Vertical Integration in Two-sided Markets*

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Abstract

This paper analyzes the effects from a vertical merger in a two-sided market on the price level, individual prices and welfare, building on Rochet and Tirole (2006), Armstrong (2006) and Weyl (2008). Several contributions are made. The organizational set-up allows for a vertical structure on both sides of the market thus enabling me to analyze both a full (symmetric) vertical integration and an asymmetrical integration (i.e. integration on one side only). Furthermore, both the cases of usage pricing and membership pricing are analyzed. Lastly, the welfare effects are derived for the case of uniformly distributed benefits. When the platform chooses its optimal price structure both the elasticity of demand and opportunity costs are taken into account as well as the rate at which downstream firms pass on increased costs to end-consumers. A full integration always lowers the price level, thereby increasing consumer and producer surplus. However, under membership pricing an asymmetric vertical merger can increase the price level by increasing the price on the opposite side of the market. Interestingly though, even when the price level increases, welfare is higher after an asymmetric merger due to higher demand on the side where the merger takes place. Thus, conventional wisdom on vertical mergers and double marginalization carry over to two-sided markets with respect to welfare effect but not necessarily with respect to price effects.

Keywords: Two-sided Markets; Network externalities; Double marginalization; Vertical Integration

JEL-Classifications: L13, L4

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1 Introduction

Today, many significant industries are based on so-called two-sided platforms. These platforms enable consumers of two distinct groups to interact with each other and thereby obtain the benefits of externalities between them. These two-sided markets include old-economy industries such as newspapers and shopping malls as well as new-economy industries such as web platforms, video game consoles, and software platforms.

The early literature on two-sided markets, Rochet and Tirole (2003), Armstrong (2006) and Rochet and Tirole (2006) established that optimal pricing in the markets differ in important aspects from those in standard markets. In particular, as a direct consequence of the network externalities, optimal prices may involve below cost pricing to one side of the market, even in the long run. Thus, individual prices on either side of the market do not necessarily track cost nor demand on that side. Furthermore, optimal prices should take into account the need for the platform to get both sides on board.

The literature warned policy makers against applying standard competition policy guidelines to these markets. The special characteristics of two-sided markets make conventional practises of antitrust policy not directly applicable. At the same time, two-sided markets are increasingly relevant, emphasizing the need for directly applicable antitrust results in this area.

The fact that optimal pricing strategies in two-sided markets differ from that of standard markets affects almost all aspects of antitrust analysis and it is the motivation of this paper. In a two-sided market price-cost margins cannot be used as a measure of market power or as a test of predatory pricing. Further, changes in individual prices cannot be viewed in isolation. One must take account of both the price structure and the price level. According to Evans (2003), a common mistake by antitrust authorities is to view a submarket in isolation or to misinterpret pricing instruments used to get the right price balance between the two sides. The author presents the case of *U.S. v. Visa USA et al.* as an illustration. Here, the economic expert of the DOJ asked the question of whether a hypothetical merger between all credit card issuers could profitably raise prices to cardholders. By focusing on the cardholder side only, two important factors were neglected. A higher price on the cardholder side should lead to fewer transaction here, which will necessarily mean fewer transaction on the merchant side. Further, a decrease in cardholder base will make the platform less attractive for merchants thus leading to a decrease in demand on the merchant side. These changes would affect profits on both sides of the market and should be accounted for in any antitrust analysis.

The focus of this paper is on vertical mergers. In most markets, goods are not sold to final consumers directly by the producer, but through intermediaries and retailers. Furthermore, to arrive at the final good, production is often carried out in several stages. Such a vertical structure gives rise to the problem of *double marginalization*, first defined by Spengler (1950). Here, the author warned antitrust authorities against looking upon vertical mergers as per se illegal. If a vertical merger eliminates the double marginalization problem it will be beneficial for both producers and consumers. Thus, eliminating the double marginalization problem in such a setting unambiguously increases welfare.

