Contracting with specialized Agents*

Benno Bühler †

University of Munich

August 15, 2007(very incomplete and preliminary version)

Abstract

This paper contributes to the literature on credence goods and analyzes a procurement problem in which two specialized agents compete for being hired to carry out a task. Two different approaches are eligible, but one of them is superior for the particular task at hand and therefore yields a higher payoff to the principal. Each firm is capable of one approach. Both firms are better informed than the principal about the superior approach and about their costs. I argue that information rents, which may potentially be earned due to cost uncertainty, induce the firms to defraud the contractor with respect to the superior approach. The contractor can mitigate the incentive problem by offering contracts to the firms that condition the payment and employment decision on both firm's recommendations. I find that rents are only paid if both firms consistently suggest the same approach. If the incremental value of the proper approach is small enough, firms' information on the optimal approach becomes worthless and the optimal contract yields the standard auction outcome. On the other hand, if the right approach is crucial, the principal optimally assigns the task only to firms that are likely to offer the appropriate approach even though this entails to pay high rents even to firms which are not hired finally.

JEL classification: D44, D82, D86

Keywords: credence goods, multidimensional auctions, asymmetric information

---

*Financial support from German Science Foundation is gratefully acknowledged.

†Department of Economics, Kaulbachstr. 45, 80539 Munich, Germany, e-mail: benno.buehler@lrz.uni-muenchen.de

1 Introduction

Ongoing specialization and its positive effects on productivity is one of the main reasons of today's prosperity. But increasing specialization also means that the variety of special products and services becomes larger and which makes it more and more difficult for a customer to figure out which fits his needs best. In contrast, specialized suppliers have a lot of expertise about their own and their direct competitors' goods and therefore could support the customer to make the right choice. But the opportunity to earn rents by selling his own good, no matter if it is the one which suits the consumer best, may induce the seller not to advice truthfully.

Consider a house owner who seeks to install either a gas, oil, wood or electricity heating. Although each type of heating serves the same purpose, each one has specific merits and drawbacks. For example, electricity heatings are cheap to install but cause high electricity costs, while oil is presently cheaper but requires expensive ovens. Hence in terms of overall costs, electricity may be best suited for well isolated buildings while this might not be true for ancient buildings with high energy wastage. As the technical components differ a lot of specialized firms offer only one type of heating and presumably know for which buildings their heating is especially suited. On the other side, a house owner typically does not know which characteristics of his house speak in favor of one or the other type of heating and thus needs to rely on the sellers' advice.

As another example, management consultancies have specialized in various approaches to make their clients more profitable: For instance, the consultancy *The Boston Consulting Group* enjoys a very good reputation for supporting their clients in opening up new markets to increase revenues. On the other hand, *McKinsey* is well known for their expertise in cutting down their clients' costs. A potential customer typically observes that profits decline and something should be done, but usually the consultancies have a better understanding if that customer should focus on bringing revenues up or costs down. Other things equal, the potential customer is interested in the consultancies knowledge of the right approach in order to determine which one to employ optimally.

In both examples the sellers are better informed about the customer's needs than he himself. This corresponds exactly to Darby and Karni's (1973) definition of *credence goods*. Credence goods comprise a large group of goods and services, like repairs, medical treatments, taxi rides etc. and enjoy increasing interest in the economic literature.[1] Formally, at the time of purchase the customer is less informed about the utility some credence good provides than the seller.

I do not allow contracts to depend on the utility a credence good provides finally to the customer, which would allow to easily solve the seller's incentive problem. This assumption is plausible as there are often unobservable stochastic factors that influence the usefulness of the purchased good. In the example above, it is hard to

[1] See e.g. Dulleck and Kerschbamer (2006) for a survey.

distinguish if low heating costs result from the right choice of the heating type or from warm weather or a parsimonious tenant. In addition Pesendorfer and Wolinsky (2003) point out that the customer's payoff may be hard to verify.

This paper proposes optimal contracts which mitigate the incentives of sellers not to advice honestly. In my model, a principal has to decide which of two specialized agents to employ to carry out a task. Each agent is specialized in a different approach. The agents possess private information with regard to their own costs and to the optimal approach. The principal offers contracts to the agents containing conditional payments and employment probabilities. Private information about costs allows agents to earn rents if they are employed. I show that the possibility to earn rents induces each agent to pretend that her approach is best suited. Hence without further payments, the principal could not learn which specialized agent to employ optimally. The principal may find it optimal to pay further rents in order to elicit the agents' private information about the optimal approach. It may be optimal to employ agents which do not offer the best suited approach in order to save on rents.

2 Related Literature

There are two big strands of literature this paper builds upon. Firstly there is a growing literature on *credence goods* which also analyzes situations where sellers have better information about customers needs than the clients themselves.

Many authors, like Pitchik and Schotter (1987) or Wolinsky (1993), (1995) assume that a customer either needs a cheap or a costly treatment and can only observe if his problem is fixed but not how it was done.[2] All problems which require only a cheap treatment can also be fixed by the costly one. This setup induces experts sometimes to recommend and to bill expensive treatments even though they conduct cheap ones if they are sufficient. Similar to my paper, Emmons (1997), (2001) and Alger and Salanie (2004) suppose that the customer can verify the treatment he gets and find that the scope for fraud is reduced significantly. Pesendorfer and Wolinsky (2003) analyze a model in which, similarly to my setup, there is exactly one proper treatment yielding an extra payoff to the customer, i.e. no treatment always solves the problem. For a more detailed survey of inefficiencies that arise in credence goods markets, see Dulleck and Kerschbamer (2006).

My paper differs from the cited articles on credence goods in two regards . I assume that the customer does not know the sellers' costs. The resulting opportunity to earn rents gives the sellers additional incentives to cheat on customers. The more important difference is that instead of analyzing how competition affects the sellers' incentives to defraud, I allow the customer to offer contracts which counter-

[2] See also Dulleck and Kerschbamer (2006) for a further discussion of the *Verifiability* assumption.

act adverse incentives of the sellers. In my model, there are exactly two specialists which compete against each other. This setup corresponds to problems which occur only rarely and hence only a very limited number of firms has the prerequisites to be able to solve it.

The methods employed to derive the optimal contracts draw on a second strand of literature. In my model sellers have two-dimensional private information (on costs and the superior treatment) which requires methods developed in the literature on multidimensional screening (e.g. Rochet and Stole (2003)) and multidimensional auctions (Armstrong 2000). To derive optimal contracts I solve similar to Armstrong (2000) a relaxed model without some of the incentive constraints first and then verify ex post that they are satisfied. In contrast to Armstrong (2000) the experts in my model have correlated information.

This paper is organized as follows: In the next section, I introduce the formal model and then present the solution technique in section 4. I suppose in section 5 as benchmark that the contractor knows the firms' signals of fit and derive the optimal contract. Afterwards, I turn again to the base model and define a relaxed problem by ignoring incentive constraints that are likely to be satisfied. After having characterized solutions of the relaxed case in section 6, I derive under which conditions these solutions carry over to the fully fletched model in section 7. I also derive solutions when incentive constraints that were ignored in the relaxed case bind. The main results are stated under the simplifying assumption that the surplus of the project is high enough so that the contractor finds it always optimal to assign the project to some firm. This assumption facilitates to introduce the method of solving the model and to convey the intuition of the results. Part A of the appendix contains a general analysis and shows that all results carry over if the project generates less surplus.

3 The model

A risk neutral contractor (or principal) needs to carry out a task and thereby would earn the payoff S. There are two risk neutral firms (or sellers) which are able to carry it out but each firm is specialized on a different approach for the task at hand. I assume that always one of both approaches is superior for the contractor's task and would increase the contractor's payoff by $\eta \in \mathbb{R}^+$ if chosen. Which approach is superior is modeled by a fit variable $F_i \in \{0, 1\}$ which takes on the value of 1 if firm i offers the superior approach and 0 otherwise. As exactly one firm offers the superior approach, we always have $f_1 + f_2 = 1$.[3] The contractor does not know which approach is superior.

Both firms possess two-dimensional private information: Each firm knows the costs to carry out its approach, which are either low (c_l) or high (c_h) with probability

[3] I use capital letters for random variables and lower case ones for realizations

β and $1-\beta$ respectively. The difference between high and low costs is δ. Each seller does not know the other firm's cost and both cost realizations are independent.[4] To save on notation, define the net value as $V_i := S - C_i$. I denote the net value of a firm as high (low) costs as L (H). As the difference between low or high net value comes from the difference in costs, we still have $H - L = \delta$. Apart from cost information, every firm receives a noisy signal $X_i \in \{0, 1\}$ that indicates if its approach is superior for the contractor's task. The probability that a firm receives a correct signal is independent of the corresponding fit variable:

$$\Pr(X_i = f_i | f_i) = 0.5 + 0.5\gamma \qquad \gamma \in (0, 1]$$

γ measures the quality of the signal. (In case of $\gamma \to 0$ firms receive white noise; γ of 1 captures complete information). Note that the realizations of the signals are negatively correlated as $\gamma > 0$ and that the realizations of the fit-signals are independent of the realizations of the costs. I define both firms' signals to be *consistent* if exactly one firm has the signal of having a superior approach, i.e. they correspond to a true fit vector that occurs with positive probability.

Definition 1. The signals (x_1, x_2) are **consistent** if $x_1 + x_2 = 1$

In summary, a firm's type θ_i consists of its value and its fit signal: $\Theta_i = V_i \times X_i$. Note that due to the correlated information structure, firms are likely to have consistent signals. Straight forward computation yields the following ex-ante probabilities $\alpha_{\theta_1,\theta_2}$ of the firms being of type θ_1 and θ_2:[5]

$$\alpha_{\theta_1,\theta_2} = \begin{cases} \frac{1}{4}\Pr(v_1)\Pr(v_2)\left(1+\gamma^2\right) & \text{if signals are consistent} \\ \frac{1}{4}\Pr(v_1)\Pr(v_2)\left(1-\gamma^2\right) & \text{if signals are inconsistent} \end{cases} \tag{1}$$

Before employing the firms, the contractor can induce them to reveal their private information. Contingent on the acquired information, the contractor can commit to a (certain) payment $m_i : \Theta_i \times \Theta_j \to \mathbb{R}$ and an probability $q_i : \Theta_i \times \Theta_j \to [0, 1]$ to employ firm i and not firm $j \neq i$.[6][7] I assume that the contractor can make payments and the employment probability for firm i contingent on information from firm 1 and firm 2. The contractor can in principle even reward a firm although he eventually employs the other one. I denote the stage where firms have possibly revealed their private information but before payments are made and the task is carried out as the *ex-interim stage*.

Another important assumption is that once firms have (truthfully) revealed their information, each of them can back out and then neither receives a payment nor

[4] Although the independence assumption is mainly made for simplicity, it can be justified if the costs reflect opportunity costs determined by capacity constraints which randomly bind or not.

[5] $\Pr(V_i)$ denotes the probability that $V_i = v_i$.

[6] Note that the first argument of both the employment probability and the payment is the type of the firm to which it accrues. The second argument is the type of the other firm.

[7] As the firms are risk neutral, it does not matter if the contract specifies a payment conditional on a firm carrying out the task or if it specifies an unconditional payment.

has to solve the customer's problem, which gives them a payoff normalized to zero. Hence to prevent agents from backing out, the expected payoffs must be nonnegative. Technically, the payment/employment scheme must be ex-interim individually rational.[8] An equivalent assumption would be that both firms have limited liability and cannot have ex-interim payoffs below zero. The contractor cannot make payments conditional on the bonus he receives from choosing the superior approach.

I denote by r_i the ex-interim payoff (or rent) of firm i before the task is carried out but after both firms have had the opportunity to pass on their information to the contractor. It consists of the monetary compensation that the firm receives from the contractor minus the probability weighted costs of solving the customer's problem. Note that I allow rents to be paid to a firm although its employment probability is zero.

$$r_i(\theta_1, \theta_2) = m_i(\theta_1, \theta_2) - q_i(\theta_1, \theta_2)c_i$$

The contractor's ex-interim payoff U_c is the sum of the probability weighted payoff from the project minus the payments to the firms:

$$\begin{aligned} U_c(\theta_1, \theta_2) = {} & q_1(\theta_1, \theta_2)\left(S + \Pr(F_1 = 1|x_1, x_2)\eta\right) \\ & + q_2(\theta_1, \theta_2)\left(S + \Pr(F_2 = 1|x_1, x_2)\eta\right) - m_1(\theta_1, \theta_2) - m_2(\theta_1, \theta_2) \end{aligned}$$

I define the *gross value* $\pi_i(\theta_1, \theta_2) \equiv v_i + Pr(F_i = 1|x_1, x_2)\eta$ to comprise the net value plus the expected payoff for the right fit given the signals of both firms. Instead of employment probabilities and payments, a contract can equivalently consist of each firm's employment probability and rent $r_i : \Theta_i \times \Theta_j \to \mathbb{R}^+$ contingent on the announced type of both firms. The contractor's expected payoff is then:

$$\begin{aligned} U_c(\theta_1, \theta_2) = {} & q_1(\theta_1, \theta_2)\pi_1(\theta_1, \theta_2) + q_2(\theta_1, \theta_2)\pi_2(\theta_1, \theta_2) \\ & - r_1(\theta_1, \theta_2) - r_2(\theta_1, \theta_2) \end{aligned} \tag{2}$$

The objective of the contractor is to find a scheme of payments and employment probabilities that maximizes the contractor's ex-ante expected payoff.

4 Optimal contracting schemes

I restrict attention to contracting schemes that are anonymous and truthfully implementable in Bayesian Nash equilibrium (BNE). The revelation principle for Bayesian Nash implementation ensures that it is impossible to implement schemes in BNE which yield a higher expected payoff than truthfully implementable ones. With respect to this model a scheme is *anonymous* if payments and employment probabilities are independent of the index of an agent.

Definition 2. A scheme is **anonymous** if it satisfies $q_1(\hat{\theta}, \tilde{\theta}) = q_2(\hat{\theta}, \tilde{\theta}) = q(\hat{\theta}, \tilde{\theta})$ and $m_1(\hat{\theta}, \tilde{\theta}) = m_2(\hat{\theta}, \tilde{\theta}) = m(\hat{\theta}, \tilde{\theta}) \quad \forall \hat{\theta}, \tilde{\theta} \in \Theta_i$.

[8] Note that this assumption rules out the application of the mechanism suggested by Cremer and McLean (1988), which would render the firms' private information concerning the fit worthless.

