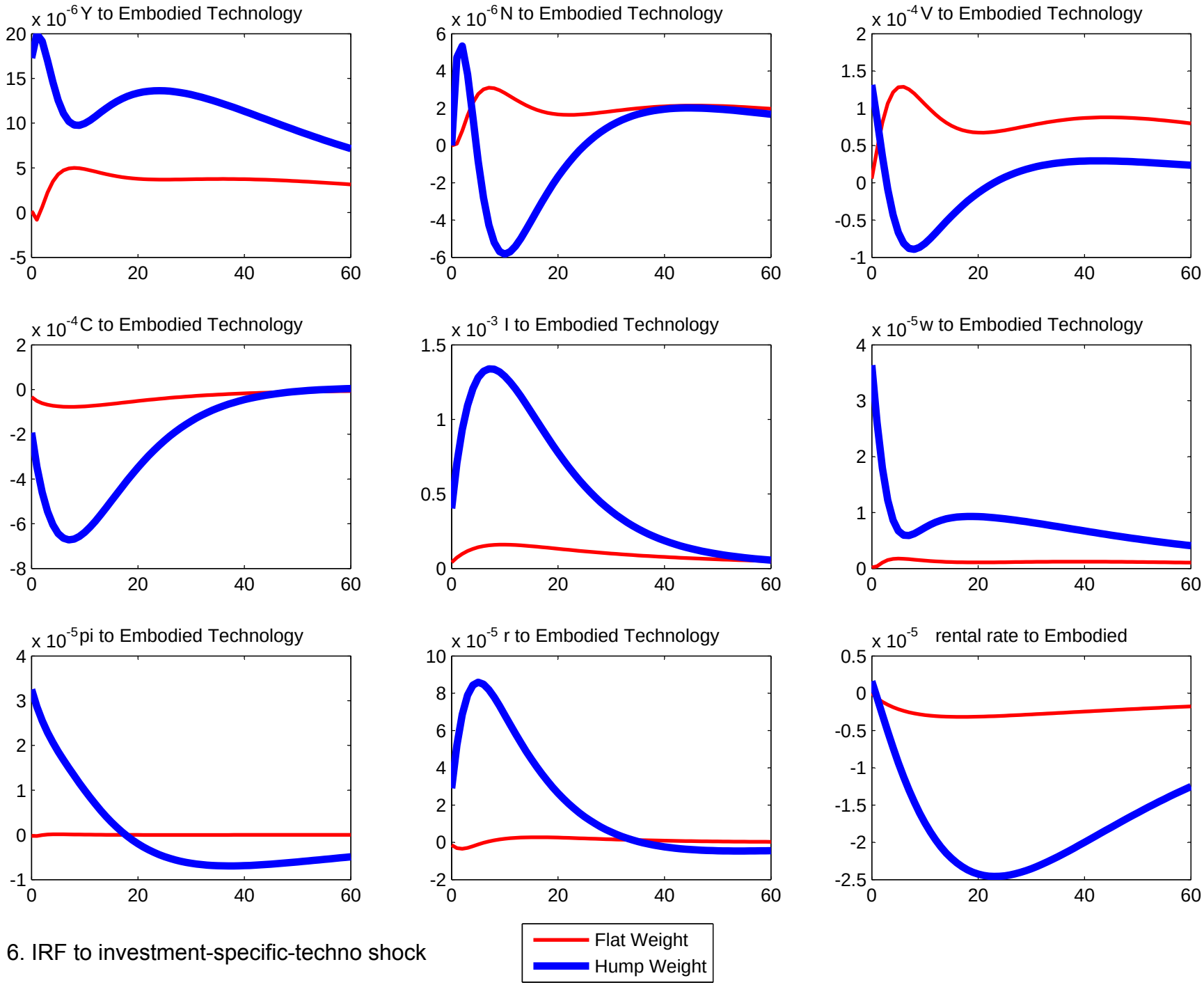

Figure 6. IRF to investment-specific-techno shock