This paper sets up a model investigating the effects of vertical integration in a two-sided market on the price level, individual prices, and consumer welfare. The paper builds on the classic models of two-sided markets in Rochet and Tirole (2003) and Armstrong (2006) and furthermore on Weyl (2008). Motivated by issues in the payment card industry, such as whether banks (through Visa) should be allowed to own debit clearing networks, Weyl (2008) combined the classic double marginalization literature with the literature on two-sided markets. With the main focus on banks and debit clearing networks he argues that since these in many ways are similar to classic vertical monopolies, the logic of double marginalization suggests that vertical integration is socially desirable. However, since the logic of double marginalization supported in standard markets had not yet been worked out in two-sided markets, this conclusion could not be immediately drawn. Weyl (2008) analyzes the case of usage pricing, which is generally believed to be the best description of the credit card industry. He finds that vertical integration in a Stackelberg organization with a downstream firm on one side of the market always decreases the price on the side where it takes place, but it could increase the price charged on the other side of the market.

This paper adds to Weyl (2008) in several ways. First, the organizational set-up is changed to allow for a vertical structure on both sides of the market. This allows for different kinds of vertical integration: full integration on both sides of the markets, or asymmetric integration on one side only. Second, both the case of pure usage pricing and pure membership pricing is considered. As mentioned in Weyl (2008) the case of pure membership pricing is interesting since it is considered a better description of markets such as software platforms, video game consoles, internet service providers etc. Third, the welfare effects of vertical integration are analyzed in the case of uniformly distributed network externalities.

In a two-sided market, a price decrease on one side of the market is not necessarily welfare improving. A price decrease on one side of the market often comes a long with a price increase on the other side of the market as a result of the seesaw effect mentioned in Rochet and Tirole (2003). If this price increase decreases demand sufficiently on one side, it is possible that the side facing a price decrease are made worse off overall if the network externality is strong. The welfare effects from price changes are for this reason different from standard one-sided markets. To understand the effects from vertical integration is it important to look beyond the effects on prices.

We find that when choosing optimal prices the platform takes into account the rate at which downstream firms pass on an increase in price to end consumers in addition to the elasticity of demand. A higher pass-through rate leads to a lower upstream mark-up. For the case of pure usage pricing conventional wisdom on double marginalization transfers well to two-sided markets. For the case of uniformly distributed transaction benefits, vertical integration will result in lower prices, and increased consumer and producer surplus. For the case of pure membership pricing the results are less clear cut. Going from the non-integrated organization to full integration unambiguously decreases prices and increases welfare. However, a shift to partial integration may increase prices on the non-integrated side.

The paper proceeds as follows. Section 2 sets up the general model framework. Section 3 solves the model for the case of pure usage pricing and section 4 investigates the other possibility, pure membership pricing. Section 5 concludes.

2 The Model

The organizational structure considered is general and allows for a vertical structure on both sides of the market, as shown by the figure below.

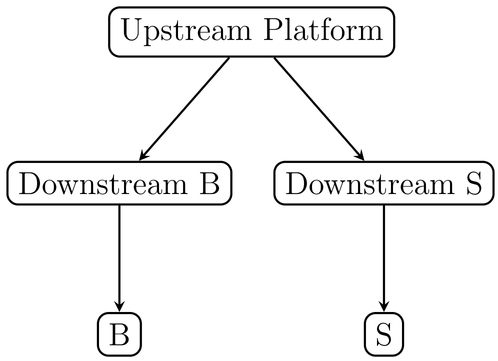

Figure 1: Non-integrated organization

A monopolist platform serves two sides of a market labeled B and S since we often think of the two sides as consisting of buyers and sellers respectively. The platform does not serve the consumers directly but through two downstream firms, DB and DS. An example of a platform could be Visa. The buyers are then cardholders and on the sellers are merchants. The downstream firm DB is the cardholders' bank and the downstream firm DS is merchants' bank.

Each downstream firm behaves as a monopolist on its own side of the market. Thus, firm DB serves consumers on side B only, and do not compete with firm DS for side S consumers. This assumption may be fitting in markets where the two downstream firms serve different purposes and are therefore not in direct competition with each other.

Consumers derive utility in two ways. First, from usage of the platform and, second, from being a "member" of the platform. More specifically, a consumer receives a fixed benefit denoted by β_i. In addition she receives utility α_i from each transaction with a member on the other side of the market. When focusing on pure usage pricing, as in Rochet and Tirole (2003), one conventionally assumes that members on each side of the market are heterogenous with respect to their per transaction benefit α_i. On the other hand, when focus is on pure membership pricing, as in Armstrong (2006), it is assumed that members are heterogenous with respect to their membership benefits β_i.