The restriction to anonymous schemes simplifies the analysis considerably. It seems reasonable for the contractor to offer both firms the same scheme because the assumption of a symmetric prior means that ex-ante both firms seem equally well suited to perform the task.

An optimal contracting scheme maximizes the contractor's expected payoff (2) subject to the resource, incentive and participation constraints, which are described below.

The resource constraints capture the assumption that after the firms have revealed their private information the task can be contracted at most once:

$$q(\theta_1, \theta_2) + q(\theta_2, \theta_1) \leq 1 \qquad \forall \theta_1, \theta_2 \in \Theta_i \tag{3}$$

If both firm announce to be of the same type, then the restriction to anonymous schemes implies that both firms are employed with equal probability.

$$q(\hat{\theta}, \hat{\theta}) \leq 0.5 \qquad \forall \hat{\theta} \in \Theta_i \tag{4}$$

The incentive constraints ensure that truth telling is optimal for each firm, given that the other firm acts truthfully. This requires:

$$E\left[r_i(\theta_i, \tilde{\theta}_j)|\theta_i\right] \geq E\left[r_i(\hat{\theta}_i, \tilde{\theta}_j) + (v_i - \hat{v}_i) q_i(\hat{\theta}_i, \tilde{\theta}_j)|\theta_i\right] \qquad \forall \hat{\theta}_i \in \Theta_i \tag{5}$$

Condition (5) states that once each firm knows its own type, telling the truth must be weakly preferred to lying for any type it may have. Multiplying (5) by the probability that a firm has a given type, these conditions can also be expressed in terms of ex ante probabilities $\alpha_{\theta_1,\theta_2}$:

$$\sum_{(\theta_i,\theta_j)\in\Theta} \alpha_{\theta_i,\theta_j} r_i(\theta_i, \theta_j) \geq \sum_{(\theta_i,\theta_j)\in\Theta} \alpha_{\theta_i,\theta_j} [r_i(\hat{\theta}_i, \theta_j) + (v_i - \hat{v}_i) q_i(\hat{\theta}_i, \theta_j)] \qquad \forall \hat{\theta}_i \in \Theta_i \tag{6}$$

As the type space of a firm contains four elements, there are three incentive constraints for every type and hence 12 incentive constraints in total. Figure 1 illustrates the incentive constraints.[9] I denote incentive constraints that prevent high (low) net value firms from pretending to have low (high) value as *downward (upward)* incentive constraints and those that prevent firms from lying only with respect to the fit-signal as *horizontal* incentive constraints. I address a single incentive constraint that prevents a firm of type θ_i from announcing being type $\hat{\theta}_i$ as incentive constraint $(\theta_i \rightarrow \hat{\theta}_i)$.

The ex-interim participation constraints ensure that after all firms have revealed their type and hence a payment and employment probability is assigned to each firm, every firm weakly prefers to accept the offer than to back out:

$$U_i(\theta_i, \theta_j) = r_i(\theta_i, \theta_j) \geq 0, \qquad \forall (\theta_i, \theta_j) \in \Theta_i \times \Theta_i \tag{7}$$

[9]Each dashed arrow represents a constraint that must be satisfied.

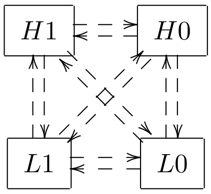

Figure 1: Incentive constraints

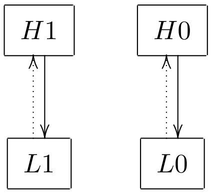

Figure 2: Benchmark incentive constraints

If all ex-interim participation constraints hold, then firms are willing to accept the contract also before they know their own type. The reason is that whatever their type will be, the ex-interim participation constraints ensure they will be better off than with their outside option.

5 The benchmark case

To isolate the effect of the firms' private information on the optimal approach, I use the case where the contractor observes the fit-signal of both firms as a benchmark.[10] I maintain the assumption that each firm does not observe its rival's type. Note that all players still have incomplete information regarding the fit. Under these modified assumptions, the contractor does not have to induce the firms to reveal their signal of fit truthfully. Hence he only needs to ensure that both firms truthfully announce their net value. Figure 2 depicts the remaining incentive constraints, where dotted lines represent constraints that are slack. Technically, the benchmark assumptions guarantee that there remains only one dimension of uncertainty in the model. Thus, the following incentive constraints have to hold:

$$E\left[r_i((H,x_i),\tilde{\theta})|\theta_i\right] \geq E\left[r_i((L,x_i),\tilde{\theta}) + \delta q_i((L,x_i),\tilde{\theta_j})|\theta_i\right] \tag{8}$$

$$E\left[r_i((L,x_i),\tilde{\theta})|\theta_i\right] \geq E\left[r_i((H,x_i),\tilde{\theta}) - \delta q_i((H,x_i),\tilde{\theta_j})|\theta_i\right] \tag{9}$$

Before analyzing the properties of optimal schemes of the benchmark case, define $R(\theta_i) \equiv E[r_i(\theta_i,\tilde{\theta}_j)|\theta_i]$.[11] Because the asymmetric information has been reduced to a single dimension, the benchmark scheme can be derived with standard techniques.

[10]To keep the benchmark case similar to the full blown model, I assume that the contractor learns the firms' signal after they have revealed their net value.

[11]Note that in contrast to the original problem, there is no need to define rents $r(\theta_i, \theta_j)$ that depend on both firms' type. The reason is that the firms do not have to be prevented from lying with respect to their fit signal in the benchmark case.

The simplifying assumption 1 ensures that the surplus of the project is high enough so that one firm is always hired regardless of the combination of firms' types. Technically, I assume that the value is high enough for all resource constraints to hold with equality. Lemma 1 characterizes generic solutions of the optimal benchmark schemes.

Assumption 1. The net value L is satisfies $L \geq \frac{\delta\beta}{1-\beta} - \frac{\eta}{2}$. In addition, picking the right approach yields strictly positive extra payoff $\eta > 0$.

Lemma 1. *Generically, any optimal scheme of the benchmark case is characterized as follows:*

i) No rents are paid to firms with a low net value.

ii) Constraint (8) binds for firms with good or bad signal of fit and $R(H, x_i) = \delta E[q_i((L, x_i), \tilde{\theta}_j)|x_i]$.

iii) If both firms provide the same net value, the firm with lower probability of fit is never employed: $q((\tilde{v}_i, 0), (\tilde{v}_i, 1)) = 0 \quad \tilde{v}_i \in L, H$.

iv) If both firms announce to be the same type then they are employed with probability 0.5: $q((\tilde{\theta}_i), (\tilde{\theta}_i) = 0.5$

v) If $\eta > \frac{\delta(1+\gamma^2)}{2\gamma(1-\beta)}$ *then* $q(L1, H0) = 1, q(H0, L1) = 0$ *and otherwise* $q(L1, H0) = 0$, $q(H0, L1) = 1$.

Proof. See appendix. □

The results with respect to the firms' private information of net value parallels the well known results of one-dimensional adverse selection: High value firms have to be paid rents that they could earn in expectation by pretending to have low net value. Part ii) of Lemma 1 states that the expected rents a firm with high value and signal x_i receives is proportional to the probability that a firm with the same signal but low net value would be employed. Define the virtual profits $\psi(\theta_i, \theta_j)$ as the gross profits adjusted for the induced expected rent payments as follows:

$$\psi(\theta_i, \theta_j) = \begin{cases} \pi(\theta_i, \theta_j) & if \quad x_i = H \\ \pi(\theta_i, \theta_j) - \delta \frac{\alpha_{(H,x_i),\theta_j}}{\alpha_{\theta_i,\theta_j}} & if \quad x_i = L \end{cases}$$

Effectively, the contractor offers contracts that maximize the sum of expected virtual profits. As the virtual profits of employing low value firms are lower than the gross profits, for a large set of parameters the contractor employs low net value firms less often than would be socially efficient. Also consistent with standard theory, there arises no allocative distortion at the top for $H1$ firms.[12]

Parts (iii) to (v) Lemma 1 establish that the contractor assigns the project efficiently with respect to the signal of fit dimension. By part iii), if two firms offer the same net value the contractor never employs firms that have a low probability

[12] However, if $\pi(L1, H0) > \pi(H0, L1) > 0$ and $\psi(L1, H0) < \psi(H0, L1)$, then $H0$ firms are employed to often compared to the first best.

of fit. This property arises because the contractor does not have to pay rents to acquire information regarding fit.

Note that the contractor finds it optimal in the benchmark case to employ low net value firms with a good fit strictly more often than those with a bad signal of fit. This directly translates into higher rents that have to be paid to $H1$ firms compared to $H0$ firms. As lying with respect to the signal is free of costs for the firms, the contractor cannot maintain this wedge of rents if firms have to be induced to announce truthfully their signal of fit. This creates additional distortions which are examined in the next section.

6 The relaxed problem

Following the standard approach, I first solve a relaxed version of the problem, which involves only a subset of the incentive constraints and check afterwards if the remaining constraints are satisfied. Specifically, I neglect upward constraints, i.e. constraints that prevent low net value firms to pretend being high value ones. Besides I will ignore horizontal constraints of high net value firms at first and then show that they are always satisfied for optimal contracts.[13] Figure 3 shows the downward constraints that may or may not bind in the relaxed case.

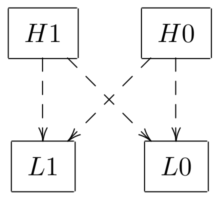

Figure 3: Relaxed case incentive constraints

The main difference compared to the benchmark case is that in the relaxed case, high net value firms may be induced to misreport their type in various directions. A firm can either lie with respect to costs or with respect to the signal or with respect to both dimensions.

To prompt a firm of type θ_i to reveal its type truthfully, its expected rents for telling the truth must weakly exceed the maximum of expected rents it would gain by deviating in any direction as captured in condition (10). This implies directly that for any fixed allocation $q(\cdot)$ the expected rents are weakly higher than in the benchmark case.

$$\sum_{\theta_j \in \Theta_j} \alpha_{\theta_i,\theta_j} r_i(\theta_i, \theta_j) \geq \max_{\theta_j \in \Theta_j} \left\{ \sum_{\theta_j \in \Theta_j} \alpha_{\theta_i,\theta_j} [r_i(\hat{\theta}_i, \theta_j) + (v_i - \hat{v}_i) q_i(\hat{\theta}_i, \theta_j)] \right\} \tag{10}$$

[13]Note that some of these constraints may hold with equality for optimal contracts, however, relaxing them would not change the optimal allocation.

More specifically, in the benchmark case low net value firms with a bad signal of fit are never employed if the other firm has reported a good fit. Under assumption 1 this means that low net value firms with good signal are more often employed than those with a bad one. Hence in any scheme of the benchmark case an $H0$ firm would gain strictly higher rents by pretending to be of type $L1$ than to be of type $L0$. This means that the contractor would have to pay strictly higher rents to a high value firm with bad fit signal in order to implement the optimal allocation of the benchmark case. In order to economize on rent-payments, the contractor may prefer to distort the allocation and to employ low net value firms with good fit less often then in the benchmark case. The analysis will show, that depending on the parameters, there may arise allocations such that all four constraints of figure 3 bind.

The following Lemma establishes that all rents for low types are zero. As there are no upward constraints considered in the relaxed problem there is no need to deter low value firms from mimicking high value ones. While there are no positive effects, paying rents to low value firms requires also higher rents to be paid to high value firms.

Lemma 2. *In any optimal scheme,* $r((L, x_i), \theta_j) = 0 \quad \forall x_i \in 0, 1, \theta_j \in \Theta_i$.

Proof. Suppose to the contrary that there exists a solution with $r((L, x_i), \theta_j) > 0$. Set $r((L, x_i), \theta_j) = 0 \quad \forall x_i \in \{0, 1\}, \theta_j \in \Theta_i$. Then all downward incentive constraints are still satisfied, horizontal incentive constraints for high value firms are unaffected and horizontal incentive constraints for low value firms are satisfied as all such firms earn zero rents. Since U_c increases by the change, this contradicts optimality. □

Lemma 3 suggests the most effective rent scheme to prevent firms from lying with respect to the signal of fit dimension. This is done by paying rents only if both firms report consistent fit signals. Intuitively, if a firm lies while the other reports truthfully, then due to the correlated signals chances are high that the reported signals are inconsistent. By paying no rents in this case, firms have little incentives to lie as this leads to zero rents with high probability.

Lemma 3. *There always exists an optimal scheme that involves* $r((v_i, x_i), (v_j, x_j)) = 0 \quad$ *if* $\quad x_i + x_j \neq 1$.