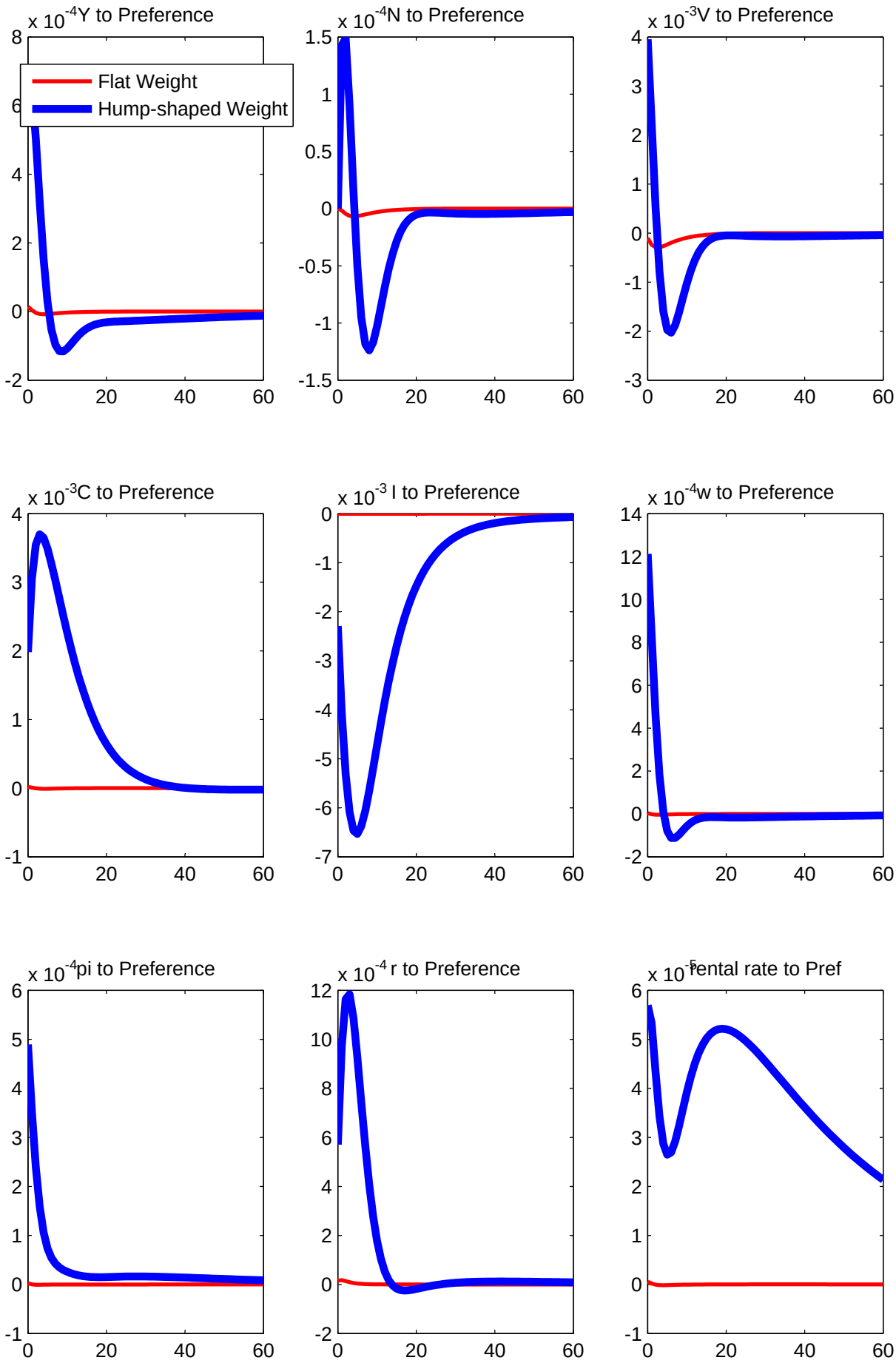

Figure 7. Model's variables responses to a one-standard-deviation preference shock.

