# Fiscal Policy, Maintenance Allowances and Expectation-Driven Business Cycles *

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**Fiscal Policy, Maintenance Allowances and the Business Cycle**

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**Abstract**

Maintenance and repair activity appears to be a quantitatively significant feature of modern industrial economies. Within a real business cycle model with arguably small aggregate increasing returns, this paper assesses the stabilizing effects of fiscal policies with a maintenance expenditure allowance. In this setup, firms are authorized to deduct their maintenance and repair expenditures from revenues in calculating pre-tax profits, as in many prevailing tax codes. While flat rate taxation does not prove useful to insulate the economy from self-fulfilling beliefs, a progressive tax can render the equilibrium unique. However, we show that the required progressivity to protect the economy against sunspot-driven fluctuations is increasing in the maintenance-to-GDP ratio. Taking into account the maintenance and repair activity of firms, and the tax deductability of the related expenditures, would then weaken the expected stabilizing properties of progressive fiscal schedules.

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# 1 Introduction

In recent years, there has been an extensive literature that examines the existence of multiple, self-fulfilling rational expectations equilibria in dynamic general equilibrium models. For example, Benhabib and Farmer [2] and Farmer and Guo [13] have shown that a one-sector real business cycle (RBC) model with sufficient aggregate increasing returns-to-scale may exhibit an indeterminate steady state (i.e. a sink) that can be exploited to generate business cycles driven by animal spirits.[1] By emphasizing expectations as an independent source of shocks, these so-called "sunspot" models create an opportunity for stabilization policies that are designed to mitigate belief-driven cycles. Following this idea,

[1] we use the terms "animal spirits", "sunspots" and self-fulfilling beliefs" interchangeably. All refer to any randomness in the economy that is not related to uncertainties about economic fundamentals such as technology, preferences and endowments.

Guo and Lansing [17] have shown that a progressive income tax policy can ensure saddle path stability in the Benhabib-Farmer-Guo model, and thereby stabilize[2] the economy against "self-fulfilling beliefs".[3] However, this literature assumes that the laissez-faire economy is subject to large and implausibly high increasing returns to scale (Burnside [7]; Basu and Fernald [1]). As pointed out by Christiano and Harrison ([10] p.20), the desirability of stabilizing the economy against sunspot fluctuations is determined by the relative magnitude of two opposing factors. First, ceteris paribus, a concave utility function implies that a sunspot equilibrium is welfare-inferior to a constant, deterministic equilibrium (*concavity* or risk-aversion effect). However, other things are not the same. The increasing returns means that by bunching hard work, consumption can be increased on average without raising the average level of employment (*bunching* effect). As a consequence, when increasing returns are strong enough, the bunching effect may dominate the concavity effect, so that volatile paths may indeed improve welfare, in comparison with stationary allocations. In that situation, one may question the desirability of any stabilization policy.

Although initial versions of these models appear to rely on empirically implausible parameter values, recent vintages are based on increasingly realistic foundations. Many authors have shown that RBC models with multiple sectors of production (Benhabib and Farmer [3]; Perli [25]; Weder [27]); Harrison [19]) or endogenous capital utilization (Wen [29]) can generate local indeterminacy with much lower degrees of increasing returns.[4] Weder [28] introduces a new formulation of the endogenous capital utilization, in which the utilization costs appear in the form of variable maintenance expenses, and shows that indeterminacy can arise at approximately constant returns to scale, challenging the viewpoint that indeterminacy is empirically implausible. In a recent paper, Guo and Lansing [18] explore the effects of introducing maintenance and repair expenditures in Wen's variable capacity utilization model, and also show that indeterminacy can occur with a mild degree of increasing returns.

As a matter of fact, the latest developments of these models allow to study indeterminacy and sunspots for close-to-constant returns, that is when the "bunching effect" is (very) weak. One suspects then expectation-driven volatility to unambiguously lead to welfare losses (by the concavity, or risk-aversion, effect) that would call for stabilization. This proves useful to re-investigate the stabilizing properties of fiscal progressivity in the close-to-constant returns to scale

[2] Here, we adopt the common view that a policy is stabilizing when it leads to saddle-point stability, hence to determinacy.

[3] See Benhabib and Farmer [4] for a survey of recent developments in this area

[4] With the noted exceptions of Benhabib and Nishimura [5]; Benhabib, Meng and Nishimura [6], and Nishimura, Shimomura and Wang [24], among others, most studies in this literature postulate constant returns-to-scale at the individual firm level. We maintain this assumption throughout the analysis.

case, where stabilization is *a priori* more desirable from a welfare standpoint.

In this paper, we investigate how the stabilizing power of fiscal progressivity, initially pushed forward in this literature by Christiano and Harrison [10] and Guo and Lansing [17], is affected when firms are authorized to deduct their maintenance expenditures from revenues in calculating pre-tax profits (as in many prevailing tax codes [5]). Because of tax ramifications of categorizing an expenditure as either maintenance and repair or investment, there are standard definitions used in the accounting literature. *Maintenance and repair* expenditures are made for the purpose of keeping the stock of fixed assets or productive capacity in good working order during the life originally intended. These include costs incurred to forestall breakdowns of equipment and structures (*maintenance*) and costs induced to restore fixed assets to a state of good working condition after malfunctioning (*repair*). Capital expenditures, or *investment spending*, are costs of all new plants, machinery and equipment which normally have a life of more than a year; these expenditures include purchases of new assets as well as major improvements or alterations to existing assets.

In a continuous-time version of the Guo and Lansing [18] maintenance expenditures model, we find that introducing maintenance allowances weakens the expected stabilizing properties of tax progressivity. Although a progressive tax can still render the equilibrium unique, we show that the required degree of progressivity to protect the economy against sunspot-driven fluctuations is increasing in the maintenance-to-GDP ratio. Put differently, the possibility for firms to deduct maintenance and repair expenditures from their pre-tax profits increases the likelihood of local indeterminacy and excess volatility due to animal spirits. Moreover, a flat tax schedule does not prove to be a useful and effective stabilizer.