Proof. See appendix. □

Building on Lemma 3, I concentrate on schemes that involve paying no rents if the announced signals are inconsistent. As each firm's net value is independent from both the fit signals and the other firm's net value, I can further restrict attention to schemes that only pay rents if both firms provide high net value.[14] Thus in the

[14]Technically, due to risk neutrality, a sufficient statistic for the payoff of a firm with respect to the rent payments $r(\cdot)$ is the expected rent paid for consistent reports: $\alpha_{(H,x_i),(L,x_j)} r((H, x_i), (L, x_j)) + \alpha_{(H,x_i),(H,x_j)} r((H, x_i), (H, x_j)) \quad x_i \in 0, 1, x_j = 1 - x_i$.

following, rents are only paid if the firms announce $(H0, H1)$ or $(H1, H0)$. Now the simplified relaxed problem looks as follows:

$$\max_{r,q} \mathrm{E}_\theta[U_c] - \alpha_{H1,H0} r(H1, H0) - \alpha_{H0,H1} r(H0, H1) \tag{11}$$

such that the following incentive constraints and the resource constraints (3) are satisfied.

$$\sum_{\theta_j \in \Theta_j} \left(\delta \alpha_{H1,\theta_j} q(L1, \theta_j)\right) - \alpha_{H1,H0} r(H1, H0) \leq 0 \tag{12}$$

$$\sum_{\theta_j \in \Theta_j} \left(\delta \alpha_{H1,\theta_j} q(L0, \theta_j)\right) - \alpha_{H1,H0} r(H1, H0) \leq 0 \tag{13}$$

$$\sum_{\theta_j \in \Theta_j} \left(\delta \alpha_{H0,\theta_j} q(L1, \theta_j)\right) - \alpha_{H0,H1} r(H0, H1) \leq 0 \tag{14}$$

$$\sum_{\theta_j \in \Theta_j} \left(\delta \alpha_{H0,\theta_j} q(L0, \theta_j)\right) - \alpha_{H0,H1} r(H0, H1) \leq 0 \tag{15}$$

$$\alpha_{H0,H0} r(H1, H0) - \alpha_{H0,H1} r(H0, H1) \leq 0 \tag{16}$$

$$\alpha_{H1,H1} r(H0, H1) - \alpha_{H1,H0} r(H1, H0) \leq 0 \tag{17}$$

The following Lemma establishes that in any optimal scheme it is weakly more attractive for high value firms to pretend being type $L1$:

Lemma 4. *In any optimal scheme, the incentive constraints* $(H1 \rightarrow L1)$ *and* $(H0 \rightarrow L1)$ *hold with equality.*

Proof. See appendix. □

Intuitively, Lemma 4 uses the fact that given the other firm's signal, the gross profit π from employing a firm with good fit signal is higher than with bad signal of fit. The incentive constraints of Lemma 4 could only be slack if the contractor often employed low net value firms without fit and thus had to pay high rents to prevent high net value firms from lying. But in this case, it would be profitable to employ also low net value firms with fit because little additional rents would have to be incurred until both constraints (12) and (14) bind again. Lemma 4 implies that in any optimal solution it is weakly more attractive for both the $H1$ and the $H0$ firm to pretend being type $L1$ instead of being type $L0$. Hence in any optimal scheme, the rents can be computed as follows:

$$\alpha_{H1,H0} r(H1, H0) = \sum_{\theta_j \in \Theta_j} \delta \alpha_{H1,\theta_j} q(L1, \theta_j) \tag{18}$$

$$\alpha_{H0,H1} r(H0, H1) = \sum_{\theta_j \in \Theta_j} \delta \alpha_{H0,\theta_j} q(L1, \theta_j) \tag{19}$$

Equations (18) and (19) directly indicate the classical result that given any allocation, the rents depend linearly on the level of uncertainty δ with respect to the net value. Hence, if there is no uncertainty concerning the costs of the project, then no firm is able to extract any rents and the private information concerning the fit becomes worthless.

While so far all results of this section are valid for general parameters, proposition 1 restricts attention to the case where the surplus of the project is sufficiently high for the project to be always carried out regardless of the firms' type. Again, the qualitative results of this proposition fully carry over to general parameters. Assumption 2 guarantees that the virtual profits are positive for any possible combination of types. Note that in the relaxed problem, the contractor has to pay weakly higher rents than in the benchmark case and therefore assumption 2 requires higher surplus of the projects than assumption 1.

Assumption 2. The net value L satisfies $L \geq \frac{2\beta\delta}{(1-\beta)(1-\gamma^2)} - \frac{\eta(1-\gamma)^2}{2(1-\beta)}$ Picking the right approach yields strictly positive extra payoff $\eta > 0$.

Proposition 1. *Suppose assumption 2 holds. Then projects are optimally assigned in the relaxed problem as follows:*

i) If both firms have identical type, then they are employed with probability $\frac{1}{2}$*:* $q(H1, H1) = q(H0, H0) = q(L1, L1) = q(L0, L0) = \frac{1}{2}$.

ii) If both firms receive the same signal of fit but differ in net value, then the firm with the higher net value is employed: $q(H1, L1) = q(H0, L0) = 1$ *and* $q(L1, H1) = q(L0, H0) = 0$.

iii) The contractor always employs high value firms which are more likely to offer the right approach: $q(H1, H0) = q(H1, L0) = 1$ *and* $q(H0, H1) = q(L0, H1) = 0$.

iv) If $\eta \leq \frac{\delta\beta}{(1-\beta)\gamma}$ *then* $q(H0, L1) = 1, q(L1, H0) = 0, q(L1, L0) = q(L0, L1) = 0.5$.

v) If $\frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right] \geq \eta > \frac{\delta\beta}{(1-\beta)\gamma}$ *then* $q(H0, L1) = q(L1, L0) = 1$ *and* $q(L1, H0) = q(L0, L1) = 0$.

vi) If $\eta > \frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right]$ *then* $q(L1, H0) = q(L1, L0) = 1$ *and* $q(H0, L1) = q(L0, L1) = 0$.

Proof of Proposition 1. The proof of this and the following propositions are mainly based on the duality theorem.[15] Note that by the dual conditions associated with $r(H1, H0)$ and $r(H0, H1)$, the dual variables of the incentive constraints must satisfy: $\lambda_{(H,x_i)\to(L,y_i)} \leq 1 \quad \forall x_i, y_i \in \{0, 1\}$.

Part i)

Assumption 2 assures that $\alpha_{(L,x_i),(L,y_i)}\pi((L, x_i), (L, y_i)) - \delta\alpha_{H1,x_i} - \delta\alpha_{H0,(L,x_i)} \geq 0 \quad \forall x_i, y_i \in \{0, 1\}$ which implies that employment probabilities for all type-combinations add up to one. In particular the dual constraints $\alpha_{(L,x_i),(L,x_i)}\pi((L, x_i), (L, x_i)) - \delta\lambda_{H1\to(L,x_i)}\alpha_{H1,(L,x_i)} - \delta\lambda_{H0\to(L,x_i)}\alpha_{H0,(L,x_i)} \geq 0 \quad \forall x_i \in \{0, 1\}$ are satisfied. This implies that the dual constraints for high value firms with equal type are also satisfied.

Part ii)

[15] see e.g. Luenberger (1989)

The dual conditions of part i) hold, imply that the dual constraints $\alpha_{(H,x_i),(L,x_i)}\pi((H,x_i),(L,x_i)) \geq \alpha_{(L,x_i),(H,x_i)}\pi((L,x_i),(H,x_i)) - \delta\lambda_{H1\to(L,x_i)}\alpha_{H1,(H,x_i)} - \delta\lambda_{H0\to(H,x_i)}\alpha_{H0,(H,x_i)}$ $\forall x_i \in \{0,1\}$ are also satisfied.

Part iii)

$\alpha_{H1,\theta_j}\pi(H1,(L,\theta_j)) \geq \alpha_{\theta_j,H1}\psi(\theta_j,H1)$ $\forall\theta_j \in L0,H0$ as $\pi(\theta) \geq \psi(\theta)$ and $\pi(H1,L0) > 0$ by assumption 2.

Part iv)

Note that the allocation assures together with (19) and (18) that all four incentive constraints (12)-(15) hold with equality and hence their dual variables may be nonzero. Choose $\lambda_{H1\to L1} = \frac{\alpha_{L1,L0}[\pi(L1,L0)-\pi(L1,L0)]}{\delta[\alpha_{H1,L0}+\alpha_{H0,L0}]}$, $\lambda_{H1\to L0} = 1-\lambda_{H1\to L1}$, $\lambda_{H0\to L0} = 0$ and $\lambda_{H0\to L1} = 1$. The hypothesis $\eta \leq \frac{\delta\beta}{(1-\beta)\gamma}$ is equivalent to $\alpha_{L1,L0}[\pi(L1,L0) - \pi(L0,L1)] \leq \delta\alpha_{H1,L0} + \delta\alpha_{H0,L0}$ and hence $\lambda_{H1\to L1} \leq 1$. $\pi(L1,L0) > \pi(L0,L1)$ always holds and thus $\lambda_{H1\to L1} > 0$. The dual variables are constructed such that the dual condition $\alpha_{L1,L0}\pi(L1,L0) - \delta\lambda_{H1\to L1}\alpha_{H1,L0} - \delta\lambda_{H0\to L1}\alpha_{H0,L0} = \alpha_{L1,L0}\pi(L0,L1) - \delta\lambda_{H1\to L0}\alpha_{H1,L1} - \delta\lambda_{H0\to L1}\alpha_{H1,L0}$ is always satisfied. As $\pi(H0,L1) = \pi(L0,L1)+\delta$, this implies that the dual condition for $q(H0,L1) = 1$ is also satisfied.

Part v)

As the allocation only involves the constraints $(H1 \to L1)$ and $(H0 \to L1)$ binding, $\lambda_{H1\to L1} = \lambda_{H0\to L1} = 1$. The hypothesis $\eta > \frac{\delta\beta}{(1-\beta)\gamma}$ is equivalent to $\alpha_{L1,L0}[\pi(L1,L0) - \pi(L0,L1)] > \delta\alpha_{H1,L0} + \delta\alpha_{H0,L0}$ and hence the dual condition for $q(L1,L0) = 1$ is satisfied. The second part of the hypothesis is equivalent to $\alpha_{L1,H0}[\pi(L1,H0) - \pi(H0,L1)] \leq \delta\alpha_{H1,H0} + \delta\alpha_{H0,H0}$ and hence the dual condition for $q(H0,L1) = 1$ is satisfied.

Part vi)

The hypothesis is equivalent to $\alpha_{L1,H0}[\pi(L1,H0)-\pi(H0,L1)] > \delta\alpha_{H1,H0}+\delta\alpha_{H0,H0}$ and hence the dual condition for $q(L1,H0) = 1$ is satisfied. This implies that the dual condition for $q(L1,L0) = 1$ must also hold. □

Proposition 1, Part i) states that whenever both firms report the same net value and the same signal of fit, each firm is charged with the project with 50% probability. This result is driven by assumption 2 which restricts the project to be profitable enough.

There is no downward distortion at the top as established in Part ii) and iii) of Proposition 1. Intuitively, in the relaxed problem the contractor does not have to incur costs to prevent low value firms from mimicking high value ones. Hence if a high value firm yields a higher gross profit than employing the second firm, it should be optimally employed.

There are two broad classes of solutions. If picking the right approach does not yield high additional value, then Proposition 1, Part iv) applies. In this case the contractor optimally commits not to take the private information with respect to the signal of fit into account. The reason is that the rents paid to a firm equal the maximum of rents it would obtain by not revealing its type truthfully. Thus the contractor can economize on rents if he commits to employ low value firms with equal probability which renders high value firms indifferent between pretending to have low value with or without fit. Formally, this class of solution involves ($H1 \rightarrow L0$) and ($H0 \rightarrow L0$) holding with equality as shown in figure 4.[16] This allocation equals those of a classical second prize auction with two bidders and independent private net value $v_i \in \{L, H\}$.[17]

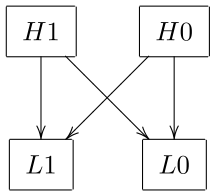

Figure 4: Binding Incentive constraints if fit is less important

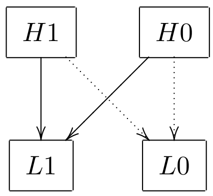

Figure 5: Binding/Slack incentive constraints if fit is important

The second class arises if the fit is important enough as characterized in Part v) and vi) of the proposition. Then it is optimal for the contractor to employ mostly firms that are likely to offer the right approach. In particular this means that low net value firms that report a good fit are more often employed than those with bad fit. This implies that high net value firms may reap higher rents by pretending to be type $L1$ rather than $L0$. Formally, this class of solution involves (13) and (15) to be slack as shown in figure 5. As a consequence, employing a $L1$ firm is very expensive at the margin, as this causes further rents to be paid to both $H1$ and $H0$ firms. If the right approach is extremely important as in part iii), the contractor may even prefer to employ low value firms with high fitting probability instead of employing high value firms without fit. For intermediate values of the bonus as described in part iv), it may be optimal to employ high value firms with bad fit in order to save on rents and gain net value. But in this case, $L1$ firms are still employed often enough so that the incentive constraints (13) and (15) are slack.

Taken together, the contractor puts less weight on the firm's announcement of its signal of fit when assigning its employment probability. If the bonus of the right approach is low enough, the probability that a firm is employed depends besides the

[16]Solid arrows indicate binding constraints, pointed arrows slack ones.

[17]In this class of solution, rents could be also paid as in a second price auction.

net values of the firms only on the fit announcement of the other firm. Intuitively, if the employment probability of firm i (positively) depends on its own announcement, then higher rents have to be paid to prevent firms with no fit from lying. On the other hand, using firm j's announcement to determine the employment probability of firm i does not induce adverse incentives for firm i and is thus cheaper information. This important conclusion is summarized by Corollary 1:

Corollary 1. *Suppose assumption 2 holds. Then employment probabilities of firms that announce a good(bad) fit signal are weakly lower(higher) in the proposed optimal scheme of the relaxed problem compared to the optimal scheme of the benchmark case.*

Proof. The employment schemes of the benchmark case and the relaxed problem possibly differ with respect to $q(L1,H0), q(H0,L1), q(L1,L0), q(L0,L1)$. For $\eta \in \left[\frac{\delta(1+\gamma^2)}{2\gamma(1-\beta)}, \frac{\delta\left[(1+\gamma^2)+\beta(1-\gamma^2)\right]}{2\gamma(1-\beta)}\right)$, in the benchmark case $q(L1,H0) = 1$, $q(H0,L1) = 0$ while in the relaxed problem $q(L1,H0) = 0$, $q(H0,L1) = 1$. They are equal otherwise.
In the benchmark case, irrespective of the parameters $q(L1,L0) = 1$, $q(L0,L1) = 0$ while in the relaxed problem for $\eta \leq \frac{\delta\beta}{(1-\beta)\gamma}$, $q(L1,L0) = q(L0,L1) = 0.5$ □

7 The full problem

Solutions of the relaxed problem may also be solutions of the full problem, if they satisfy the incentive constraints that were ignored in the relaxed problem. The following inequality constraints (20)-(23) describe in this order the upward incentive constraints $(L1 \to H1)$, $(L0 \to H1)$, $(L1 \to H0)$, $(L0 \to H0)$:

$$\alpha_{L1,H0}\left[r(H1,H0) - r(L1,H0)\right] - \sum_{\theta_j \in \Theta_j} \left(\delta\alpha_{L1,\theta_j} q(H1,\theta_j)\right) \leq 0 \tag{20}$$

$$\alpha_{L0,H0}\left[r(H1,H0) - r(L0,H0)\right] - \sum_{\theta_j \in \Theta_j} \left(\delta\alpha_{L0,\theta_j} q(H1,\theta_j)\right) \leq 0 \tag{21}$$

$$\alpha_{L1,H1}\left[r(H0,H1) - r(L1,H0)\right] - \sum_{\theta_j \in \Theta_j} \left(\delta\alpha_{L1,\theta_j} q(H0,\theta_j)\right) \leq 0 \tag{22}$$

$$\alpha_{L0,H1}\left[r(H0,H1) - r(L0,H1)\right] - \sum_{\theta_j \in \Theta_j} \left(\delta\alpha_{L0,\theta_j} q(H0,\theta_j)\right) \leq 0 \tag{23}$$

Lemma 5 establishes that whenever an optimal solution of the relaxed problem satisfies the incentive constraint (23), it also satisfies all other constraints of the full problem.