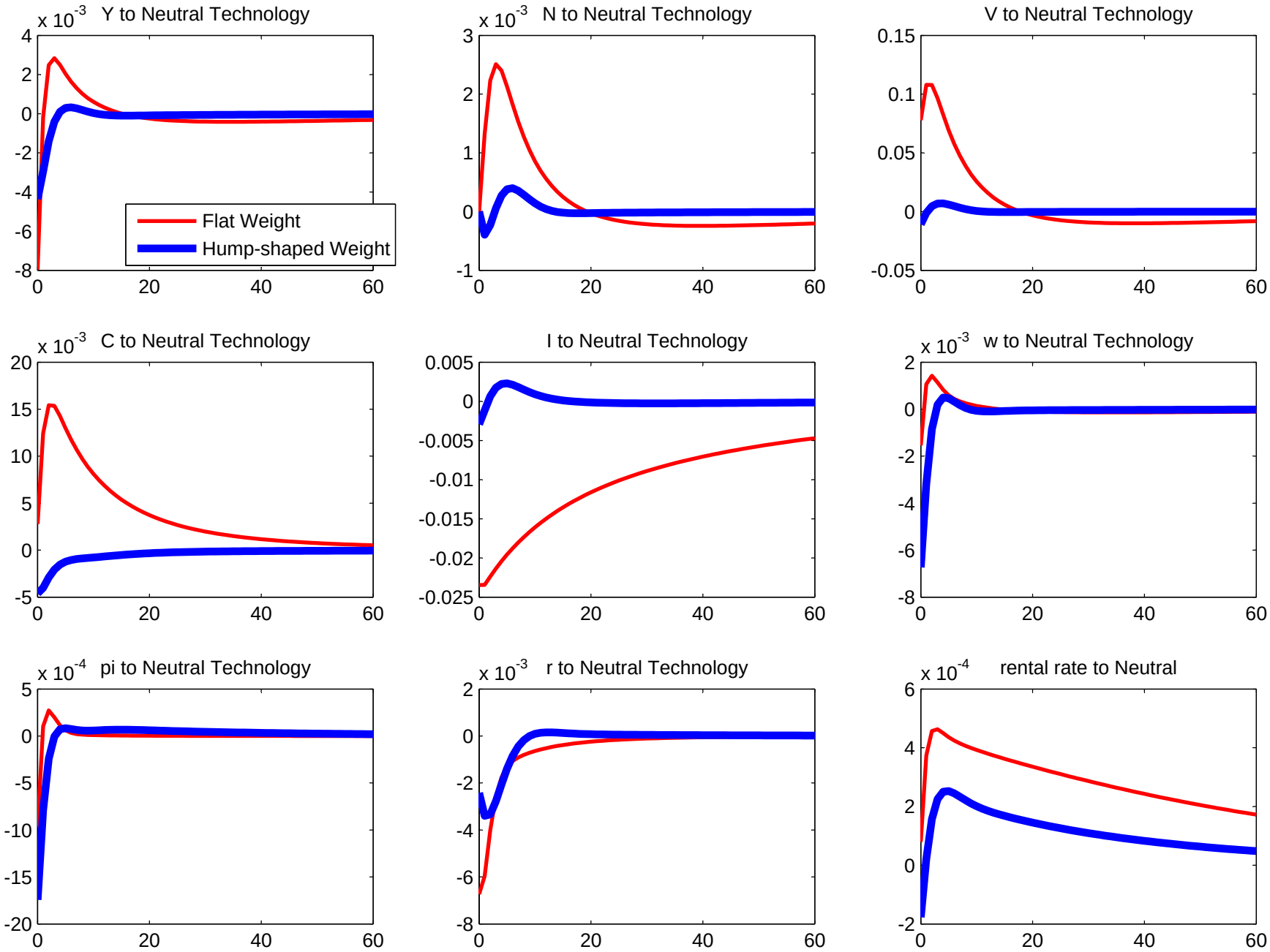

Figure 8. Model's variables responses to a one-standard-deviation neutral technology shock.