It has been argued and documented (see for instance Mc Grattan and Schmitz [23]), that maintenance expenditures are "too big to ignore", strongly procyclical and important potential substitutes for investment. This substitutability feature can be used to provide an intuitive discussion of the basic mechanism driving our result. Let us suppose agents have optimistic expectations about, say, a higher return on capital in the next period. Firms will naturally want to invest more in the form of capital. But, due to the fiscal scheme progressivity, they know they will have to face in that case a higher tax rate. Thus, instead of investing in new physical capital (equipments or structures), firms prefer to substitute maintenance to investment. The consequent reduction in the tax base implies that a higher level of fiscal

[5] As an illustration, in the United States "it has been held that expenses for small parts of a large machine, made in order to keep the machine in efficient working condition, were deductible expenses and not capital expenditures even though they may have a life of two or three years" (Commerce Clearing House, Chicago, Standard Federal Tax Reports, 1999, p.22, 182 [9])

progressivity will be needed to stabilize the economy against belief-driven cycles.

Our result can be linked to a parallel strand of the literature, investigating the stabilizing properties of non-linear tax schedules in constant returns to scale, segmented asset markets economies (see for instance Lloyd-Braga, Modesto and Seegmuller [22]). In a monetary economy with constant returns to scale, Dromel and Pintus [11] show that tax progressivity reduces, in parameter space, the likelihood of local indeterminacy, sunspots and cycles. However, considering plausibly low levels of tax progressivity does not ensure saddle-point stability and preserves as robust the occurrence of sunspot equilibria and endogenous cycles. Exploiting a different mechanism, our paper gives also support to the view that low levels of tax progressivity may not be able to ensure the determinacy of equilibria.

The remainder of this paper proceeds as follows. The next section presents the model, while section 3 analyses dynamics and (in)determinacy conditions, showing how fiscal progressivity may lead to saddle-path stability. Some concluding remarks are gathered in section 4.

# 2 The Economy

This paper introduces fiscal policy, depreciation allowance and maintenance expenditures deductions into a continuous-time version of the Guo and Lansing [18] model. The decentralized economy consists of an infinite lived representative household that supplies labor, taking the real wage as given. The household owns a representative firm, acting in his best interest while making decisions about production, investment, maintenance and capital utilization.

## 2.1 Firms

There is a continuum of identical competitive firms, with the total normalized to unity, acting so as to maximize a discounted stream of profits. The representative firm $i$ is endowed with $k_0$ units of capital and produces an homogeneous final good $y_i$ using the following Cobb-Douglas technology

$$y_i = \bar{e}(u_i k_i)^{\alpha} n_i^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{1}$$

where $k_i$ and $n_i$ are firm $i$'s usage of physical capital (equipment and structures) and labor hours, respectively[6]. The variable $u_i \in (0,1)$ designates the capital utilization rate. Although each firm is competitive, we assume that the economy as a whole is affected by organizational synergies that cause the output of the $i$th firm to be higher if all other firms in the economy are producing more. These productive external effects, denoted by $\bar{e}$, are outside of the scope of the market, and cannot be traded. Taken as given by each firm, they are specified as

$$\bar{e} = (\bar{u}\bar{k})^{\alpha\eta}\bar{n}^{(1-\alpha)\eta}, \qquad \eta \geq 0 \tag{2}$$

where $\bar{u}\bar{k}$ and $\bar{n}$ are economy-wide average levels of utilized capital and production labor inputs, respectively. We look at a symmetric equilibrium, in which all firms would take the same actions such that $u_i = \bar{u} = u$, $k_i = \bar{k} = k$ and $n_i = \bar{n} = n$, for all $t$. As a result, equation (2) can be substituted into equation (1) to obtain the following aggregate production technology, that may display increasing returns-to-scale:

$$y = \left[(uk)^{\alpha} n^{1-\alpha}\right]^{1+\eta} \tag{3}$$

where $1+\eta$ characterizes the degree of aggregate increasing returns. When $\eta = 0$, the model boils down to the standard Ramsey formulation with constant returns-to-scale at both private and social levels.

We assume an endogenous capital depreciation rate, $\delta \in (0,1)$, such that

$$\delta = \chi \frac{u^{\theta}}{(\frac{m}{k})^{\phi}}, \qquad \chi > 0, \quad \theta > 1, \quad \phi \geq 0 \tag{4}$$

where $m$ represents maintenance and repair expenditures. The ratio $m/k$ denotes the magnitude of the maintenance and repair per unit of capital. When the depreciation elasticity to maintenance ($\phi$) is positive, a rise in maintenance activity will lower capital depreciation. On the other hand, an increase in the capital utilization rate $u_t$ will speed up the depreciation. If $\phi = 0$, the model resembles the one analyzed in Wen [29], while if $\theta \to \infty$, it reduces to an economy with constant utilization like in the standard Benhabib-Farmer-Guo setup.

## 2.2 Households

The economy is populated by a large number of identical Ramsey households, each endowed with one unit of time, choosing their consumption $c_t$ and labor supply $n_t$ so as to maximize:

$$\int_0^{\infty} e^{-\rho t}\left\{\log[c] - A\frac{n^{1+\gamma}}{1+\gamma}\right\}dt, \tag{5}$$

[6] To save on notation, time dependence of all variables will be dropped in the sequel.

where $\rho > 0$ is the discount rate, A is a scaling parameter, $\gamma \geq 0$ is the inverse of the intertemporal elasticity of substitution in labor supply. The representative consumer owns the inputs and rents them to firms through competitive markets. We can write down the consolidated budget constraint as:

$$\dot{k} = (1-\tau)x - c, \quad \text{with} \quad x = y - \delta k - m \tag{6}$$

where $1 > \tau \geq 0$ is the tax rate imposed on the income net of capital depreciation and maintenance expenditures. We assume the capital stock is predetermined $k(0) = k_0$, and both consumption and capital are non-negative $k \geq 0$, $c \geq 0$.