Lemma 5. *Any optimal solution of the relaxed problem that satisfies the incentive constraint $(L0 \to H0)$ is also an optimal solution of the full problem.*

Proof. See Appendix □

Lemma 5 builds on the fact that any optimal solution of the relaxed problem also satisfies the constraints ($L1 \rightarrow H1$) and ($L0 \rightarrow H1$). ($L1 \rightarrow H1$) is satisfied because given the other firm's type and the signal of fit, firms that offer a high net value are more often employed then those with a low net value. ($L0 \rightarrow H1$) is satisfied because ($L0$) firms pretending to have high net value and a good fit will be rewarded the project very often and thus would have to incur high costs. ($L1 \rightarrow H0$) holds if ($L0 \rightarrow H0$) is satisfied because ($L1$) firms pretending to have high net value and a good fit will be paid rents only if the other firm reports a good fit signal, and it is unlikely that both firms receive a good signal of fit.

By Lemma 5, the most critical incentive constraint is ($L0 \rightarrow H0$). It may be violated if the optimal solution of the relaxed case involves $L1$ firms to be employed more often than $H0$ firms. Intuitively in this case high rents have to be paid to $H0$ firms and their employment probability is low. Low employment probabilities imply that low net value firms need not incur mimicking costs often. This makes it attractive for $L0$ firms to pretend having high net value in order to reap the rents, as low value firms would not earn rents otherwise.

Plugging equation (19) that determines $r(H0, H1)$ in the incentive constraint (23) and using $\frac{1-\beta}{\beta}\alpha_{(H,x_i),\theta_j} = \alpha_{(L,x_i),\theta_j}$ yields condition (24).

$$\begin{aligned}\alpha_{L0,L0}\left[q(L1,L0) - q(H0,L0)\right] + \alpha_{L0,L1}\left[q(L1,L1) - q(H0,L1)\right] \\ +\alpha_{L0,H0}\left[q(L1,H0) - q(H0,H0)\right] \leq 0\end{aligned} \tag{24}$$

Any optimal solution of the relaxed case that additionally satisfies this condition also solves the fully constrained problem. Effectively, inequality (24) imposes an upper bound on the difference between rents paid to low and high net value firms respectively. Plugging the schemes proposed in section 6 into condition (24) yields the following result:

Corollary 2. *Suppose assumption 2 holds. The solution of the reduced problem also solves the full problem if additionally* $\eta \leq \frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right]$.

Proof. Verifying if the optimal allocations of Proposition 1 satisfy condition (24) yields the result. □

If solutions of the relaxed problem violate inequality (24), all incentive constraints described by (6) have to be taken into account when solving the model. However, this occurs only if the bonus of the right approach is sufficiently high. Then with low net value and a good signal of fit are relatively more employed than firms with a high net value but a bad signal of fit. In this case, Lemma 6 suggests that the solution of the full problem is governed by four binding incentive constraints.

Lemma 6. *If the parameters are such that the solution of the relaxed problem violates the incentive condition* ($L0 \rightarrow H0$)*, then any solution of the full prob-*

lem involves the incentive constraints $(H1 \to L1)$*,* $(H0 \to L1)$*,* $(L1 \to L0)$ *and* $(L0 \to H0)$ *to bind.*

Proof. Ignore for the moment all incentive conditions except those mentioned in the Lemma. It is then shown in the appendix that they are satisfied if the following four conditions hold with equality.

- Constraint $(H1 \to L1)$:
  The proof parallels that of Lemma 4.

- Constraint $(H0 \to L1)$:
  The proof parallels that of Lemma 4.

- Constraint $(L0 \to H0)$:
  Suppose that the constraint does not bind. Then the solution of the problem would equal that of the relaxed problem, which would violate this condition by Lemma 5 which is a contradiction.

- Constraint $(L1 \to L0)$:
  Suppose that it this condition did not bind. Then $\alpha_{L0,H1} r(L0, H1) > \alpha_{L0,H0} r(L1, H0) \geq 0$. Hence $r(L0, H1)$ could be reduced by $\epsilon > 0$ small enough s.t. this condition is still satisfied. After this reduction the other constraints would still hold. As a reduction in rents would imply an increase of expected profits this contradicts optimality.

□

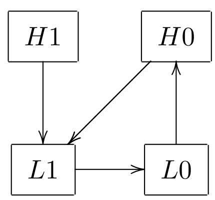

Figure 6: Binding incentive constraints in full problem if fit is important

Interestingly, the binding incentive constraints $(H0 \to L1)$, $(L1 \to L0)$ and $(L0 \to H0)$ which are addressed in Lemma 6 have a circular character, as shown in figure 6. Intuitively, paying higher rents to an $H0$ agent to keep him from mimicking being of type $L1$ also requires to pay higher rents to a firm with type $L0$. This in turn makes it necessary to pay also higher rents to an $L1$ firm, which then countervails the objective of making a $H0$ firm to reveal its type truthfully. This circularity makes it extremely costly to induce truthful behavior by paying rents. Hence, besides offering very high rents, the contractor may employ less frequently $L1$ firms or more often $H0$ firms compared to the relaxed case. For any optimal allocation, the following Lemma states the minimal rents. Of course, the rents equal those computed by (18) and (19) if Corollary 2 applies.

Lemma 7. *In any optimal allocation of the full problem, the rents are computed as follows:*

$$r(L0,L1) = \max\left\{0, \frac{\delta(1+\gamma^2)}{\gamma^2 \alpha_{L0,H1}} \sum_{\theta_j \in \Theta_j} \alpha_{L0,\theta_j}\left[q(L1,\theta_j) - q(H0,\theta_j)\right]\right\} \tag{25}$$

$$r(L1,H0) = \frac{1-\gamma^2}{1+\gamma^2} r(L0,H1) \tag{26}$$

$$r(H1,H0) = \alpha^{-1}_{H1,H0} \sum_{\theta_j \in \Theta_j} \delta\alpha_{H1,\theta_j} q(L1,\theta_j) + r(L1,H0) \tag{27}$$

$$r(H0,H1) = \alpha^{-1}_{H0,H1} \sum_{\theta_j \in \Theta_j} \delta\alpha_{H0,\theta_j} q(L1,\theta_j) + \frac{1-\gamma^2}{1+\gamma^2} r(L1,H0) \tag{28}$$

Proof. Imposing the constraints $(H1 \to L1)$, $(H0 \to L1)$, $(L1 \to L0)$ and $(L0 \to H0)$ to hold with equality and solving the system yields the result. □

Following Lemma 6 it is possible to ignore all constraints not mentioned in the Lemma if picking the right approach is crucial. Hence all dual variables of these conditions may be set to zero. The dual variable $\lambda_{X\to Y}$ reflects the imputed expected rents which are paid in order to satisfy incentive constraint $(X \to Y)$. Define the *virtual profits* $\psi(\theta_i,\theta_j)$ for extreme parameters as follows:

$$\psi(\theta_i, theta_j) = \begin{cases} \pi(\theta_i,\theta_j) & if \quad \theta_i \in \{L0, H1\} \\ \pi(\theta_i,\theta_j) + \delta\frac{\alpha_{L0,\theta_j}}{\alpha_{\theta_i,\theta_j}}\lambda_{L0\to H0} & if \quad \theta_i = H0 \\ \pi(\theta_i,\theta_j) - \frac{\delta}{\alpha_{\theta_i,\theta_j}}\left[\alpha_{H1,\theta_j} + \alpha_{H0,\theta_j}\lambda_{H0\to L1}\right] & if \quad \theta_i = L1 \end{cases} \tag{29}$$

If the right approach is crucial, then employment probabilities of $(H1)$ and $(L0)$ firms do not enter in any of the binding constraints. This means that no further rents have to be borne for employing these firms and hence the virtual profit equals the gross profit. The virtual profit of $H0$ firms is higher than the gross profit, because employing them makes it less attractive for $L1$ firms to pretend being of type $H1$. From equation (29) one sees that only firms of type $L1$ have a virtual value which may be dramatically lower than in the benchmark case.

Similar to the relaxed case, all previous results of this section except Corollary 2 are valid for general parameters. For simplicity, Proposition 2 again restricts attention to the case of sufficiently high surplus of the project such that for each combination of types, some firm is charged with the project finally. Again, the qualitative results of this proposition fully carry over to general parameters.

Assumption 3 guarantees that the virtual profits for any possible combination of types are positive. As the full problem involves weakly higher rents than the relaxed case, assumption 3 requires a higher surplus of the project than assumption 2.

Assumption 3. The net value L satisfies

$$L \geq \frac{1}{2}\left[\frac{\left(1+2\beta\gamma^2-\gamma^2\right)\left(1+\gamma^2\right)\delta}{\gamma^2(1-\beta)\left(1-\gamma^2\right)} - \eta\right]$$

Picking the right approach yields strictly positive extra payoff $\eta > 0$.

Proposition 2. *Suppose assumption 3 holds. Then projects are optimally assigned in the full problem as follows:*

i) If both firms have identical type, they are employed with probability $\frac{1}{2}$: $q(H1,H1) = q(H0,H0) = q(L1,L1) = q(L0,L0) = \frac{1}{2}$.

ii) If both firms receive the same signal of fit but differ in net value, the firm with higher net value is employed: $q(H1,L1) = q(H0,L0) = 1$ and $q(L1,H1) = q(L0,H0) = 0$.

iii) The contractor always employs high value firms which are more likely to offer the right approach: $q(H1,H0) = q(H1,L0) = 1$ and $q(H0,H1) = q(L0,H1) = 0$.

iv) If $\eta \leq \frac{\delta\beta}{(1-\beta)\gamma}$ then $q(H0,L1) = 1, q(L1,H0) = 0, q(L1,L0) = q(L0,L1) = \frac{1}{2}$.

v) If $\frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right] \geq \eta > \frac{\delta\beta}{(1-\beta)\gamma}$ then $q(H0,L1) = q(L1,L0) = 1$ and $q(L1,H0) = q(L0,L1) = 0$.

vi) If $\frac{\delta(1+\gamma^2)^2}{4\gamma^3(1-\beta)} - \frac{\delta\beta(1+\gamma^2)}{2\gamma} \geq \eta > \frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right]$ then $q(L1,H0) = q(H0,L1) = \frac{1}{2}$, $q(L1,L0) = 1$ and $q(L0,L1) = 0$.

vii) If $\eta > \frac{\delta(1+\gamma^2)^2}{4\gamma^3(1-\beta)} - \frac{\delta\beta(1+\gamma^2)}{2\gamma}$ then $q(L1,H0) = q(L1,L0) = 1$ and $q(H0,L1) = q(L0,L1) = 0$.

Proof.
Part i-iii) For parameters that satisfy the condition iv) or v) the proof parallels that of part i)-iii) of Proposition 1.
If eta is as required in parts vi) or vii), then it is shown below that $\lambda_{H0\to L1} \leq \frac{1+\gamma^2}{2\beta\gamma^2}$ and $\lambda_{H1\to L1} \leq 1$. Assumption 3 is equivalent to $\alpha_{L1,L1}pi(L1,L1) \geq \delta\left[\alpha_{H1,L1} + \alpha_{H0,L1}\frac{1+\gamma^2}{2\beta\gamma^2}\right]$ and therefore assures that the dual condition for $q(L1,L1)$ holds. This implies that the other dual conditions of part i)-iii) also hold.
Part iv) and v)
Follows from Proposition 1 and Corollary 2.
Part vi)
Set the dual variables of binding incentive constraints as follows:

$$\lambda_{H0\to L1} = \frac{\beta\delta\left(1+\gamma^2\right)^2 - 2\eta\gamma(1-\beta)}{\delta\left[\gamma^2(2\beta-1)-1\right]}$$

$$\lambda_{L1\to L0} = \frac{\left((\beta-1)\gamma^2 - \beta - 1\right)\Delta + 2\beta\gamma\left(\gamma^2-1\right)\eta}{\left((2\beta-1)\gamma^2-1\right)\Delta}$$

$$\lambda_{L0\to H0} = \frac{\beta\left[\left((\beta-1)\gamma^2-\beta-1\right)\Delta - 2(\beta-1)\gamma\eta\right]}{(\beta-1)\left((2\beta-1)\gamma^2-1\right)\Delta}$$

$$\lambda_{H1\to L1} = 1$$

These dual variables are constructed such that $\forall\eta \in \left(\frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right], \frac{\delta(1+\gamma^2)^2}{4\gamma^3(1-\beta)} - \frac{\delta\beta(1+\gamma^2)}{2\gamma}\right]$ the dual condition for $q(L1,H0) = q(H0,L1) = 0.5$, namely $\psi(L1,H0) = \psi(H0,L1)$, is satisfied. This condition implies that the dual condition for $q(L1,L0) = 1$ also holds. If $\eta = \frac{\delta}{2}\left[\frac{1+\beta}{\gamma(1-\beta)} + \gamma\right]$ then $\lambda_{H0\to L1} = 1$, $\lambda_{L0\to H0} = 0$ and $\lambda_{L1\to L0} = 1 + \frac{2\beta}{(1-\beta)(\gamma^2+1)}$ all these variables

are monotonically increasing in η. On the other hand, for $\eta = \frac{\delta(1+\gamma^2)^2}{4\gamma^3(1-\beta)} - \frac{\delta\beta(1+\gamma^2)}{2\gamma}$ the dual variables adopt the values stated in part vii). For all values in between, it can be verified easily that the dual condition for $r(L0, H1) = 0$, namely $\lambda_{L1\to L0} \leq \frac{1}{1-\beta} + \frac{\beta(1-\gamma^2)}{(1-\beta)(1+\gamma^2)}\lambda_{H0\to L1}$ holds. Besides, $\psi(L1, H0) = \psi(H0, L1)$ implies that the dual condition for $q(L1, L0) = 1$ also holds.
Part vii)
The allocation of part vii) is such that all four rent payments are strictly positive. Using the four associated dual equations yields the following dual variables:

$$\lambda_{H0\to L1} = \frac{1+\gamma^2}{2\beta\gamma^2}$$
$$\lambda_{L1\to L0} = \frac{1+\gamma^2}{2(1-\beta)\gamma^2}$$
$$\lambda_{L0\to H0} = \frac{1+(1-2\beta)\gamma^2}{2(1-\beta)\gamma^2}$$
$$\lambda_{H1\to L1} = 1$$

For $\eta > \frac{\delta(1+\gamma^2)^2}{4\gamma^3(1-\beta)} - \frac{\delta\beta(1+\gamma^2)}{2\gamma}$, direct computation shows that the dual constraint $\psi(L1, H0) \geq \psi(H0, L1)$ is satisfied. Besides, $\psi(L1, H0) > \psi(H0, L1)$ implies that the dual condition for $q(L1, L0) = 1$ also holds. □

Proposition 2 fully characterizes the optimal allocation of the full problem. Due to the assumption that the surplus of the project is high enough so that it is always assigned to some firm, Part i)-iii) remain unchanged compared to Proposition 1. Part iv) and v) also equal those of Proposition 1 by corollary 2.