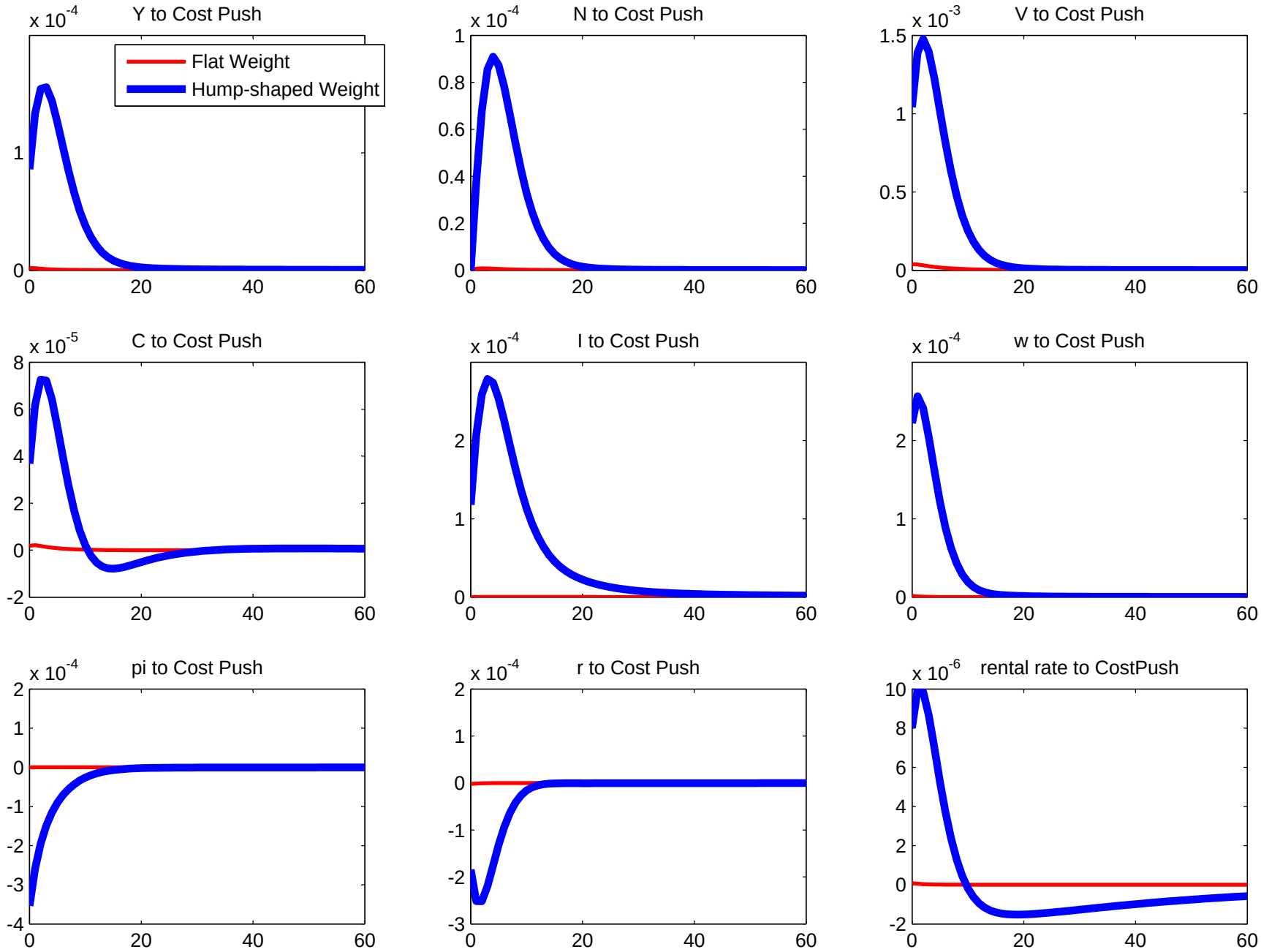

Figure 9. Model's variables responses to a one-standard-deviation cost-push shock.