## 2.3 Government

The government chooses tax policy $\tau$ and balances the public budget at each point in time. Hence, the instantaneous government budget constraint is $g = \tau x$, where $g$ represents government spending on goods and services that are assumed not to contribute to either production or household utility. The aggregate resource constraint of the economy is given by:

$$c + \dot{k} + \delta k + g + m = y$$

The government is assumed to set $\tau$ according to the following tax schedule:

$$\tau = 1 - \nu\left(\frac{\bar{x}}{x}\right)^{\psi}, \qquad \nu \in (0,1); \qquad \psi \in (0,1) \tag{7}$$

where $\bar{x}$ denotes a base level of income, net of depreciation and maintenance, that is taken as given. Here, $\bar{x}$ is set to the steady-state level of that income. The parameters $\nu$ and $\psi$ govern the level and slope of the tax schedule, respectively. When $\psi > 0$, the tax rate $\tau$ increases with the household's taxable income, that is, households with taxable income above $\bar{x}$ face a higher tax rate than those with income below $\bar{x}$. When $\psi = 0$, all households face the same tax rate $1 - \nu$ regardless of their taxable income. For sake of simplicity, we only consider here flat and progressive taxation. [7]

In making decisions about how much to consume, work, invest in new capital, and spend on maintenance of existing capital over their lifetimes, households take into account the way in which the tax schedule affects their earnings. To understand the progressivity feature of the above tax schedule, it is useful to distinguish between the average and

[7] However, the present analysis could easily be adapted to the case of weak regressivity (i.e. when $\psi < 0$), as in Guo and Lansing [17]. For instance, our results still hold if $\psi \in ((\alpha_k - 1)/\alpha_k, 1)$, $\alpha_k$ being the elasticity of aggregate output with respect to capital, to be precisely defined later on in the text.

marginal tax rates. The average tax rate $\tau$, given by (7), is equal to the total taxes paid by each household divided by its taxable income $x$. The marginal tax rate $\tau_m$ is defined as the change in taxes paid divided by the change in taxable income. The expression for $\tau_m$ is

$$\tau_m = \frac{\partial(\tau x)}{\partial x} = 1 - (1-\psi)\nu\left(\frac{\bar{x}}{x}\right)^{\psi} \tag{8}$$

We require $\tau < 1$ to prevent government from confiscating all productive resources, and $\tau_m < 1$ so that households have an incentive to supply labor and capital services to firms. From (7) and (8), we notice that $\tau_m = \tau + \nu\psi(\bar{x}/x)^{\psi}$. Therefore, the marginal tax rate will be above the average tax rate when $\psi > 0$. In this case, the tax schedule is said to be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the value $(1-\nu)$ and the tax schedule is said to be "flat".

## 2.4 Intertemporal Equilibria

Households' decisions follow from maximizing (5) subject to the budget constraint (6), given the initial capital stock $k(0) \geq 0$. Straightforward computations yield the following first-order conditions:

$$n \quad : \qquad Acn^{\gamma} = (1-\tau_m)(1-\alpha)\frac{y}{n} \tag{9}$$

$$u \quad : \qquad \frac{\alpha}{\theta}\frac{y}{k} = \delta \tag{10}$$

$$m \quad : \qquad 1 = \phi\frac{\delta k}{m} \tag{11}$$

$$k \quad : \qquad \frac{\dot{c}}{c} = (1-\tau_m)\frac{\alpha\big[\theta - (1+\phi)\big]}{\theta}\frac{y}{k} - \rho \tag{12}$$

where (9) equates the slope of the representative household's indifference curve (utility trade-off between leisure and consumption) to the after-tax real wage. Equation (12) is the consumption Euler equation. Equation (10) shows that the firm utilizes capital to the point where the marginal benefit of more output is equal to the marginal cost of faster depreciation. Equation (11) shows that the firm undertakes maintenance activity to the point where one unit of goods devoted to maintenance is equal to the marginal reduction in the firm's depreciation expense. Notice that the household's decisions regarding labor supply and capital investment are governed by the marginal tax rate $\tau_m$.

We may rewrite the budget constraint, from (6), as:

$$\dot{k}/k = (1-\tau)x/k - c/k. \tag{13}$$

Equations (3), (9)-(12) and (13) characterize the dynamics of intertemporal equilibria with perfect foresight, given $k(0)$.

The transversality condition writes as:

$$\lim_{t \to +\infty} e^{-\rho t} \frac{k}{c} = 0 \tag{14}$$

It is easily checked that the transversality constraint is met in the following analysis, as we consider orbits that converge towards an interior steady state. From equation (10), we get $\delta k = (\alpha/\theta)y$ which gives, when plugged into (11),

$$m = \frac{\phi\alpha}{\theta} y$$

The equilibrium maintenance-to-GDP ratio is constant, and maintenance expenditures perfectly correlated with output. This reminds us the procyclicality of maintenance documented by McGrattan and Schmitz [23].

As we want to characterize the reduced-form social technology as a function of $k$ and $n$, we use equations (7) and (10) to solve [8] for $u$.

$$u = \left[ \left( \frac{\phi}{\theta} \right)^{\phi} \left( \frac{\alpha y}{k} \right)^{1+\phi} \right]^{\frac{1}{\theta}}$$

Then we substitute this optimal rate of capacity utilization into (3) to finally get

$$y = B k^{\alpha_k} n^{\alpha_n}$$

where the $B$, $\alpha_k$ and $\alpha_n$ write as:

$$B = \left[ \left( \frac{\phi}{\theta} \right)^{\phi} \alpha^{(1+\phi)} \right]^{\frac{\alpha(1+\eta)}{\theta - \alpha(1-\eta)(1+\phi)}}$$

$$\alpha_k = \frac{\alpha(1+\eta)(\theta - 1 - \phi)}{\theta - \alpha(1+\eta)(1+\phi)} \tag{15}$$

$$\alpha_n = \frac{(1-\alpha)(1+\eta)\theta}{\theta - \alpha(1+\eta)(1+\phi)} \tag{16}$$

We restrict our attention to the case where $\alpha_k < 1 \Leftrightarrow 1 > \alpha(1+\eta)$, so that the externality on capital is not strong enough to generate sustained endogenous growth. We further assume that $\theta - 1 - \phi > 0$ to guarantee $\alpha_k > 0$. Equations (15) and (16) together imply $\partial(\alpha_k + \alpha_n)/\partial\phi > 0$ whenever $\eta > 0$. Hence, a higher degree of aggregate increasing returns can be achieved through an rise in $\phi$.