If the approach is very important, as in Part vi) and vii) of Proposition 2, the upward incentive constraints bind and the optimal allocation differs from the solution of the relaxed problem. In Part vi), $L1$ firms are less often and $H0$ firms are more often employed in comparison to the relaxed case although the gross profit $\pi(L1, H0)$ is higher than $\pi(H0, L1)$. By distorting the allocation, the contractor effectively deters $L0$ firms from mimicking $H0$ ones without having to incur any rents to low value firms. In Part vii), the value of the right approach is extremely high and justifies not to distort the allocation and always to employ the firm which offers the highest gross profit even though this invokes very high rent payments. This case is illustrated in Corollary 3. Note that although the contractor has all the bargaining power, both firms earn strictly positive rents regardless of their type.

Corollary 3. *If assumption 3 holds and* $\eta > \frac{\delta(1+\gamma^2)^2}{4\gamma^3(1-\beta)} - \frac{\delta\beta(1+\gamma^2)}{2\gamma}$, *the contractor always employs the firm with the highest probability of fit and pays positive rents to both firms, regardless of their type.*

Proof. Follows directly from Lemma 7 and Proposition 2, Part vii). □

8 Conclusion

This paper shows that rents which arise due to cost uncertainty may induce specialized firms to excessively recommend the approach they are specialized on. Adverse incentives of sellers to provide wrong information on the appropriateness of their services may be mitigated by rewarding firms which are not hired or by committing to take their recommendations less into account than would be efficient. Nevertheless, the sellers' private information allows them in most cases to extract higher rents than in a benchmark case without asymmetric information on the appropriateness of their approach. If the bonus gained by the proper approach is neither too low nor too high, the need to pay additional information rents has detrimental effects on allocative efficiency.

If the additional bonus from choosing the superior approach is low compared to the base surplus, the optimal contracts generate the same outcome as a standard auction with two participants and private information on the fit becomes irrelevant in optimum. In contrast, if the proper approach yields high additional value, the contractor employs only firms signaling to offer the appropriate approach even though this requires paying information rents to firms indicating to work with the wrong approach and which are not employed finally.

Coming back to the introductory examples: It is common practice that clients engage multiple consultancies to analyze their problems and make suggestions how to proceed. After the suggestions are made, the client awards one consultancy with the project. In hope to acquire a profitable project, consultancies often agree to carry out the introductory screening at low fares. But exactly this practice exacerbates the adverse incentives of the consultancies for claiming to offer the superior approach against better judgment. Hence if the contractor deems the proper approach to be crucial, he should theoretically offer a high payment in case a consultancy is not awarded with the project finally.

Sometimes, other means to prevent adverse incentives in credence good markets are feasible. A widespread practice is to consult a neutral expert who cannot gain from selling the credence good on which he gives advice. This practice is feasible if the costs to diagnose customers' needs and to acquire the knowledge of products are low compared to the cost uncertainty of the agents. Another mechanism to alleviate fraud is that sellers might build up a reputation as honest adviser which helps them to acquire new customers or to attract new orders from existing ones.

This illustrates that there is still scope to further research on credence goods. One step which might constitute a good advice for further work might be to analyze cost uncertainty in a credence goods market equilibrium framework.

Appendix

A: General results

Benchmark case

Lemma 8 the general counterpart of Lemma 1:

Lemma 8. *Any optimal scheme of the benchmark case has the following properties: if $L > \delta\frac{1-\beta}{\beta}$ is satisfied:*
i) No rents are paid to firms with a low net value.
ii) Constraint (8) binds for firms with good or bad signal of fit and $R(H, x_i) = \delta E[q_i((L, x_i), \tilde{\theta_j})|x_i]$.
iii) Firms with high net value are weakly more often employed then firms with low net value:
$q((H, x_i), \theta_j) \geq q((L, x_i), \theta_j) \quad \forall \theta_j \in \Theta_j$
iv) If both firms provide the same net value, the firm with lower probability of fit is never employed: $q((\tilde{v}_i, 0), (\tilde{v}_i, 1)) = 0$.
v) Generically, in all optimal schemes firms are equally often employed if both firms report inconsistently a good or a bad signal of fit: $q((\tilde{v}_i, 0), (\tilde{v}_i, 0)) = q((\tilde{v}_i, 1), (\tilde{v}_i, 1))$.
vi) Holding the other firm's type constant, a firm is weakly more often employed if it reports a good signal of fit instead of a bad one: $q((\tilde{v}_i, 1), \theta_j) \geq q((\tilde{v}_i, 0), \theta_j) \quad \forall \theta_j \in \Theta_j$.

Proof of Lemma 8.
I first prove all parts of the Lemma ignoring constraint (9) and then prove afterwards that all solutions that satisfy properties i) to v) also satisfy (9).

Part i)
Suppose to the contrary that an optimal scheme involves $R(L, x_i) > 0$. After setting $R(L, x_i) = 0$, incentive constraint (8) still holds and thus the new solution is feasible but U_c has increased. Hence $R(L, x_i) > 0$ cannot be optimal.

Part ii)
Suppose that (8) did not bind for some (H, x_i). Then $R(H, x_i)$ could be decreased by ϵ small enough such that (8) still hold. This would increase U_c and thus contradicts optimality.

Part iii)
Define

$$\psi(\theta_i, \theta_j) = \begin{cases} \alpha_{\theta_i,\theta_j}\pi(\theta_i, \theta_j) & if \quad x_i = H \\ \alpha_{\theta_i,\theta_j}\pi(\theta_i, \theta_j) - \delta\alpha_{(H,x_i),\theta_j} & if \quad x_i = L \end{cases}$$

A necessary condition for $q((L, x_i), \theta_j) > 0$ is $\alpha_{(L,x_i),\theta_j}\pi((L, x_i), \theta_j) -$

$\delta\alpha_{(H,x_i),\theta_j} \geq \max\{0, \psi(\theta_j, (L, x_i))\}$. But this implies that $\alpha_{(H,x_i),\theta_j}\pi((H, x_i), \theta_j) >$ $\max\{0, \psi(\theta_j, (H, x_i))\}$ which is a sufficient condition for $q((H, x_i), \theta_j) = 1$.

Part iv)

Note that $\psi((v_i, 1), \theta_i) > \psi((v_i, 0), \theta_i)$. Hence the necessary condition $\psi((v_i, 0), (v_i, 1) \geq \max\{0, \psi((v_i, 1), (v_i, 0))\}$ for $q((v_i, 0), \theta_i) > 0$ is never satisfied.

Part v)

Note that $\psi((\tilde{v}_i, 0), (\tilde{v}_i, 0)) = \psi((\tilde{v}_i, 1), (\tilde{v}_i, 1))$. Hence iff $\psi((\tilde{v}_i, 0), (\tilde{v}_i, 0)) > 0$ then $\psi((\tilde{v}_i, 1), (\tilde{v}_i, 1)) > 0$

To prove that any scheme that involves $R(H, x_i) = \delta E[q_i((L, x_i), \tilde{\theta_j})|x_i]$ and $q((H, x_i), \theta_j) \geq q((L, x_i), \theta_j) \quad \forall\theta_j \in \Theta_j$ always satisfies (9), observe that

$$E\left[r_i((H, x_i), \tilde{\theta}) - \delta q_i((H, x_i), \tilde{\theta_j})|\theta_i\right] = \delta E\left[q_i((L, x_i), \tilde{\theta_j}) - q_i((H, x_i), \tilde{\theta_j})|\theta_i\right] \leq 0$$

which means that indeed (9) holds.

Part vi)

Note that $\psi((\tilde{v}_i, 1), \theta_j) \geq \psi((\tilde{v}_i, 0), \theta_j) \quad \forall\theta_j \in \Theta_j$ and $\psi((\theta_i, \tilde{v}_j, 0)) \geq \psi((\theta_i, \tilde{v}_j, 1)) \quad \forall\theta_j \in \Theta_j$. Hence $\psi((\tilde{v}_i, 0), \theta_j) \geq \psi((\theta_j, \tilde{v}_i, 0)$ implies $\psi((\tilde{v}_i, 1), \theta_j) \geq \psi((\theta_j, \tilde{v}_i, 1)$. □

Relaxed Problem

Note that Lemma 2, Lemma 3 and Lemma 4 are formulated and proven for unrestricted L and thus carry over completely.

The result of Lemma 4 implies that the difference of payoffs between mimicking a $L1$ instead of a $L0$ firm is in every optimal scheme nonnegative. By deducting (13) from (12) and (15) from (14) and inserting $\alpha_{H1,H0} = \frac{\beta}{1-\beta}\alpha_{H1,L0}$ one gets:

$$\alpha_{H1,L0}\left[q(L1, L0) + \frac{\beta}{1-\beta}q(L1, H0) - q(L0, L0)\right] + \alpha_{H1,L1}\left[q(L1, L1) - q(L0, L1)\right] \geq 0 \tag{30}$$

$$\alpha_{H0,L0}\left[q(L1, L0) + \frac{\beta}{1-\beta}q(L1, H0) - q(L0, L0)\right] + \alpha_{H0,L1}\left[q(L1, L1) - q(L0, L1)\right] \geq 0 \tag{31}$$

Equations (30) and (31) denote the difference in expected payoff for $H1$ and $H0$ firms respectively. Lemma 9 and Proposition 3 are the general counterparts to Proposition 1.

Lemma 9. *i) In any optimal solution* $q(L1, L0) + \frac{\beta}{1-\beta}q(L1, H0) - q(L0, L0) \geq 0$.

ii) Either $(H1 \to L0)$ *and* $(H0 \to L0)$ *bind, or* $(H1 \to L0)$ *bind while* $(H0 \to L0)$ *is slack or both* $(H1 \to L0)$ *and* $(H0 \to L0)$ *are slack.*

iii) If $(H1 \to L0)$ *and* $(H0 \to L0)$ *bind, then* $q(L1, L0) + \frac{\beta}{1-\beta}q(L1, H0) - q(L0, L0) = 0$ *and* $q(L1, L1) - q(L0, L1) = 0$

Proof of Lemma 9. Part i)

Suppose to the contrary that $q(L1,L0)+\frac{\beta}{1-\beta}q(L1,H0)-q(L0,L0)<0$. Then $q(L1,L1)-q(L0,L1)>0$ must be true for (31) and (30) to hold. $q(L1,L1)-q(L0,L1)>0$ implies that $q(L0,L1)<q(L1,L1)\leq 0.5$. Similarly, $q(L1,L0)+\frac{\beta}{1-\beta}q(L1,H0)-q(L0,L0)<0$ implies that $q(L1,L0)<q(L0,L0)\leq 0.5$. Hence $q(L1,L0)+q(L0,L1)<1$ and thus $\mu_{L1,L0}=0$. Now consider the following cases:

a) Both (30) and (31) are slack. Then the dual variables have $\lambda_{H1\to L1}=1$ and $\lambda_{H0\to L1}=1$. $q(L1,L1)>0$ requires the dual constraint $\alpha_{L1,L1}\pi(L1,L1,)\geq\delta(\alpha_{H1,L1}+\alpha_{H0,L1})$. $q(L1,L0)+q(L0,L1)<1$ requires $\alpha_{L1,L0}\pi(L1,L0,)\leq\delta(\alpha_{H1,L1}+\alpha_{H0,L1})$, which is impossible as $\alpha_{L1,L0}\pi(L1,L0,)>\alpha_{L1,L1}\pi(L1,L1)$.

b) Equation (30) holds with equality, (31) is slack. Then $\lambda_{H0\to L1}=1$ and $\lambda_{H0\to L0}=0$. Now the dual constraint for $q(L1,L0)$ is $\alpha_{L1,L0}\pi(L1,L0)\leq\delta(\alpha_{H1,L0}\lambda_{H1\to L1}+\alpha_{H0,L0})$. This requires $\alpha_{L1,L1}\pi(L1,L1)<\delta(\alpha_{H1,L1}\lambda_{H1\to L1}+\alpha_{H0,L1})$. But $q(L1,L1)>0$ requires $\alpha_{L1,L1}\pi(L1,L1)\geq\delta(\alpha_{H1,L1}\lambda_{H1\to L1}+\alpha_{H0,L1})$, which is impossible.

c) (30) is slack and (31) holds with equality. This case cannot occur because $\alpha_{H1,H0}>\alpha_{H0,H0}$ and $\alpha_{H1,H1}<\alpha_{H0,H1}$ and hence if (31) holds with equality, (30) must be violated.

Part ii)

Suppose to the contrary, that (30) holds with equality and (31) is slack. Because $\alpha_{H1,H0}>\alpha_{H0,H0}$ and $\alpha_{H1,H1}<\alpha_{H0,H1}$ this would require that $q(L1,L0)+\frac{\beta}{1-\beta}q(L1,H0)-q(L0,L0)<0$ and $q(L1,L1)-q(L0,L1)>0$. But according to Part i) of this Lemma it is impossible.