As with utility, we distinguish between to different kinds of prices. Usage pricing and membership pricing. The end-users pay to their respective downstream firm a fixed price P_i^D for membership and an additional price p_i^D per transaction. Likewise, each downstream firm pays to the upstream firm a fixed price P_i^U and a per transaction price p_i^U.

When a consumer k on side i pays both a membership fee P_i and a per transaction fee p_i she receives net utility

$$U_i^k = (\alpha_i^k - p_i)N_j + \beta_i^k - P_i \tag{1}$$

where N_j is the number of members on the other side of the market. Now define an agent's *gross* utility as

$$u_i = -p_i N_j - P_i$$

One can view the platform as offering a gross utility u_i instead of a set of prices. Then, the number of agents N_i who join the platform will be a function of this utility:

$$N_B = \phi_B(u_B) \qquad ; \qquad N_S = \phi_S(u_S)$$

In line with Rochet and Tirole (2006), it will be assumed that the number of transactions is given by the product $N_B N_S$. The platform incurs a fixed cost, C_i, per member joining the platform on side i. Furthermore, there is a transaction cost, c_i, every time a transaction is carried out. The cost of each downstream firm is simply the price paid to the upstream platform.

The timing of the model is as follows: the upstream firm chooses fixed- and variable fees to charge downstream firms B and S. The downstream firms observe these prices, and then each of the firms decide on their own fixed- and variable prices, which are the prices facing consumers.

With these specifications, the profit of the upstream platform is given by

$$\pi^U = (P_B^U - C_B)\phi_B(u_B) + (P_S^U - C_S)\phi_S(u_S) + (p_B^u + p_S^u - c_B - c_S)\phi_B(u_B)\phi_S(u_S) \tag{2}$$

The first two terms are profits from membership payments and the last term is profit from transactions between the groups.

Each downstream firm serves one side only. The profit function of a downstream firm is given by

$$\pi_i^D = (P_i^D - P_i^U)\phi_i(u_i) + (p_i^D - p_i^u)\phi_i(u_i)\phi_j(u_j)$$

The optimal prices are found using backwards induction. In order to isolate the effect from vertical integration these prices will be compared to prices emerging in more integrated organizations: a semi-integrated organization with a downstream unit on side B only and a fully integrated organization in which the platform serves consumers directly on both sides. These organizations are shown in the figures below.

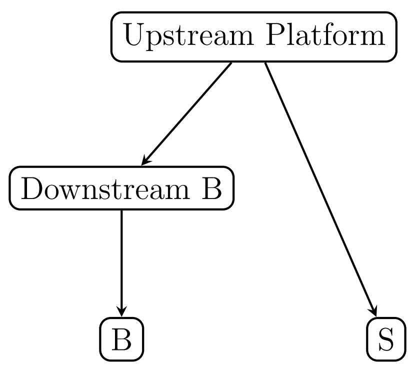

Figure 2: Semi-integrated organization

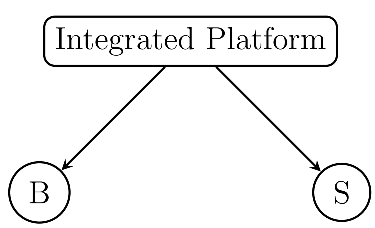

Figure 3: Integrated Organization

There are thus two kinds of integration possible. A full integration where the downstream units are removed jointly on both sides of the market. This will be labeled *symmetric integration*. The other option is integration on one side only, which is labeled *asymmetric integration*. This can be carried out in two different ways. Integration from the non-integrated organization to the semi-integrated organization. That is, integration on one side, preserving the downstream unit on the other side of the market. Or, integration from the semi-integrated organization to the fully integrated organization, going from one downstream unit to none.

The focus will be divided between usage pricing and membership pricing. Usage pricing is generally thought to be the best description of markets such as payment cards and internet auction sites such as eBay. With these platforms, you pay every time you want to interact with a member on the other side of the market. Membership pricing on the other hand better describes markets such as software platforms, video game consoles, night clubs etc. Here, you pay a fixed price and then you can use the platform as much as you like.