[8] Since the parameter $\chi$ has no independent influence on the model's steady-state and dynamics around the steady-state, we simply set $\chi = 1/\theta$. Note that if we set $\phi$ to zero (when capital depreciation is inelastic to the maintenance activity), we recover the same optimal capacity utilization as in Wen [29].

# 3 Analysis of the Dynamics

In Appendix A, we linearize equations (3) and (12)-(13) around an interior steady state. Straightforward computations yield the expressions for the trace $T$ and the determinant $D$ of the Jacobian matrix of the dynamical system (cf. Appendix B).

**Proposition 3.1** (Linearized Dynamics around a Steady State)**.**

*Linearized dynamics for deviations $\hat{c}-\hat{c}^*$ and $\hat{k}-\hat{k}^*$ are determined by linear map such that, in steady-state:*

$$\begin{aligned} T &= \frac{\rho(1+\eta)\Big\{(\gamma+1)\Big[\theta-\alpha(1+\phi)\Big]-(1-\alpha)\theta\Big\}}{\Big[\theta-(1+\phi)\alpha(1+\eta)\Big]\Big[\gamma+1-(1-\psi)\alpha_n\Big]} \\ D &= \frac{(\gamma+1)\Big[\alpha_k(1-\psi)-1\Big]}{\gamma+1-(1-\psi)\alpha_n}\frac{\rho^2\Big[\theta-\alpha(1+\phi)\Big]}{\alpha(1-\psi)\Big[\theta-(1+\phi)\Big]}, \end{aligned} \tag{17}$$

## 3.1 Local Determinacy with Progressive Taxes

Indeterminacy is defined as follows.

**Definition 3.1** (Indeterminacy of the Steady State)**.**

*The equilibrium is indeterminate if there exists an infinite number of perfect foresight equilibrium sequences.*

The variable $k$ is predetermined since $k_0$ is given by the initial conditions of the economy while $c_0$ is free to be determined by the behavior of the agents in the economy. Suppose that the steady-state $\{k^*, c^*\}$ is completely stable in the sense that all equilibrium trajectories which begin in the neighborhood of $\{k^*, c^*\}$ converge back to the steady state. In this case, there will be a continuum of equilibrium path $\{k(t), c(t)\}$, indexed by $c_0$, since any path that converges to $\{k^*, c^*\}$ necessarily satisfies the transversality condition. Completely stable steady states giving rise to a continuum of equilibria are termed indeterminate and in this case the stable manifold has dimension 2. Indeterminacy requires that both eigenvalues of the Jacobian matrix have negative real parts (the steady-state is a sink).

— Figure 1 about here —

Alternatively, if there is a one-dimensional manifold in $\{k, c\}$ space with the property that trajectories that begin on

this manifold converge to the steady-state but all other trajectories diverge then the equilibrium will be locally unique in the neighborhood of the steady-state. In this case, for every $k_0$ in the neighborhood of $k^*$ there will exist a unique $c_0$ in the neighborhood of $c^*$ that generates a trajectory converging to $\{k^*, c^*\}$. This $c_0$ is the one that places the economy on the stable branch of the saddle point $\{k^*, c^*\}$.

— Figure 2 about here —

Since the Trace of the Jacobian measures the sum of the roots and the Determinant measures the product we can use information on the sign of the Trace and the Determinant to check the dimension of the stable manifold of the steady-state $\{k^*, c^*\}$. Indeterminacy can be restated as $T < 0 < D$. Similarly the steady-state is saddle-path stable if $D < 0$, and is unstable (a source) if $T > 0$ and $D > 0$. Since the eventual fate of trajectories that diverge from the steady state cannot be determined from the properties of the Jacobian evaluated at the steady state, we will not further elaborate on the source case [9].

— Figure 3 about here —

Let us notice that $\nu$, the parameter characterizing the level of the fiscal schedule, does not appear neither in the Trace, nor in the Determinant (although it affects the steady state, cf. A1). Put differently, $\nu$ only affects the level of the steady-state, but not the dynamics around it. Hence, when the tax progressivity parameter is set to zero, the flat-rate fiscal structure does not seem to have any effect on the dynamics, in the neighborhood of a stationary state. This result complements some recent conclusions underlining that flat-rate taxation does *not* promote macroeconomic stability (see e.g., among others, Dromel and Pintus [11], Dromel and Pintus [12]).

Our main task is now to underline the conditions such that the steady-state is locally determinate or indeterminate. Direct inspection of equations (17) gives the following Proposition:

**Proposition 3.2** (Local Stability of the Steady-State)**.**

[9] Trajectories may eventually violate non-negativity constraints or may settle down to a limit cycle or to some more complicated attracting set

*Assume* $0 < \alpha_k < 1$. *Then the following holds:* $(1-\psi)\alpha_n > \gamma + 1$ *is a necessary and sufficient condition for the occurrence of local indeterminacy.* ***Proof.*** *C.f. Appendix C.*

**Corollary 3.1** (Local Indeterminacy and Sunspots)**.**

*Assume* $0 < \alpha_k < 1$. *Then, the following holds:*

*in the economy without taxes or with linear taxes, (that is, when* $\psi = 0$*), the local dynamics of consumption* $c$ *and capital* $k$ *given by Eqs. (3) and (12)-(13) around the positive steady-state* $(c^*, k^*)$ *exhibit local indeterminacy (that is* $T < 0 < D$*) if an only if* $\alpha_n > \gamma + 1$.

**Corollary 3.2** (Saddle-Path Stability through Progressive Taxation)**.**

*Assume* $0 < \alpha_k < 1$. *Then, the following holds:*

*in the economy with progressive income taxes (that is, when* $\psi \in (0,1)$*, the local dynamics of* $c$ *and* $k$ *given by Eqs. (3) and (12)-(13) exhibit saddle-path stability (that is,* $D < 0$ *if and only if*
$\psi > \psi_{\min} = \frac{(1-\alpha)(1+\eta)\theta - (\gamma+1)\left[\theta - \alpha(1+\eta)(1+\phi)\right]}{(1-\alpha)(1+\eta)\theta}$.