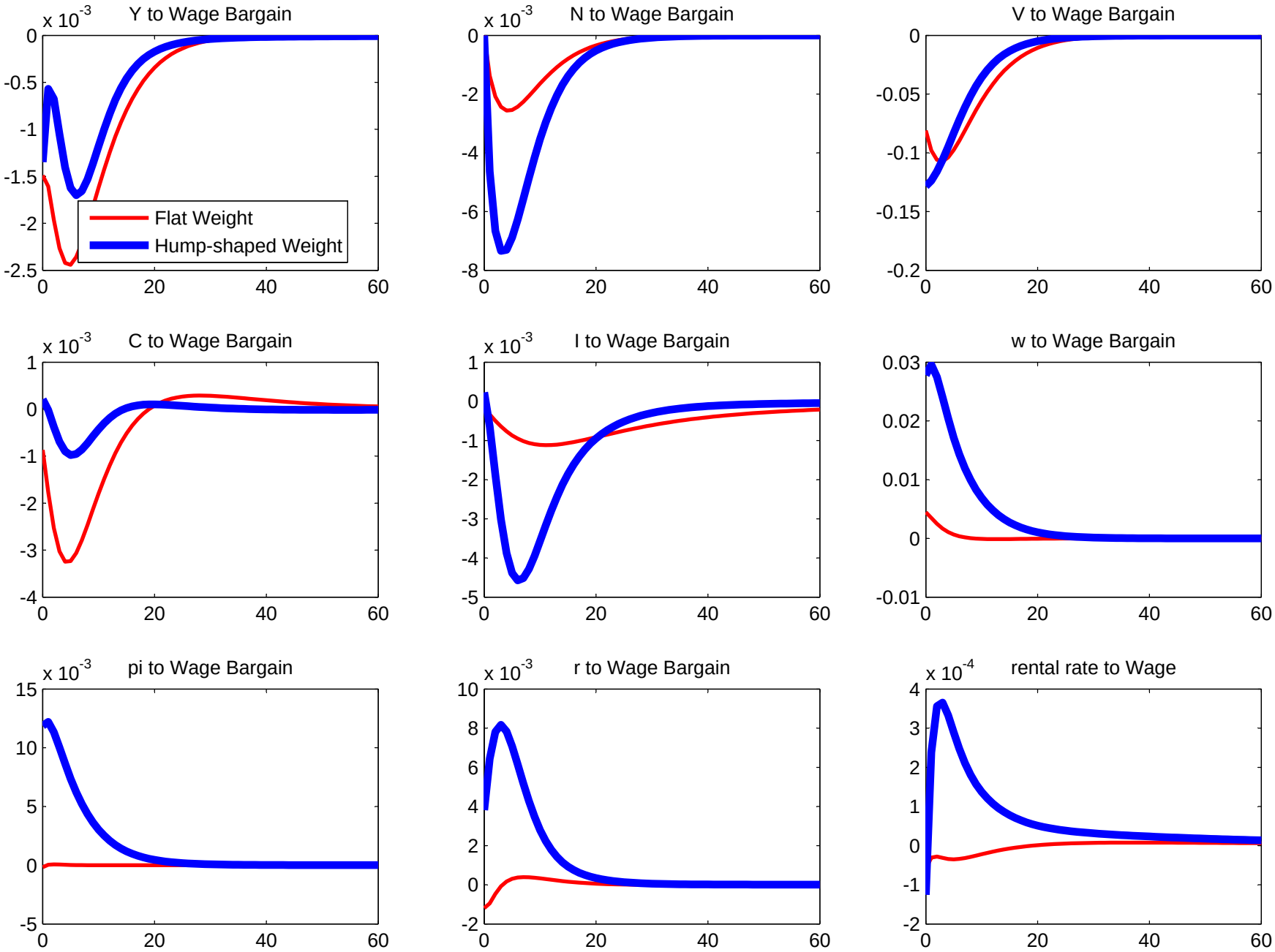

Figure 10. Model's variables responses to a one-standard-deviation wage bargaining shock.