The model is solved first for the case of usage pricing and then in section 4 for membership pricing.

3 Pure Usage Pricing

This section will analyze the case of pure usage pricing. The platform charges a price per transaction, p_i, but does not charge for membership, i.e. $P_i = 0$. For simplicity we follow Rochet and Tirole (2003) and assume that end-users differ only with respect to their per-transaction benefits α_i. Transaction benefits on either side are distributed according to a function $F_i(\alpha_i)$. Furthermore there are no fixed costs and benefits $(C_i = \beta_i = 0)$. Utility of agent k on side i is then given by

$$u_i^k = \left(\alpha_i^k - p_i\right) N_j$$

Consequently end-consumers use the platform only if $\alpha_i^k \geq p_i$ and demand is given by

$$N_i = pr(\alpha_i \geq p_i) \equiv \phi_i(p_i) \tag{3}$$

Notice that with pure usage pricing demand does not depend on participation on the other side of the market.

With the above specification of demand, the platform's profit is given by

$$\pi^U = \left(p_B^U + p_S^U - c\right)\phi_B\left(p_B^D\right)\phi_S\left(p_S^D\right)$$

and downstream profit on side i is given by

$$\pi_i^D = \left(p_i^D - p_i^U\right)\phi_i\left(p_i^D\right)\phi_j\left(p_j^D\right)$$

The downstream firms maximize profits taking the upstream price as given. This results in the following first order condition

$$p_i^D - p_i^U = -\frac{\phi_i\left(p_i^D\right)}{\partial\phi_i(p_i^D)/\partial p_i^D}$$

The downstream firms set their price as a mark-up over upstream prices. In this way the downstream prices are functions of upstream prices, $p_i^D(p_i^U)$. Inserting this into the platform's profit function and maximizing with respect to prices yields

$$p_B^U + p_S^U - c = \phi_B\left[\frac{\partial\phi_B}{\partial p_B^D}\frac{\partial p_B^D}{\partial p_B^U}\right] = \phi_S\left[\frac{\partial\phi_S}{\partial p_S^D}\frac{\partial p_S^D}{\partial p_S^U}\right] \tag{4}$$

Now, let $\sigma_i = -\frac{\partial\phi_i/\partial p_i^D}{\phi_i}$ denote the *semi-elasticity*. Then (4) can be rewritten as

$$p_B^U + p_S^U - c = \frac{1}{\sigma_B}\frac{\partial p_B^D}{\partial p_B^U} = \frac{1}{\sigma_S}\frac{\partial p_S^D}{\partial p_S^U}$$

$\frac{\partial p_B^D}{\partial p_B^U}$ indicates how much a change in the upstream price affects the downstream price. This is the so-called pass-through rate as argued in Weyl and Fabinger (2009). It is the rate at which downstream firms pass on an increase in upstream prices to end-consumers. Denote this pass-through rate by ρ_i and we can rewrite the platform's optimal price structure as

$$p_B^U + p_S^U - c = \frac{1}{\sigma_B\rho_B} = \frac{1}{\sigma_S\rho_S}$$

A similar price structure is found in Weyl (2008). It is the price structure obtained in Rochet and Tirole (2006) but with the pass-through rate in the denominator. In its optimal price structure, the platform takes into account the elasticity of demand, opportunity costs, *and* the rate at which downstream firms pass on increases in their cost to end-consumers. For a concave demand function, the pass-through rate is a positive number smaller than 1. When the pass-through rate is small, the platform can raise its price and only see a slight change in final demand. This pulls in the direction of a higher upstream mark-up. If the pass-through rate is large, a small increase in the upstream mark-up results in a large increase in the downstream price and a resulting drop in demand. This pulls in the direction of a lower upstream mark-up.

When the platform integrates with the downstream firm on side S the pricing incentives will change. This semi-integrated organization is identical to the organization labeled "Spencer-Stackelberg" organization in Weyl (2008). The platform's profit function in this organization is given by

$$\pi^U = \left(p_B^U + p_S^U - c\right)\phi_B\left(p_B^D(p_B^U)\right)\phi_S\left(p_S^U\right)$$

The remaining downstream firm's profit function is unchanged. The platform's optimal price structure will in this organization be

$$p_B^U + p_S^U - c = \frac{1}{\sigma_B \rho_B} = \frac{1}{\sigma_S}$$

Which is the same price structure found in Weyl (2008).