A particular threshold of fiscal progressivity is thus able to immunize the economy from local indeterminacy. However, it is straightforward to show that $\psi_{\min}$ is decreasing in $\theta$ while increasing in $\phi$ (cf. Appendix D). When the tax code displays some capital depreciation allowance and maintenance/repair deductions (as in the US tax code) the required degree of fiscal progressivity to protect the economy against sunspot-driven fluctuations is increasing in the equilibrium maintenance-to-GDP ratio. Consequently, the possibility for firms to deduct maintenance and repair expenditures from their pre-tax profit tends to weaken the stabilizing power of progressive fiscal schemes established by Guo and Lansing [17] and Guo [15] among others.

As shown in Fig. 4 and Fig. 5, the geometrical locus depicting the sensitivity of $\psi_{\min}$ to $\eta$ is upward sloping and concave. An increase in the equilibrium maintenance ratio (achieved though an increase in $\phi$ or a decrease in $\theta$) translates this locus upward (its slope remains exactly the same, regardless of the level of the maintenance-to-GDP ratio) (cf. Appendix E). Hence, for a given level of externalities in the economy, the tax deduction on maintenance and repair expenditures makes local indeterminacy more likely, since a higher level of fiscal progressivity is required to insulate the economy from belief-driven fluctuations.

— Figure 4 and 5 about here —

The recent empirical literature has shown that maintenance and repair activity is "too big to ignore" (Mc Grattan and Schmitz [23]): for instance, in Canada, expenditures devoted to maintenance and repair of existing equipment and structures averaged 6.1 percent of GDP from 1961 to 1993. Accordingly, in our theoretical setup, one could possibly expect an arguably high tax progressivity threshold necessary to eliminate indeterminacy.

## 3.2 Intuition

To gain insight into the mechanism that drives our result, it is useful to analyse how maintenance activity affects the equilibrium elasticity of social output with respect to labor $\alpha_n$ (cf. Appendix F). It is easy to check that $\alpha_n$ is decreasing in $\theta$, and increasing in $\phi$, thus increasing in the equilibrium maintenance-to-GDP ratio.

The sufficient condition for saddle-path stability in this model is

$$(1-\psi)\alpha_n - 1 < \gamma$$

We clearly see that the higher $\psi$, the lower the left-hand-side, and the lower the likelihood of indeterminacy. Given the fact that the elasticity of output with respect to labor is higher when firms undertake maintenance activity, and a fortiori even more when the maintenance-to-GDP ratio is increased, $\psi_{min}$ (the level of fiscal progressivity needed to render the equilibrium unique) is also higher.

The procyclicality of maintenance expenditures, assumed in the model, explains intuitively the excess of volatility added to the economy. On the empirical ground, these procyclical properties have been well established and documented by Mc Grattan and Schmitz [23]. Using some unique survey data for Canada, these authors find that detrended maintenance and repair expenditures in Canada are strongly procyclical, exhibiting a correlation coefficient with GDP of 0.89. The Canadian survey also suggests that the activities of maintenance and repair and investment are to some degree close *substitutes* for each other. For example, during slumps, maintenance and repair spending falls in a lower extent than investment spending does. Similarly, during booms, maintenance and repair expenditures increase less than investment does. The standard deviation of maintenance and repair expenditures only represents 60 percent of the investment spending standard deviation, the difference being even sharper in the manufacturing industry. This tends to push forward the idea that during crises, new capital acquisitions are postponed, and existing equipment/structures are maintained and repaired to a larger extent. In other words, there would be a good deal of substitutability over the

business cycle between the activities of maintenance and of investment.

This substitutability property can be used to provide an intuitive discussion of the basic mechanism driving our result. Let us suppose agents have optimistic expectations about, say, a higher return on capital in the next period. Firms will naturally want to invest more in the form of capital. But, due to the fiscal scheme progressivity, they know they will have to face in that case a higher tax rate. Thus, instead of investing in new physical capital (equipments or structures), firms will prefer to substitute maintenance to investment. The consequent reduction in the tax base implies that a higher level of fiscal progressivity will be needed to stabilize the economy against belief-driven cycles.

# 4 Conclusion

Maintenance and repair activity appears to be a quantitatively significant feature of modern industrial economies. Within a real business cycle model with arguably small aggregate increasing returns, this paper assesses the stabilizing effects of fiscal policies with a maintenance expenditure allowance. In this setup, firms are authorized to deduct their maintenance and repair expenditures from revenues in calculating pre-tax profits, as in many prevailing tax codes. While flat rate taxation does not prove useful to insulate the economy from self-fulfilling beliefs, a progressive tax can render the equilibrium unique. However, we show that the required progressivity to protect the economy against sunspot-driven fluctuations is increasing in the maintenance-to-GDP ratio. Taking into account the maintenance and repair activity of firms, and the tax deductability of the related expenditures, would then weaken the expected stabilizing properties of progressive fiscal schedules.

Some directions for further research seem natural. A calibration and simulation exercise would be useful in assessing the stabilizing level of fiscal progressivity in this economy and its plausibility. Indeed it remains to be seen if, for plausible values of increasing returns and realistic progressivity features, self-fulfilling beliefs can be a reasonable explanation for the excess of aggregate volatility. In addition, it seems relevant to introduce in this setup, following Guo [15], different progressivity features for labor and capital income, consistent with many OECD countries tax codes. Also, this paper (as many of the contributions in the area) consider fiscal progressivity with a continuously increasing marginal tax rate, which is not a shared feature by most actual tax schedules, as casual observation suggests. Considering linearly progressive taxation instead (as in Dromel and Pintus [12]) could be of interest, in order to get closer to the tax codes

with brackets prevailing in most developed economies.