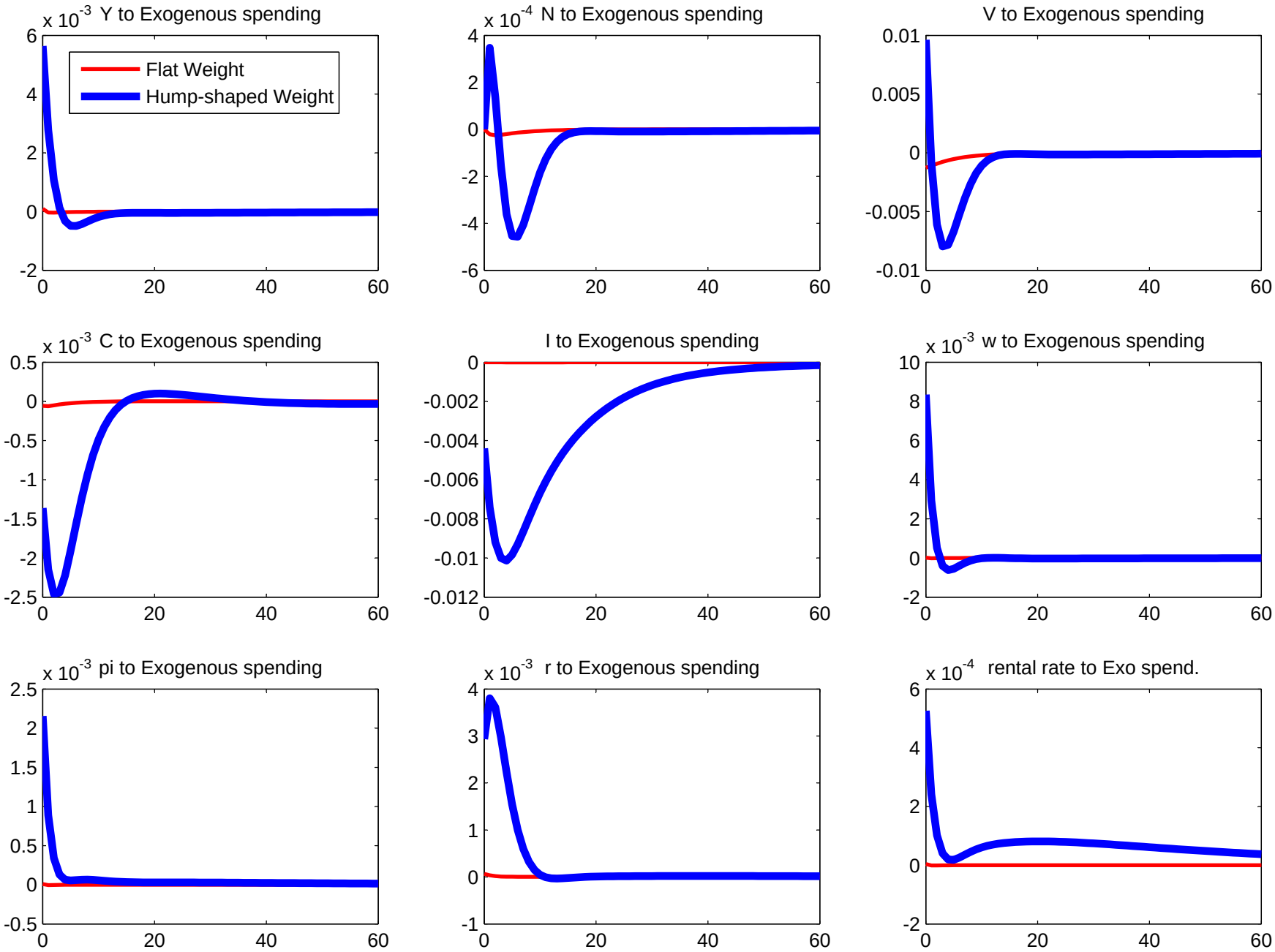

Figure 11. Model's variables responses to a one-standard-deviation exogenous spending shock.