Part iii)

Suppose to the contrary that (31) and (30) hold with equality but $q(L1,L0)+\frac{\beta}{1-\beta}q(L1,H0)-q(L0,L0)>0$ and hence $q(L1,L1)-q(L0,L1)<0$. Then as $\alpha_{H1,H0}>\alpha_{H0,H0}$ and $\alpha_{H1,H1}<\alpha_{H0,H1}$ condition (30) would be violated which contradicts the hypothesis. □

Proposition 3. *i) For η high enough, the contractor employs always the firm with the highest conditional probability of a fit. The optimal scheme involves $q(L1,H0)=q(L1,L0)=1, q(L1,L1)=q(L0,L0)=0.5, q(L0,L1)=0$, i.e.*

For η high enough, the optimal scheme involves both incentive constraints (13) and (15) slack.

ii) For η low enough, the optimal scheme involves $q(L1,H0)=0$.

iii) If additionally $L>\delta\frac{1-\beta}{\beta}$ then the optimal scheme involves $q(L1,L0)=q(L1,L1)=q(L0,L0)=q(L0,L1)=0.5$, i.e. if both firms announce to have low net value, then each firm is employed with probability 0.5 irrespective of the fit signal.

iv) If additionally $L < \delta\frac{1-\beta}{\beta}$, then the optimal scheme involves $q(L1, L0) = q(L1, L1) = q(L0, L0) = q(L0, L1) = 0$, i.e. firms with low net value are never employed.

Proof of Proposition 3.
Part i):
The proof of optimality is based on the standard duality theorem. First note that the with the employment probabilities given in the proposition, both incentive constraints (13) and (15) slack. As in addition feasibility requires $r(H1, H0) > 0, r(H0, H1) > 0$, the only permissible choice of dual variables for the incentive constraints is: $\lambda_{H1\to L1} = \lambda_{H0\to L1} = 1, \lambda_{H1\to L0} = \lambda_{H0\to L0} = 0$. Now define the dual constraints of all binding resource constraints with $q(\theta_i, \theta_j) > 0$ as follows:

$$\mu_{\theta_i,\theta_j} = \alpha_{\theta_i,\theta_j}\pi(\theta_i, \theta_j) - \delta\alpha_{H1,\theta_j}\lambda_{H1\to\theta_i} - \delta\alpha_{H0,\theta_j}\lambda_{H0\to\theta_i}$$

Note that for η high enough, all those variables are positive as required. Now consider the dual condition for $q(L0, L1) = 0$: $\mu_{L1,L0} = \alpha_{L1,L0}\pi(L1, L0) + \delta\alpha_{H1,L0} + \delta\alpha_{H0,L0} > \alpha_{L0,L1}\pi(L0, L1)$. This condition is always satisfied as $\pi(L1, L0) - \pi(L0, L1)$ grows arbitrarily in η. Using the same argument, it can be shown that the dual condition for $q(L0, H1) = 0$ is also satisfied.

Part ii)
Suppose to the contrary that $q(L1, H0) > 0$. The associated dual constraint requires that $\alpha_{L1,H0}\left(\pi(L1, H0) - \pi(H0, L1)\right) \geq \delta\alpha_{H1,H0}\lambda_{H1\to L1} + \delta\alpha_{H0,H0}\lambda_{H0\to L1}$ which is impossible because $\pi(L1, H0)-\pi(H0, L1)$ is negative for fixed δ and η low enough while $\alpha_{H1,H0}\lambda_{H1\to L1} + \alpha_{H0,H0}\lambda_{H0\to L1} \geq 0$.

Part iii)
As all incentive constraints hold with equality, no associated dual variable has to be set equal to zero. Choose the following dual variables for the incentive constraints:

$$\lambda_{H1\to L1} = 1$$

$$\lambda_{H0\to L1} = \frac{\alpha_{L1,L0}\pi(L1, L0) - \alpha_{L0,L1}\pi(L0, L1) + \delta\alpha_{H0,L1} - \delta\alpha_{H1,L0}}{\delta\left(\alpha_{H0,L0} + \alpha_{H0,L1}\right)}$$

$$\lambda_{H0\to L0} = \frac{-\alpha_{L1,L0}\pi(L1, L0) + \alpha_{L0,L1}\pi(L0, L1) + \delta\alpha_{H0,L0} + \delta\alpha_{H1,L0}}{\delta\left(\alpha_{H0,L0} + \alpha_{H0,L1}\right)}$$

$$\lambda_{H1\to L1} = 0$$

Note that for η small enough, those dual variables are nonnegative. Furthermore we have for $\eta \to 0$ that $\lambda_{H0\to L1} \searrow 0$ and $\lambda_{H0\to L0} \nearrow 1$. Note that these variables satisfy the following dual conditions associated stemming from $r(L1, L0) > 0$ and $r(L0, L1) > 0$: $\lambda_{H1\to L0}+\lambda_{H1\to L1} = 1$ and $\lambda_{H0\to L0}+\lambda_{H0\to L1} = 1$. Next note that we have $q(L1, L0)+q(L0, L1) = 1$ and hence the corresponding dual variable $\mu_{L1,L0}$ can adopt any nonnegative value. Choose $\mu_{L1,L0} = \alpha_{L1,L0}\pi(L1, L0)-\delta\alpha_{H1,L0}\lambda_{H1\to L1}-$

$\delta\alpha_{H0,L0}\lambda_{H0\to L1}$ so that the associated dual constraint for $q(L1, L0) > 0$ is satisfied. Now we have

$$\lim_{\eta\searrow 0}\mu_{L1,L0} = \alpha_{L1,L0}L - \delta\alpha_{H1,L0} = \alpha_{L1,L0}\left(L - \delta\frac{1-\beta}{\beta}\right) > 0$$

Hence by continuity there must exist some $\hat{\eta}$ low enough which satisfies: $\eta < \hat{\eta} \Rightarrow \mu_{L1,L0} \geq 0$. It can be easily verified that the construction of $\mu_{L1,L0}$ also satisfies the dual constraint that arises from $q(L0, L1) > 0$.
Now turn to the resource constraint $q(L1, L1) \leq 0.5$ which also holds with equality. Choose $\mu_{L1,L1} = \alpha_{L1,L1}\pi(L1, L1) - \delta\alpha_{H1,L1}\lambda_{H1\to L1} - \delta\alpha_{H0,L1}\lambda_{H0\to L1}$ and note that $\lim_{\eta\searrow 0}\mu_{L1,L1} = \alpha_{L1,L1}L - \delta\alpha_{H1,L1} = \alpha_{L1,L1}\left(L - \delta\frac{1-\beta}{\beta}\right) > 0$. Hence the same argument as above applies.
For the binding resource constraint $q(L0, L0) \leq 0.5$ choose $\mu_{L0,L0} = \alpha_{L0,L0}\pi(L0, L0) - \delta\alpha_{H1,L0}\lambda_{H1\to L0} - \delta\alpha_{H0,L0}\lambda_{H0\to L0}$ and note that $\lim_{\eta\searrow 0}\mu_{L0,L0} = \alpha_{L0,L0}L - \delta\alpha_{H0,L0} = \alpha_{L0,L0}\left(L - \delta\frac{1-\beta}{\beta}\right) > 0$. Hence the same argument as above applies.

Part iv) Suppose to the contrary that there exists some $q(Lx, Ly) > 0, \quad x, y \in \{0, 1\}$ and define $z = 1 - x$. The associated dual constraints for an optimal solution are:

$$\begin{aligned}
\delta\alpha_{Hx,Ly}\lambda_{Hx\to Lx} + \delta\alpha_{Hz,Ly}\lambda_{Hz\to Lx} + \mu_{Lx,Ly} = &\quad \alpha_{Lx,Ly}\pi(Lx, Ly)\\
\delta\alpha_{Hx,Ly}\lambda_{Hx\to Lz} + \delta\alpha_{Hz,Ly}\lambda_{Hz\to Lz} + \mu_{Lz,Ly} \geq &\quad \alpha_{Lz,Ly}\pi(Lz, Ly)
\end{aligned}$$

Feasibility requires that $r(L1, L0) > 0$ and $r(L0, L1) > 0$ and hence $\lambda_{Hx\to Lx} + \lambda_{Hx\to Lz} = 1$ and $\lambda_{Hz\to Lx} + \lambda_{Hz\to Lz} = 1$ must be satisfied. Adding both constraints yields

$$\delta\alpha_{Hx,Ly} + \delta\alpha_{Hz,Ly} + \mu_{Lz,Ly} + \mu_{Lx,Ly} \geq \alpha_{Lx,Ly}\pi(Lx, Ly) + \alpha_{Lz,Ly}\pi(Lz, Ly)$$

This is impossible because $\lim_{\eta\searrow 0}\delta\alpha_{Hx,Ly} + \delta\alpha_{Hz,Ly} - \alpha_{Lx,Ly}\pi(Lx, Ly) - \alpha_{Lz,Ly}\pi(Lz, Ly) < 0$ by hypothesis and both $\mu_{Lx,Ly}$ and $\mu_{Lz,Ly}$ must be nonnegative. Hence there do not exist nonnegative dual variables which permit $q(Lx, Ly) > 0$. □

Proposition 4 is the general counterpart to Corollary 1:

Proposition 4. *Generically, employment probabilities for firms that announce a good fit signal are weakly higher in an optimal scheme of the benchmark case than in any of the optimal schemes of the relaxed problem. Generically, an optimal scheme in the benchmark case in which employment probabilities for firms that announce a good fit signal are weakly higher than in any of the optimal schemes of the relaxed case.*

Proof. Employment probabilities for firms with high net value.
Suppose that $q(H1, \tilde{\theta}_j) > 0$. Then $\pi(H1, \tilde{\theta}_j) \geq 0$ and $\pi(H1, \tilde{\theta}_j) \geq \pi(\tilde{\theta}_j, H1)$. But

these two conditions guarantee that $q^B(H1, H1) = 0.5$ and $q^B(H1, \tilde{\theta}_j) = 1$ for $\tilde{\theta}_j \in H0, L0, L1$.

Employment probabilities for firms with low net value.
Consider two cases:

- The incentive constraint (13) is slack. Then for any optimal solution with $q(L1, L0) > 0$ the corresponding dual constraint is $\alpha_{H1,L0}\pi(H1, L0) \geq \delta\alpha_{H1,L0} + \delta\alpha_{H0,L0}$. This implies $\alpha_{H1,L0}\pi(H1, L0) > \delta\alpha_{H1,L0}$ which is sufficient for $q^B(H1, L0) = 1$.

- The incentive constraint (13) binds. Then by Lemma 9, Part ii), $q(L1, L0) + \frac{\beta}{1-\beta}q(L1, H0) - q(L0, L0) = 0$ and $q(L1, L1) - q(L0, L1) = 0$. Hence if $q(L1, \tilde{\theta}_j) > 0$ then also $q(L0, \tilde{\theta}_j) > 0$ for $\tilde{\theta}_j \in \{L0, L1, H0\}$. The corresponding dual constraints are

$$\alpha_{L1,\tilde{\theta}_j}\pi(L1, \tilde{\theta}_j) \geq \delta\alpha_{H1,\tilde{\theta}_j}\lambda_{H1\to L1} + \delta\alpha_{H0,\tilde{\theta}_j}\lambda_{H0\to L1}$$

and

$$\alpha_{L0,\tilde{\theta}_j}\pi(L0, \tilde{\theta}_j) \geq \delta\alpha_{H1,\tilde{\theta}_j}\lambda_{H1\to L0} + \delta\alpha_{H0,\tilde{\theta}_j}\lambda_{H0\to L0}$$

Adding up both inequalities and using $\lambda_{H1\to L1} + \lambda_{H1\to L0} = 1$ and $\lambda_{H0\to L1} + \lambda_{H0\to L0} = 1$ yields

$$\alpha_{L1,\tilde{\theta}_j}\pi(L1, \tilde{\theta}_j) + \alpha_{L0,\tilde{\theta}_j}\pi(L0, \tilde{\theta}_j) \geq \delta\alpha_{H1,\tilde{\theta}_j} + \delta\alpha_{H0,\tilde{\theta}_j}$$

To show that the last inequality implies $\alpha_{L1,\tilde{\theta}_j}\pi(L1, \tilde{\theta}_j) > \delta\alpha_{H1,\tilde{\theta}_j}$ suppose to the contrary that this $\alpha_{L1,\tilde{\theta}_j}\pi(L1, \tilde{\theta}_j) \leq \delta\alpha_{H1,\tilde{\theta}_j}$. Then $\pi(L0, \tilde{\theta}_j) < \pi(L1, \tilde{\theta}_j)$ implies $\alpha_{L0,\tilde{\theta}_j}\pi(L0, \tilde{\theta}_j) < \delta\alpha_{H0,\tilde{\theta}_j}$ which is a contradiction. The implication $\alpha_{L1,\tilde{\theta}_j}\pi(L1, \tilde{\theta}_j) > \delta\alpha_{H1,\tilde{\theta}_j}$ is sufficient to have $q^B(\alpha_{L1,\tilde{\theta}_j})$.

□

Lemma 10 analyzes the optimal employment probabilities of firms with low net value and high probability of fit, given that the second firm offers a high net value but a low probability of fit. In this case there is a tradeoff between the bonus minus rents on the one hand and the difference in net values on the other. Lemma 10, Part ii) establishes that $q(L1, H0) > 0$ is usually chosen only if there is a high bonus for choosing the right approach. Then it is optimal to employ only firms with a high probability of fit and incur high rent payments.

Lemma 10. *Suppose* $\pi(H0, L1) > 0$. *If* $\alpha_{L1,H0}[\pi(L1, H0) - \pi(H0, L1)] - \delta[\alpha_{H1,H0} + \alpha_{H0,H0}] > 0$ *then* $q(L1, H0) = 1$ *and otherwise* $q(L1, H0) = 0$. *If* $q(L1, H0) = 1$ *then both incentive constraints* $(H1 \to L0)$ *and* $(H0 \to L0)$ *are slack. ALTE VERSION i) In any optimal scheme, if* $q(L1, H0) > 0$ *then* $q(L0, L1) = 0$ *and* $q(H0, L1) = 0$.
ii) If $q(L1, H0) > 0$ *and* $\pi(H0, L1) > 0$ *then* $q(L1, H0) = 1$, $q(L1, L0) = 1$ *and the incentive constraints (14) and (13) are slack.*

Proof of Lemma 10.