# Appendix

## A Linearized Dynamics

The dynamics of intertemporal equilibria with perfect foresight, given $k(0)$, are characterized by

$$\begin{aligned}\frac{\dot{c}}{c} &= (1-\tau_m)\alpha\frac{\left[\theta-(1+\phi)\right]}{\theta}\frac{y}{k}-\rho\\ \frac{\dot{k}}{k} &= (1-\tau)\frac{(y-\delta k-m)}{k}-\frac{c}{k}\end{aligned}$$

To facilitate our analysis, we make the following logarithmic transformation of variables: $\hat{c}=\log(c)$, $\hat{k}=\log(k)$ and $\hat{y}=\log(y)$. With this transformation, the equilibrium conditions (9)-(14)can be rewritten as

$$\begin{aligned}\dot{\hat{c}} &= \alpha\nu(1-\psi)\bar{x}^{\psi}\frac{\left[\theta-(1+\phi)\right]}{\theta}\frac{\left[\theta-\alpha(1+\phi)\right]}{\theta}^{-\psi}e^{(1-\psi)\hat{y}-\hat{k}}-\rho\\ \dot{\hat{k}} &= \nu\bar{x}^{\psi}\frac{\left[\theta-\alpha(1+\phi)\right]}{\theta}^{1-\psi}e^{(1-\psi)\hat{y}-\hat{k}}-e^{\hat{c}-\hat{k}}\end{aligned}$$

where $\hat{c}=\log(c)$, $\hat{k}=\log(k)$ and $\hat{y}=\log(y)$.

We can obtain, from the static condition in (12), the following equation:

$$[\gamma+1-(1-\psi)\alpha_n]\hat{n}=\log\frac{\Gamma}{A}+(1-\psi)\log B+(1-\psi)\alpha_k\hat{k}-\hat{c}, \tag{18}$$

where $\Gamma=(1-\alpha)\nu(1-\psi)\bar{x}^{\psi}\left[\frac{\theta-\alpha(1+\phi)}{\theta}\right]^{-\psi}$. This yields, by using equation (18):

$$(1-\psi)\hat{y}-\hat{k}=\xi_0+\xi_1\hat{k}+\xi_2\hat{c},$$

where

$$\xi_0 = (1-\psi)Z \tag{19}$$

$$\xi_1 = \frac{(1-\psi)\alpha_n+(\gamma+1)[\alpha_k(1-\psi)-1]}{\gamma+1-(1-\psi)\alpha_n} \tag{20}$$

$$\xi_2 = -\frac{\alpha_n(1-\psi)}{\gamma+1-(1-\psi)\alpha_n} \tag{21}$$

where $Z=\log B+\frac{\alpha_n}{\gamma+1-(1-\psi)\alpha_n}\left[\log\frac{\Gamma}{A}+(1-\psi)\log B\right]$. By rewriting equations (12)-(13) in logs and using equations (19)-(21), it is easy to get:

$$\begin{aligned}\dot{\hat{c}} &= \alpha\nu(1-\psi)(\bar{x})^{\psi}\frac{\theta-(1+\phi)}{\theta}\frac{\left[\theta-\alpha(1+\phi)\right]}{\theta}^{-\psi}e^{\xi_0+\xi_1\hat{k}+\xi_2\hat{c}}-\rho,\\ \dot{\hat{k}} &= \nu(\bar{x})^{\psi}\frac{\left[\theta-\alpha(1+\phi)\right]}{\theta}^{1-\psi}e^{\xi_0+\xi_1\hat{k}+\xi_2\hat{c}}-e^{\hat{c}-\hat{k}}\end{aligned} \tag{22}$$

It is straightforward to show that, under our assumptions, the differential equations (22) possess a steady state $\hat{c}^*, \hat{k}^*$. More precisely, $\dot{\hat{c}} = 0$ yields, from the first equation of system (22):

$$\exp\left(\xi_0 + \xi_1 \hat{k}^* + \xi_2 \hat{c}^*\right) = \frac{\rho}{\alpha\nu(1-\psi)(\bar{x})^{\psi} \frac{\theta-(1+\phi)}{\theta} \left[\frac{\theta-\alpha(1+\phi)}{\theta}\right]^{-\psi}} = \Omega$$

On the other hand, $\dot{\hat{k}} = 0$ then yields, from the second equation of system (22):

$$\exp\left(\hat{c}^* - \hat{k}^*\right) = \frac{[\theta - \alpha(1+\phi)]\rho}{\alpha(1-\psi)[\theta - (1+\phi)]} = \Upsilon$$

One can then easily establish that the two latter equations have solutions $\hat{c}^*$ and $\hat{k}^*$, provided that the scaling parameter $A$ is appropriately chosen. More precisely, one can set, without loosing generality, $\hat{k}^* = 0$ (that is, $k^* = 1$) by fixing $A = \Upsilon^{-1}\Omega^{\frac{(1-\psi)\alpha_n - (\gamma+1)}{\alpha_n(1-\psi)}} B^{\frac{\gamma+1}{\alpha_n}} \Gamma$.

# B The Jacobian Matrix

The steady-state Jacobian matrix is derived from:

$$\begin{bmatrix} \dot{\hat{k}} \\ \\ \dot{\hat{c}} \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} \\ \\ j_{21} & j_{22} \end{bmatrix} \begin{bmatrix} \hat{k} - \hat{k}^* \\ \\ \hat{c} - \hat{c}^* \end{bmatrix}$$

where

$$j_{11} = \frac{\rho\big[\theta - \alpha(1+\phi)\big]}{\alpha(1-\psi)\big[\theta - (1+\phi)\big]}(\xi_1 + 1)$$

$$j_{12} = \frac{\rho\big[\theta - \alpha(1+\phi)\big]}{\alpha(1-\psi)\big[\theta - (1+\phi)\big]}(\xi_2 - 1)$$

$$j_{21} = \xi_1 . \rho$$

$$j_{22} = \xi_2 . \rho$$

# C Proof of Proposition 3.2

Assume $0 < \alpha_k < 1$. We can analyse the sign of the steady-state Jacobian's Trace as follows:

$$T = \frac{\overbrace{\rho(1+\eta)}^{>0}\{(\gamma+1)\big[\theta - \alpha(1+\phi)\big] - (1-\alpha)\theta\}}{\underbrace{\big[\theta - (1+\phi)\alpha(1+\eta)\big]}_{>0}\big[\gamma + 1 - (1-\psi)\alpha_n\big]}$$