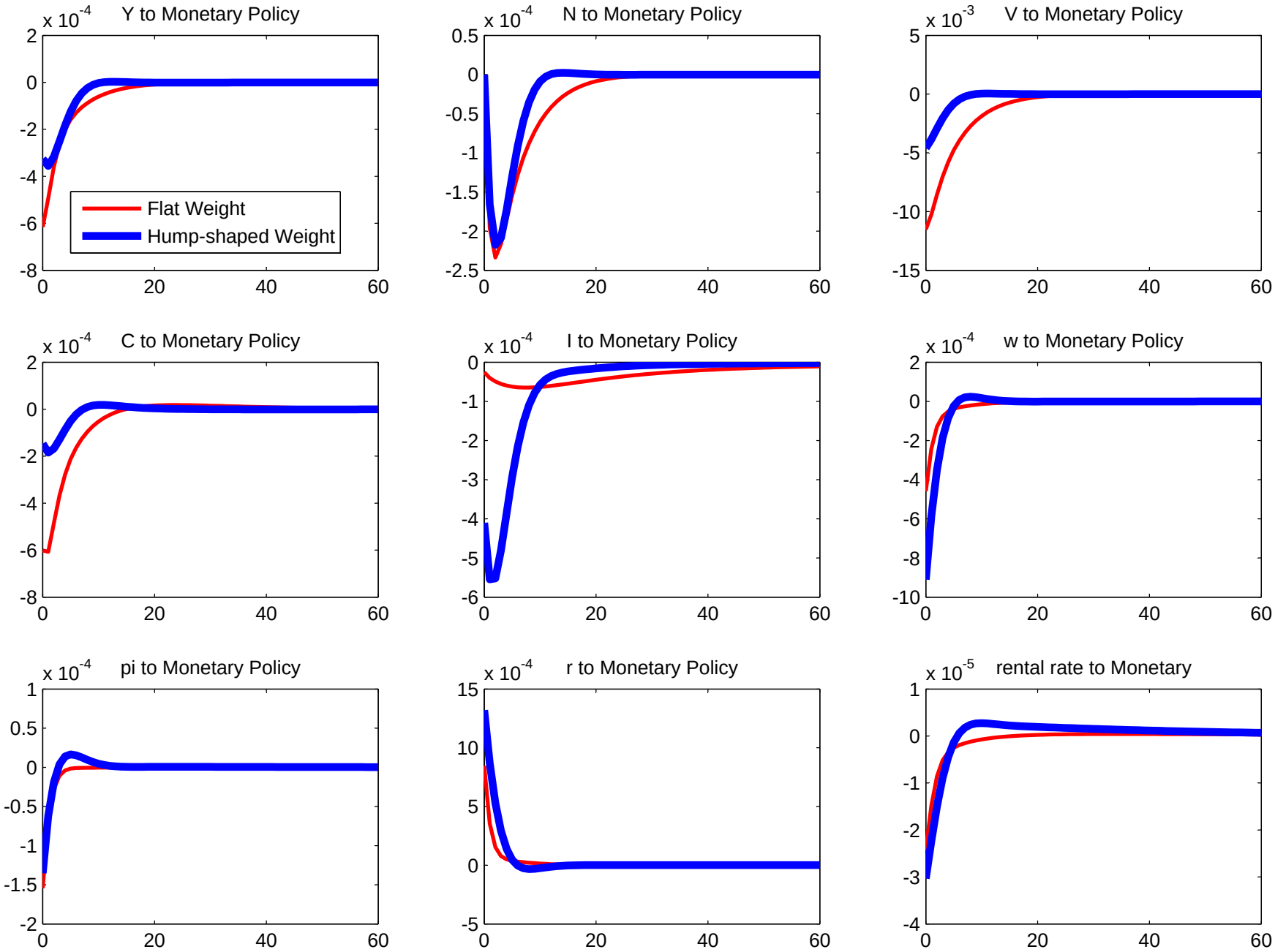

Figure 12. Model's variables responses to a one-standard-deviation monetary policy shock.