Part i):

Suppose to the contrary that $q(L1, H0) > 0$ and $q(L0, L1) > 0$. The associated dual constraint for $q(L1, H0) > 0$ is

$$\alpha_{L1,H0}\pi(L1, H0) = \max\{\alpha_{L1,H0}\pi(H0, L1), 0\} + \delta\alpha_{H1,H0}\lambda_{H1\to L1} + \delta\alpha_{H0,H1}\lambda_{H0\to L1}$$

Using $\pi(L1, H0) = \pi(L1, L0)$ and $\pi(H0, L1) > \pi(L0, L1)$ yields:

$$\alpha_{L1,L0}\pi(L1, L0) > \alpha_{L1,L0}\pi(L0, L1) + \delta\alpha_{H1,L0}\lambda_{H1\to L1} + \delta\alpha_{H0,L1}\lambda_{H0\to L1}$$

On the other hand, the dual constraint for $q(L0, L1) > 0$ is:

$$\begin{aligned}\alpha_{L0,L1}\pi(L0, L1) &= \mu_{L1,L0} + \delta\alpha_{H1,L1}\lambda_{H1\to L0} + \delta\alpha_{H0,L1}\lambda_{H0\to L0}\\ &\geq \alpha_{L1,L0}\pi(L1, L0) - \delta\alpha_{H1,L0}\lambda_{H1\to L1} - \delta\alpha_{H0,L0}\lambda_{H0\to L1}\\ &+ \delta\alpha_{H1,L1}\lambda_{H1\to L0} + \delta\alpha_{H0,L1}\lambda_{H0\to L0}\end{aligned}$$

This implies:

$$\alpha_{L1,L0}\pi(L1, L0) \leq \alpha_{L0,L1}\pi(L0, L1) + \delta\alpha_{H1,L0}\lambda_{H1\to L1} + \delta\alpha_{H0,L0}\lambda_{H0\to L1}$$

which is impossible.

To see that $q(H0, L1) = 0$, suppose to the contrary that $q(L1, H0) > 0$ and $q(H0, L1) > 0$. The dual constraint for $q(H0, L1) > 0$ is $\pi(H0, L1) = \max\{\pi(L1, H1), 0\}$. The dual constraint for $q(L1, H1) > 0$ requires that $\pi(L1, H1) > 0$, hence the constraint simplifies to $\pi(H0, L1) = \pi(L1, H1)$. Plugging this into the dual constraint for $q(L1, H1) > 0$ yields immediately a contradiction.

Part ii):

Suppose to the contrary that $q(L1, H0) > 0$ and $\pi(H0, L1) > 0$ but $q(L1, L0) < 1$. From Part i) $q(L0, L1) = 0$, so that $q(L1, L0) + q(L0, L1) > 1$ and hence the dual variable $\mu_{L1,L0} = 0$. The associated dual constraint for $q(L1, H0) > 0$ is
$\alpha_{L1,H0}\pi(L1, H0) = \alpha_{L1,H0}\pi(H0, L1) + \delta\alpha_{H1,H0}\lambda_{H1\to L1} + \delta\alpha_{H0,H1}\lambda_{H0\to L1}$
This implies $\alpha_{L1,L0}\pi(L1, L0) > \delta\alpha_{H1,L0}\lambda_{H1\to L1} + \delta\alpha_{H0,L1}\lambda_{H0\to L1}$
which means that $\mu_{L1,L0}$ must be greater than zero - a contradiction. Hence $q(L1, L0) = 1$ and $q(L0, L1) = 0$ which renders both (14) and (13) slack. □

The Full Problem

Note that the formulation and the proof of Lemma 5 applies fully to general parameters. Proposition 5 gives sufficient conditions which guarantee that solutions of the relaxed problem also satisfy inequality (24) and is the counterpart to Corollary 2.

Proposition 5. *If* $\alpha_{L1,H0}\pi(L1, H0) - \delta(\alpha_{H0,H0} + \alpha_{H1,H0}) < \max\{0, \alpha_{L1,H0}\pi(H0, L1)\}$ *is satisfied and either* $\pi(H0, L1) > 0$ *or* $\pi(L1, L1) - \delta(\alpha_{H0,L1} + \alpha_{H1,L1}) < 0$ *or* $\alpha_{L0,L1} - \alpha_{L0,H0} < 0$ *holds, then the optimal solution of the relaxed problem also solves the fully constrained problem.*

Proof. First note that any solution of the relaxed problem involves $r(L0, H1) = r(L1, H0) = 0$. The condition $\alpha_{L1,H0}\pi(L1, H0) - \delta(\alpha_{H0,H0} + \alpha_{H1,H0}) < \max\{0, \alpha_{L1,H1}\pi(H0, L1)\}$ implies that the relaxed solution involves $q(L1, H0) = 0$. It also implies that whenever $\alpha_{L1,L0}\pi(L1, L0) - \delta(\alpha_{H0,L0} + \alpha_{H1,L0}) > 0$ then $\pi(H0, L1) > 0$ and hence $\pi(H0, L0) > 0$ which is a sufficient condition for $q(H0, L0) = 1$. This means that the first bracket in condition 24 is always non-positive.

- If additionally $\pi(H0, L1) > 0$ then $q(H0, L1) = 1$ and hence inequality (19) is satisfied for all feasible values of $q(L1, L1)$.

- If additionally $\pi(L1, L1) - \delta(\alpha_{H0,L1} + \alpha_{H1,L1}) < 0$ then any solution of the relaxed problem involves $q(H0, L1) \geq q(L1, L1) = 0$ and hence inequality (19) is satisfied.

- If additionally $\alpha_{L0,L1} - \alpha_{L0,H0} < 0$ then even in the case $q(L1, L1) = 0.5$, which implies $q(H0, L0) = 1$ and $q(H0, H0) = 0.5$, inequality (19) is satisfied.

□

Lemma 6 of section 7 also applies to general parameters and asserts formally which of the incentive constraints bind if the solution of the relaxed problem violates inequality (24). A very important case of extreme parameters is when the choice of the appropriate approach is very important. The following proposition states that under sufficiently high η the contractor finds it optimal to incur very high rents in order to employ always the firm whose approach is more likely to be superior.

Proposition 6. *For η high enough, the optimal solution has the following properties:*
i) The contractor always employs the firm which is more likely to offer the right approach: $q(H1, H0) = q(H1, L0) = q(L1, H0) = q(L1, L0) = 1$ and $q(H0, H1) = q(L0, H1) = q(H0, L1) = q(L0, L1) = 0$.
ii) If both firms receive signals that are not consistent but differ in net value, then the firm with the higher net value is employed: $q(H1, L1) = q(H0, L0) = 1$ and $q(L1, H1) = q(L0, H0) = 0$.
iii) If both firms have identical type, then they are employed with probability $\frac{1}{2}$: $q(H1, H1) = q(H0, H0) = q(L1, L1) = q(L0, L0) = \frac{1}{2}$.
iv) The contractor pays positive rents to both firms, irrespective of their type.

Proof. Part i)
Note that for given δ, the unique generic solution is characterized completely by this proposition. Using all nonnegative employment probabilities and the four rent variables as basis one can derive straight forward the following dual variables for

the binding incentive constraints.

$$\lambda_{H0\to L1} = \frac{1+\gamma^2}{2\beta\gamma^2}$$
$$\lambda_{L1\to L0} = \frac{1+\gamma^2}{2(\beta-1)\gamma^2}$$
$$\lambda_{L0\to H0} = \frac{1+(1-2\beta)\gamma^2}{2(\beta-1)\gamma^2}$$
$$\lambda_{H1\to L1} = 1$$

Note that all dual variables are defined in terms of fixed parameters. Define the dual variable for the resource constraint $q(L1,H0)+q(H0,L1)\leq 1$ as follows: $\mu_{L1,H0} = \alpha_{L1,H0}\pi(L1,H0)-\delta(\alpha_{H1,H0}\lambda_{H1\to L1}+\alpha_{H0,H0}\lambda_{H0\to L1})$. Note that this satisfies the dual constraint for $q(L1,H0)$ by construction and for high η and hence $\pi(L1,H0)$ high enough $\mu_{L1,H0}\geq 0$ as required. In addition $\mu_{L1,H0}-\alpha_{H0,L1}\pi(H0,L1) = 0.5\eta\left(1-\beta^2\right)\gamma^2\alpha_{L1,H0}-\delta\alpha_{H0,L1}-\delta(\alpha_{H1,H0}\lambda_{H1\to L1}+\alpha_{H0,H0}\lambda_{H0\to L1})$ which is positive for η high enough. This means that indeed $q(H0,L1)=0$ must be true. Using the same argument, one can show that the dual constraints for $q(L1,L0)=1$ and $q(L0,L1)=0$ are also satisfied.
Part iv) and the level of the rents can be determined uniquely by the four binding constraints stated in Lemma 6, Part ii).

□

B: Proofs of Lemmas and Propositions

Proof of Lemma 3. Suppose $q(\cdot,\cdot),\tilde{r}(\cdot,\cdot)$ is a feasible scheme with $r((v_i,x_i),(v_j,x_j))\neq 0$ and $x_i+x_j\neq 1$ for some (x_i,x_j) . Then all incentive constraints are satisfied. Now define $\hat{r}(\cdot,\cdot)$ as follows: [18]

$$\hat{r}(H1,H0) = \frac{1}{\alpha_{H1,H0}}\sum_{\theta_j\in\Theta_i}\left(\alpha_{H1,\theta_j}\tilde{r}(H1,\theta_j)\right)$$
$$\hat{r}(H1,\theta_j) = 0 \qquad \forall \quad \theta_j\neq H0$$
$$\hat{r}(H0,H1) = \frac{1}{\alpha_{H0,H1}}\sum_{\theta_j\in\Theta_i}\left(\alpha_{H0,\theta_j}\tilde{r}(H0,\theta_j)\right)$$
$$\hat{r}(H0,\theta_j) = 0 \qquad \forall \quad \theta_j\neq H1$$

Note that by construction $\alpha_{H1,H0}\hat{r}(H1,H0) = \sum_{\theta_j\in\Theta_i}(\alpha_{H1,\theta_j}\tilde{r}(H1,\theta_j))$ and $\alpha_{H0,H1}\hat{r}(H0,H1) = \sum_{\theta_j\in\Theta_i}(\alpha_{H0,\theta_j}\tilde{r}(H0,\theta_j))$. Hence all downward incentive constraints also hold under $\hat{r}(\cdot,\cdot)$ when $q(\cdot,\cdot)$ remains unchanged. Now consider the

[18]Note that for the proof to go through it is important that positive rents are only paid for consistent fit-signals. In principle it is also possible to construct schemes with $\hat{r}(H1,L0)\neq 0$.

horizontal incentive constraint $H1 \to H0$:

$$\begin{aligned}
&\alpha_{H1,H0}\hat{r}(H1,H0)\\
&= \sum_{\theta_j \in \Theta_i} \alpha_{H1,\theta_j}\tilde{r}(H1,\theta_j)\\
&\geq \sum_{\theta_j \in \Theta_i} \frac{\alpha_{H1,\theta_j}}{\alpha_{H0,\theta_j}}\alpha_{H0,\theta_j}\tilde{r}(H0,\theta_j)\\
&\geq \sum_{\theta_j \in \Theta_i} \frac{\alpha_{H1,H1}}{\alpha_{H0,H1}}\alpha_{H0,\theta_j}\tilde{r}(H0,\theta_j)\\
&= \frac{\alpha_{H1,H1}}{\alpha_{H0,H1}}\alpha_{H0,H1}\hat{r}(H0,H1)\\
&= \alpha_{H1,H1}\hat{r}(H0,H1)
\end{aligned}$$

The first inequality follows because by hypothesis the incentive constraint holds for $\tilde{r}(\cdot,\cdot)$, the next inequality is due to the definition of $\alpha_{\theta_i,\theta_j}$. The second inequality is strict if $\gamma > 0$.

Exactly the same argument can be used to show that the incentive constraint $(H0 \to H1)$ still holds for $\hat{r}(\cdot,\cdot)$. As all rents for low value agents are zero in optimum, horizontal incentive constraints for low value firms are always satisfied.

Note that by construction of $\hat{r}(\cdot,\cdot)$, the expected payoff U_c remains unchanged. Hence the new scheme is feasible and optimal.

□

Proof of Lemma 4. Step 1: Show that (12) must hold with equality. Suppose to the contrary that (12) is slack. Distinguish two cases:

- Incentive constraint (13) is slack.
  Then there exists an ϵ small enough, s.t. $r(H1,H0)$ can be lowered by ϵ and both (12) and (13) still hold, which contradicts optimality.

- Incentive constraint (13) binds.
  Deducting (13) from (12), applying Lemmas 2, inserting $q(L0,H0) = q(L0,H1) = q(L1,H1) = 0$ and rearranging yields:

$$\alpha_{H1,L0}\left(q(L1,L0) - q(L0,L0)\right) + \alpha_{H1,L1}\left(q(L1,L1) - q(L0,L1)\right) + \alpha_{H1,L0}q(L1,H0) < 0$$

  This condition is only satisfied if $q(L1,L0) - q(L0,L0) < 0$ or $q(L1,L1) - q(L0,L1) < 0$ or both. By the resource constraint $q(L1,L0) + q(L0,L1) \leq 1$ this is only possible if $q(L1,L0) < 1$. Hence it is possible to increase $q(L1,L0)$ by some ϵ small enough and increase $r(H0,H1)$ by $\epsilon\delta\frac{\alpha_{H0,L0}}{\alpha_{H0,H1}}$ which satisfies all incentive constraints and changes U_c by $\epsilon\left(\alpha_{L1,L0}\pi(L1,L0) - \delta\alpha_{H0,L0}\right)$. Note that in order for (12) to be slack, $q(L0,L0) > 0$ or $q(L0,L1) > 0$. If $q(L0,L0) > 0$, the dual constraints require $\alpha_{L0,L0}\pi(L0,L0) \geq \delta\alpha_{H1,L0}$. But by definition of $\pi(\cdot,\cdot)$ and $\alpha_{\cdot,\cdot}$ this implies $\alpha_{L1,L0}\pi(L1,L0) - \delta\alpha_{H0,L0} > 0$. If

$q(L0,L1) > 0$, the dual constraint yields $\alpha_{L0,L1}\pi(L0,L1) \geq \delta\alpha_{H1,L1}$ which also implies $\alpha_{L1,L0}\pi(L1,L0) - \delta\alpha_{H0,L0} > 0$. If $q(L1,L0) + q(L0,L1) = 1$, then $q(L0,L1)$ has to be decreased by ϵ and $r(H1,H0)$ can be decreased by $\epsilon\delta\frac{\alpha_{H1,L1}}{\alpha_{H1,H0}}$ which changes U_c by $\epsilon\alpha_{L1,L0}\left[\pi(L1,L0) - \pi(L0,L1)\right]$ which is always positive. Hence in all cases the change would be profitable which contradicts optimality.

Step 2: Show that (14) must hold with equality. Suppose to the contrary that (14) is slack. Distinguish two cases:

- Incentive constraint (15) is slack.
  Then there exists an ϵ small enough, s.t. $r(H0,H1)$ can be lowered by ϵ and both (14) and (15) still hold, which contradicts optimality.