$(\gamma+1)\big[\theta - \alpha(1+\phi)\big] - (1-\alpha)\theta$ can be re-written as $\underbrace{\gamma\big[\theta - \alpha(1+\phi)\big]}_{>0} + \underbrace{\alpha(\theta - 1 - \phi)}_{>0} > 0$. We get $T < 0$, which is a necessary condition for local indeterminacy, whenever $\gamma + 1 < (1-\psi)\alpha_n$. It is easy to check with the steady-state Jacobian's determinant that this necessary condition is also sufficient:

$$D = \frac{\overbrace{(\gamma+1)}^{>0}\overbrace{\big[\alpha_k(1-\psi) - 1\big]}^{<0}}{\gamma + 1 - (1-\psi)\alpha_n}\frac{\overbrace{\rho^2\big[\theta - \alpha(1+\phi)\big]}^{>0}}{\underbrace{\alpha(1-\psi)}_{>0}\underbrace{\big[\theta - (1+\phi)\big]}_{>0}}$$

The other necessary condition for indeterminacy, namely $D > 0$, is obtained whenever $\gamma + 1 < (1-\psi)\alpha_n$. Hence, $\gamma + 1 < (1-\psi)\alpha_n$ is a Necessary and Sufficient Condition for the occurrence of local indeterminacy.

□

# D Sensitivity of $\psi_{\min}$ to $\phi$ and $\theta$

Since $\frac{\partial \psi_{\min}}{\partial \theta} = -\frac{\alpha(\gamma+1)(1+\phi)}{(1-\alpha)\theta^2} < 0$ and $\frac{\partial^2 \psi_{\min}}{\partial \theta^2} = \frac{2\alpha(\gamma+1)(1+\phi)\theta}{(1-\alpha)\theta^3} > 0$, $\psi_{\min}$ is convexly decreasing in $\theta$. Moreover, as $\frac{\partial \psi_{\min}}{\partial \phi} = \frac{\alpha(\gamma+1)}{(1-\alpha)\theta} > 0$ and $\frac{\partial^2 \psi_{\min}}{\partial \phi^2} = 0$, $\psi_{\min}$ is linearly increasing in $\phi$.

# E Sensitivity of $\psi_{\min}$ to $\eta$

When the equilibrium maintenance-to-GDP ratio is set to zero ($\phi = 0$ and or $\theta \to \infty$), the level of fiscal progressivity $\bar{\psi}_{\min}$ needed to ensure saddle-path stability is $\bar{\psi}_{\min} = \frac{(1-\alpha)(1+\eta)-(\gamma+1)}{(1-\alpha)(1+\eta)}$. Since $\frac{\partial \bar{\psi}_{\min}}{\partial \eta} = \frac{\gamma+1}{(1-\alpha)(1+\eta)^2} > 0$ and $\frac{\partial^2 \bar{\psi}_{\min}}{\partial^2 \eta} = -\frac{2(\gamma+1)}{(1-\alpha)(1+\eta)^3} < 0$, $\bar{\psi}_{\min}$ is concavely increasing. If $\bar{\psi}_{\min} = 0$, then $\eta_{\min} \mid_{\psi_{\min}=0} = \frac{\gamma+\alpha}{1-\alpha}$.

As mentioned earlier in the text, when firms do undertake maintenance activity, and deduct the related expenditures from their pre-tax profit, the fiscal progressivity level required to ensure saddle-path stability is $\psi_{\min} =$

$\frac{(1-\alpha)(1+\eta)\theta-(\gamma+1)\left[\theta-\alpha(1+\eta)(1+\phi)\right]}{(1-\alpha)(1+\eta)\theta}$. Since $\frac{\partial\psi_{\min}}{\partial\eta} = \frac{\gamma+1}{(1-\alpha)(1+\eta)^2} = \frac{\partial\bar{\psi}_{\min}}{\partial\eta} > 0$ and $\frac{\partial^2\psi_{\min}}{\partial^2\eta} = -\frac{2(\gamma+1)}{(1-\alpha)(1+\eta)^3} < 0 = \frac{\partial^2\bar{\psi}_{\min}}{\partial^2\eta}$, we notice that $\psi_{\min}$ as a function of $(\eta)$ exhibits the same slope, whether or not maintenance and repair activity is effective.

If $\psi_{\min} = 0$, then $\eta_{\min} \mid_{\psi_{\min}=0} = \frac{(\gamma+1)[\theta-\alpha(1+\phi)]-\theta(1-\alpha)}{(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)}$. It is easily checked that since $\frac{\partial\eta|_{\psi_{\min}=0}}{\partial\theta} = \frac{(\gamma+1)^2\alpha(1+\phi)}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^2} > 0$ and $\frac{\partial^2\eta|_{\psi_{\min}=0}}{\partial\theta^2} = -\frac{(\gamma+1)^2\alpha(1+\phi)2(1-\alpha)}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^3} < 0$, $\eta \mid_{\psi_{\min}} = 0$ is concavely increasing in $\theta$. Moreover, since $\frac{\partial\eta|_{\psi_{\min}=0}}{\partial\phi} = -\frac{(\gamma+1)^2\alpha\theta}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^2} < 0$ and $\frac{\partial^2\eta|_{\psi_{\min}=0}}{\partial\phi^2} = \frac{(\gamma+1)^3\alpha^2\theta 2}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^3} > 0$, $\eta \mid_{\psi_{\min}} = 0$ is convexly decreasing in $\phi$.

Given the equilibrium maintenance ratio writes as $\frac{m}{y} = \frac{\phi\alpha}{\theta}$, we know that an increase in this indicator can be achieved though an increase in $\phi$ or a decrease in $\theta$. Consequently, when the equilibrium maintenance-to-GDP ratio rises, $\eta \mid_{\psi_{\min}} = 0$ falls.

# F Sensitivity of $\alpha_n$ to $\phi$ and $\theta$

Since $\frac{\partial\alpha_n}{\partial\theta} = -\frac{\alpha(1+\eta)(1-\phi)}{[\theta-\alpha(1-\eta)(1+\phi)]^2} < 0$ and $\frac{\partial^2\alpha_n}{\partial\theta^2} = \frac{2\alpha(1+\eta)(1+\phi)[\theta-\alpha(1-\eta)(1+\phi)]}{[\theta-\alpha(1-\eta)(1+\phi)]^4} > 0$, $\alpha_n$ is convexly decreasing in $\theta$. Moreover, as $\frac{\partial\alpha_n}{\partial\phi} = \frac{\theta\alpha(1-\alpha)(1+\eta)^2}{[\theta-\alpha(1-\eta)(1+\phi)]^2} > 0$ and $\frac{\partial^2\alpha_n}{\partial\phi^2} = \frac{2\theta\alpha(1-\alpha)\alpha^2(1+\eta)^3[\theta-\alpha(1-\eta)(1+\phi)]}{[\theta-\alpha(1-\eta)(1+\phi)]^4} > 0$, $\alpha_n$ is convexly increasing in $\phi$.

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Figure 1: Local Indeterminacy: the Steady-State is a Sink

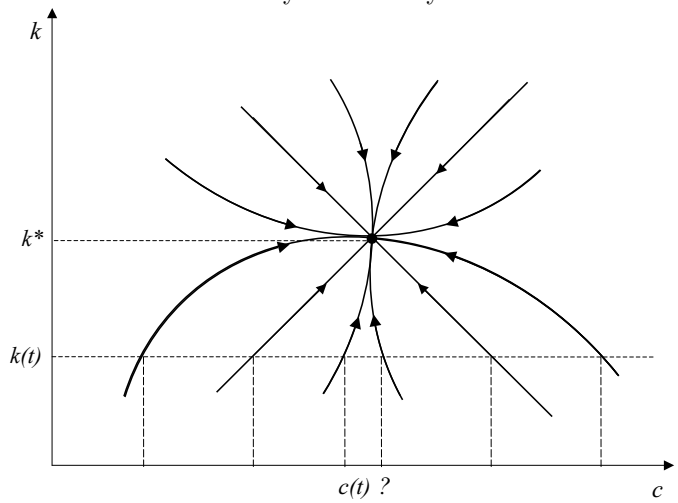


Figure 2: Saddle-Path Stability: the Steady-State is Locally Determinate

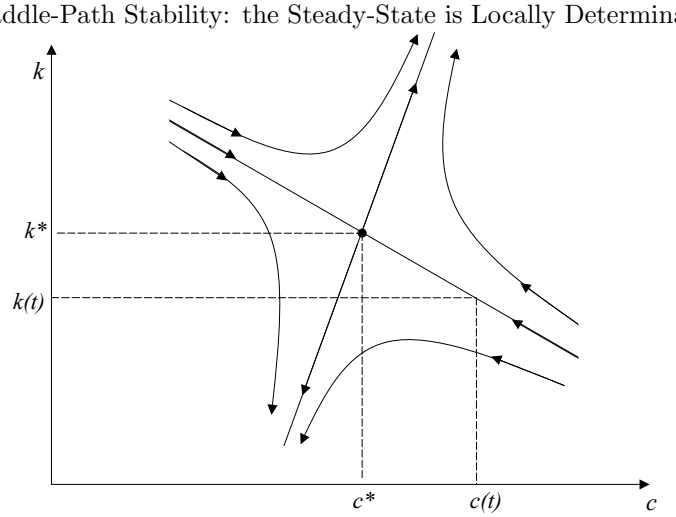


Figure 3: Jacobian's Trace-Determinant Diagram: Stability Regimes of Steady State in Continuous Time

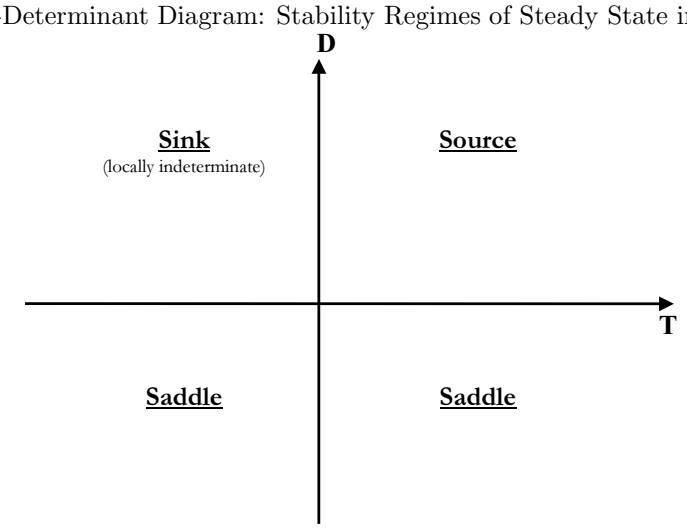

Figure 4: Sensitivity of $\psi_{\min}$ with respect to $\eta$ when the equilibrium maintenance-to-GDP ratio is set to zero

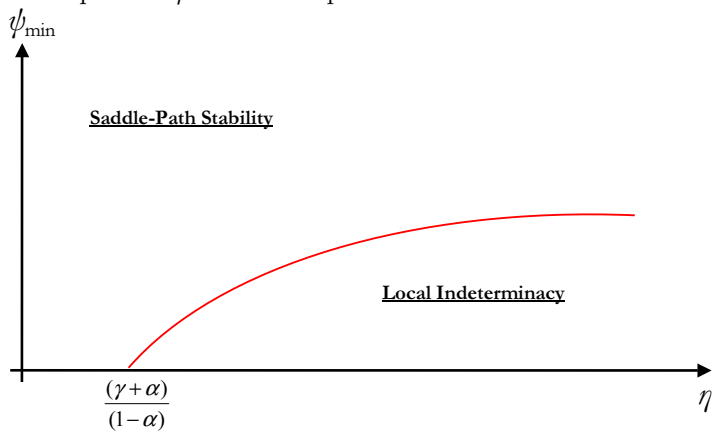


Figure 5: Sensitivity of $\psi_{\min}$ with respect to $\eta$ when the equilibrium maintenance-to-GDP ratio rises