- Incentive constraint (15) binds.
  Deducting (15) from (14) and rearranging yields:

$$\alpha_{H0,L1}\left(q(L1,L0) - q(L0,L0)\right) + \alpha_{H0,L1}\left(q(L1,L1) - q(L0,L1)\right) < 0$$

  Now exactly the same argument as before can be applied.

□

Proof of Lemma 5. I show that under the requirements of the lemma, all incentive constraints (20)-(23) are satisfied.

- Constraint $(L1 \to H1)$:
  In any solution of the relaxed problem $r(H1,H0)$ is determined by equation (18), $r(L1,H0) = 0$ and hence the constraint $(L1 \to H1)$ becomes:

$$\frac{\alpha_{L1,H0}}{\alpha_{H1,H0}} \sum_{\theta_j\in\Theta_j} \left(\delta\alpha_{H1,\theta_j} q(L1,\theta_j)\right) - \sum_{\theta_j\in\Theta_j} \left(\delta\alpha_{L1,\theta_j} q(H1,\theta_j)\right) \leq 0$$

  Using $\frac{1-\beta}{\beta}\alpha_{H1,\theta_j} = \alpha_{L1,\theta_j}$ yields:

$$\sum_{\theta_j\in\Theta_j} \alpha_{L1,\theta_j}\left(q(L1,\theta_j) - q(H1,\theta_j)\right) \leq 0$$

  and in any optimal solution of the relaxed case $q(H1,\theta_j) \geq q(L1,\theta_j) \quad \forall\theta_j \in \Theta_j$.

- Constraint $(L0 \to H1)$:
  Inserting equation (18) in (21) and setting $r(L0,H1) = 0$ yields:

$$\frac{\alpha_{L0,H0}}{\alpha_{H1,H0}} \sum_{\theta_j\in\Theta_j} \delta\alpha_{H1,\theta_j} q(L1,\theta_j) - \sum_{\theta_j\in\Theta_j} \delta\alpha_{L0,\theta_j} q(H1,\theta_j) \leq 0$$

$$\Leftrightarrow \frac{1-\gamma^2}{1+\gamma^2} \sum_{\theta_j\in\Theta_j} \delta\alpha_{L1,\theta_j} q(L1,\theta_j) - \sum_{\theta_j\in\Theta_j} \delta\alpha_{L0,\theta_j} q(H1,\theta_j) \leq 0$$

  This condition holds as $\frac{1-\gamma^2}{1+\gamma^2}\alpha_{L1,\theta_j} \leq \alpha_{L0,\theta_j} \quad \forall\theta_j \in \Theta_j$ and because $(L0 \to H1)$ is satisfied.

- Constraint $(L1 \to H0)$:
  By hypothesis of this Lemma, constraint $(L0 \to H0)$ is satisfied and hence:

$$
\begin{aligned}
&\alpha_{L0,H1} r(H0,H1) - \sum_{\theta_j \in \Theta_j} \delta \alpha_{L0,\theta_j} q(H0,\theta_j) \leq 0 \\
&\Rightarrow \alpha_{L1,H1} r(H0,H1) - \frac{1-\gamma^2}{1+\gamma^2} \sum_{\theta_j \in \Theta_j} \delta \alpha_{L0,\theta_j} q(H0,\theta_j) \leq 0 \\
&\Rightarrow \alpha_{L1,H1} r(H0,H1) - \sum_{\theta_j \in \Theta_j} \delta \alpha_{L1,\theta_j} q(H0,\theta_j) \leq 0
\end{aligned}
$$

  The last conclusion follows because $\frac{1-\gamma^2}{1+\gamma^2}\alpha_{L0,\theta_j} \leq \alpha_{L1,\theta_j} \quad \forall \theta_j \in \Theta_j$. The last line represents inequality (22).

□

Proof of Lemma 6,Part ii).

- Constraint $(H1 \to H0)$:
  Suppose $(H0 \to L1)$ and $(H1 \to L1)$ bind.

$$
\begin{aligned}
&\alpha_{H1,H0} r(H1,H0) \\
&= \alpha_{H1,H0} r(L1,H0) + \delta \sum_{\theta_j \in \Theta_i} \alpha_{H1,\theta_j} q(L1,\theta_j) \\
&= \alpha_{H1,H1} \left[\frac{\alpha_{H1,H0}}{\alpha_{H1,H1}} r(L1,H0) + \delta \sum_{\theta_j \in \Theta_i} \frac{\alpha_{H1,\theta_j}}{\alpha_{H1,H1}} q(L1,\theta_j) \right] \\
&\geq \alpha_{H1,H1} \left[\frac{\alpha_{H0,H0}}{\alpha_{H0,H1}} r(L1,H0) + \delta \sum_{\theta_j \in \Theta_i} \frac{\alpha_{H0,\theta_j}}{\alpha_{H0,H1}} q(L1,\theta_j) \right] \\
&= \alpha_{H1,H1} r(H0,H1)
\end{aligned}
$$

  The last inequality comes from $\frac{\alpha_{H1,\theta_j}}{\alpha_{H1,H1}} \geq \frac{\alpha_{H0,\theta_j}}{\alpha_{H0,H1}} \quad \forall \theta_j \in \Theta_i$.

- Constraint $(H1 \to L0)$:
  Suppose that $(H1 \to L1)$ and $(L1 \to L0)$ bind. Then $\alpha_{H1,H0} r_(H0, H1) = \delta \sum_{\theta_j \in \Theta_i} \alpha_{H1,\theta_j} q(L1,\theta_j) + \alpha_{H1,H1} r(L1,H1)$. Hence it remains to show that $\sum_{\theta_j \in \Theta_i} \alpha_{H1,\theta_j} q(L1,\theta_j) \geq \sum_{\theta_j \in \Theta_i} \alpha_{H1,\theta_j} q(L0,\theta_j)$. On the other hand, $(H0 \to L1)$ and $(L0 \to H0)$ binding imply that $\sum_{\theta_j \in \Theta_i} \alpha_{L0,\theta_j} q(L1,\theta_j) - q(H0,\theta_j) \geq 0$. Note that in any solution $q(L1,H1) = q(H0,H1) = 0$ and $q(L0,\theta_j) \leq q(H0,\theta_j)$ as $\psi(L0,\theta_j) < \psi(H0,\theta_j)$. Hence if $\alpha_{L0,L0}[q(L1,L0) - q(H0,L0)] + \alpha_{L0,H0}[q(L1,L0) - q(H0,L0)] \geq 0$ the desired result follows. By the binding constraints mentioned previously, this condition has to be true if $\alpha_{L0,L1}[q(L1,L1) - q(H0,L1)] \leq 0$. For $\alpha_{L0,L1}[q(L1,L1) - q(H0,L1)] > 0$, this implies $\psi(H0,L0) > 0$, $\psi(H0,H0) > 0$ and hence $q(H0,L0) = 1$, $q(H0,H0) = 0.5$. It also implies that $\psi(H0,L1) < \psi(L1,H0) =$

$\psi(L1, L0)$ and hence $q(L0, L1) = 0$ and $q(L1, L0) = 1$. Hence in this case $q(L1, L0) - q(H0, L0) = 0$ and $q(L1, H0) - q(H0, H0) \geq 0$, which also implies $\alpha_{L0,L0}[q(L1, L0) - q(H0, L0)] + \alpha_{L0,H0}[q(L1, L0) - q(H0, L0)] \geq 0$

- Constraint $(H0 \to H1)$:
  Supposing $(H0 \to L1)$ and $(H1 \to L1)$ bind, using the same techniques as in the previous paragraph one can show that $(H0 \to H1)$ is always satisfied.

- Constraint $(H0 \to L0)$:
  Suppose $(L0 \to H0)$ binds. Then $\alpha_{H0,H1} r(H0, H1) = \alpha_{H0,H1} r(L0, H1) + \delta \sum_{\theta_j \in \Theta_i} \alpha_{H0,\theta_j} q(H0, \theta_j)$. As $\psi(H0, \theta_j) > \psi(L0, \theta_j)$ and hence $q(H0, \theta_j) \geq (L0, \theta_j)$, it directly follows that $(H0 \to L0)$ is satisfied.

- Constraint $(L1 \to H1)$:
  First note that the implication $\psi(L1, \theta_j) \geq \max\{0, \psi(\theta_j, L1)\} \Rightarrow \psi(H1, \theta_j) \geq \max\{0, \psi(\theta_j, H1)\}$ is always true because $\psi(\theta_j, L1) = \psi(\theta_j, H1)$ and $\psi(H1, \theta_j) > \psi(L1, \theta_j)$. Hence $q(H1, \theta_j) \geq q(L1, \theta_j$. Now suppose that $(H1 \to L1)$ binds. Then:

$$\begin{aligned} r(L1, H0) &= r(H1, H0) - \frac{\delta}{\alpha_{H1,H0}} \sum_{\theta_j \in \Theta_j} \alpha_{H1,\theta_j} q(L1, \theta_j) \\ &= r(H1, H0) - \frac{\delta}{\alpha_{L1,H0}} \sum_{\theta_j \in \Theta_j} \alpha_{L1,\theta_j} q(L1, \theta_j) \\ &\geq r(H1, H0) - \frac{\delta}{\alpha_{L1,H0}} \sum_{\theta_j \in \Theta_j} \alpha_{L1,\theta_j} q(H1, \theta_j) \end{aligned}$$

  Where the last inequality holds because $q(H1, \theta_j) \geq q(L1, \theta_j$.

- Constraint $(L1 \to H0)$:
  Suppose $(L1 \to L0)$ binds and $(L0 \to H0)$ is satisfied, then:

$$\begin{aligned} &\alpha_{L1,H0} r(L1, H0) = \alpha_{L1,H1} r(L0, H1) \\ &\geq \alpha_{L1,H1} r(H0, H1) - \delta \sum_{\theta_j \in \Theta_i} \frac{\alpha_{L1,H1}}{\alpha_{L0,H1}} \alpha_{L0,\theta_j} q(L1, \theta_j) \\ &\geq \alpha_{L1,H1} r(H0, H1) - \delta \sum_{\theta_j \in \Theta_i} \alpha_{L1,\theta_j} q(L1, \theta_j) \end{aligned}$$

  The first inequality is because $(L0 \to H0)$ is satisfied and the second inequality holds because $\frac{\alpha_{L1,H1}}{\alpha_{L0,H1}} \alpha_{L0,\theta_j} \geq \alpha_{L1,\theta_j} \quad \forall \theta_j \in \Theta_i$.

- Constraint $(L0 \to L1)$:
  If $(L1 \to L0)$ binds this directly implies that $r(L1, H0) = \frac{\alpha_{L1,H1}}{\alpha_{L1,H0}} r(L0, H1) < r(L0, H1)$ and hence $(L0 \to L1)$ is satisfied.

- Constraint $(L0 \to H1)$:
  Pretending to be type $H1$ changes the expected payoff for an $L1$ firm as

follows:

$$
\begin{aligned}
&\alpha_{L0,H0}r(H1,H0) - \delta \sum_{\theta_j \in \Theta_i} \alpha_{L0,\theta_j} q(H1,\theta_j) - \alpha_{L0,H1}r(L0,H1) \\
=&\frac{\alpha_{L0,H0}}{\alpha_{H1,H0}} \delta \sum_{\theta_j \in \Theta_i} \alpha_{H1,\theta_j} q(H1,\theta_j) - \delta \sum_{\theta_j \in \Theta_i} \alpha_{L0,\theta_j} q(H1,\theta_j) \\
&+ \alpha_{L0,H0}r(L1,H0) - \alpha_{L0,H1}r(L0,H1) \\
\leq&\delta \sum_{\theta_j \in \Theta_i} \alpha_{L0,\theta_j} q(H1,\theta_j) - \delta \sum_{\theta_j \in \Theta_i} \alpha_{L0,\theta_j} q(H1,\theta_j) \\
\leq&0
\end{aligned}
$$

The first equality uses the fact that $(H1 \to L1)$ binds. The next inequality is because $\frac{\alpha_{L0,H0}\alpha_{H1,\theta_j}}{\alpha_{H1,H0}} \leq 1 \quad \forall \theta_j \in \Theta_i$ and in optimum $r(L1,H0) \leq r(L0,H1)$. The last inequality is because in optimum $q(H1,\theta_j) \geq q(L1,\theta_j) \quad \forall \theta_j \in \Theta_i$. □

References

Alger, I., and F. Salanie (2004): "A Theory of Fraud and Over-Consumption in Experts Markets," Discussion paper, Boston College Department of Economics, available at http://ideas.repec.org/p/boc/bocoec/495.html.

Armstrong, M. (2000): "Optimal Multi-Object Auctions," *Review of Economic Studies*, 67(3), 455 – 481.

Cremer, J., and R. P. McLean (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 56(6), 1247 – 57.

Darby, M. R., and E. Karni (1973): "Free Competition and the Optimal Amount of Fraud," *Journal of Law and Economics*, 16(1), 67 – 88.

Dulleck, U., and R. Kerschbamer (2006): "On Doctors, Mechanics and Computer Specialists," *Journal of Economic Literature*, 44(1), 5–42.

Emons, W. (1997): "Credence goods and fraudulent experts," *RAND Journal of Economics*, 28(1), 107–119.

——— (2001): "Credence Goods Monopolists," *International Journal of Industrial Organization*, 19(3-4), 375–389.

Luenberger, D. G. (1989): *Linear and nonlinear programming*. Addison-Wesley, Reading, Mass, 2. edn.

Pesendorfer, W., and A. Wolinsky (2003): "Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice," *Review of Economic Studies*, 70(2), 417 – 437.

Pitchik, C., and A. Schotter (1987): "Honesty in a Model of Strategic Information Transmission," *American Economic Review*, 77(5), 1032 – 36.

Rochet, J.-C., and L. A. Stole (2003): "The Economics of Multidimensional Screening," in *Advances in Economics and Econometrics: Theory and Applications - Eight World Congress*, ed. by M. Dewatripont, L. P. Hansen, and S. J. Turnowsky. Cambridge University Press.

Wolinsky, A. (1993): "Competition in a Market for Informed Experts' Services," *RAND Journal of Economics*, 24(3), 380 – 398.

——— (1995): "Competition in Markets for Credence Goods," *Journal of Theoretical and Institutional Economics*, 151(1), 117 – 131.