Lastly, in the completely integrated organization in which the platform serves both sides of the market directly, profit is

$$\pi^U = \left(p_B^U + p_S^U - c\right)\phi_B\left(p_B^U\right)\phi_S\left(p_S^U\right)$$

and the optimal price structure is

$$p_B^U + p_S^U - c = \frac{1}{\sigma_B} = \frac{1}{\sigma_S}$$

which is the price structure found in Rochet and Tirole (2006).

Uniformly Distributed Transaction Benefits

With the aim of arriving at a closed form solution, we proceed by assuming a specific distribution of transaction benefits. More specifically, we will assume that transaction benefits are distributed uniformly on the interval $[0, \bar{\alpha}_i]$.

In addition it is assumed that $\bar{\alpha}_B + \bar{\alpha}_S > c$. This will ensure non-negative demand in equilibrium.

With this form of distribution, demand on side i is given by

$$N_i = 1 - \frac{1}{\bar{\alpha}_i} p_i^D$$

Final prices to consumers in the non-integrated organization are then given by

$$p_B^D = \tfrac{5}{6}\bar{\alpha}_B - \tfrac{1}{6}\bar{\alpha}_S + \tfrac{1}{6}c \qquad , \qquad p_S^D = \tfrac{5}{6}\bar{\alpha}_S - \tfrac{1}{6}\bar{\alpha}_B + \tfrac{1}{6}c$$

The more the average consumer values transactions with the other side of the market, the higher is the price charged on that side. Furthermore, the price facing side i is decreasing in the average benefits on side j, which is in line with the literature. The side with lower transaction benefits will be offered a lower price. The low price attracts members, which makes the platform more valuable for high-benefit consumers on the other side of the market. The platform will then be able to charge these consumers a high price.

Optimal prices in the semi-integrated organization, labeled $\tilde{p}$ are given by

$$\tilde{p}_B^D = \tfrac{5}{6}\bar{\alpha}_B - \tfrac{1}{6}\bar{\alpha}_S + \tfrac{1}{6}c \qquad , \qquad \tilde{p}_S = \tfrac{2}{3}\bar{\alpha}_S - \tfrac{1}{3}\bar{\alpha}_B + \tfrac{1}{3}c$$

Notice that the only change in price is on the side where integration takes place.

Lastly, for the integrated organization prices are labeled $\hat{p}$ and are given by

$$\hat{p}_B = \tfrac{2}{3}\bar{\alpha}_B - \tfrac{1}{3}\bar{\alpha}_S + \tfrac{1}{3}c \qquad , \qquad \hat{p}_S = \tfrac{2}{3}\bar{\alpha}_S - \tfrac{1}{3}\bar{\alpha}_B + \tfrac{1}{3}c$$

Notice that with usage pricing there is no difference between the two types of asymmetric integration. Going from the non-integrated organization to the semi-integration is the same as going from the semi-integrated organization to the fully integrated organization. It does not matter whether there is a downstream unit left on the other side or not.

The next step is to compare the prices for the different organizations. The change in price on side i when going from the non-integrated organization to the semi-integrated organization is labeled $\Delta_i^{N \to SI}$ and likewise for non-integrated to integrated and semi-integrated to integrated. We have:

$$\Delta_S^{N \to SI} = \Delta_B^{SI \to I} = \Delta_B^{N \to I} = \Delta_S^{N \to SI} = \frac{1}{6}\left(\bar{\alpha}_B + \bar{\alpha}_S - c\right)$$
$$\Delta_B^{N \to SI} = \Delta_S^{SI \to I} = 0$$

Prices always (weakly) decrease as a result of integration. This holds since we assumed that $\bar{\alpha_B} + \bar{\alpha_S} - c > 0$ to ensure non-negative demand. Hence we conclude that the prices are (weakly) lower under semi-integration and full integration.

The effect on the price level, conventionally defined as the sum of the prices charged to the two sides, is then immediately clear. Since prices either decrease or remain unchanged following integration, the price level must decrease following integration. That is, the price level is lower under semi-integration and even lower under full integration compared to the non-integrated organization.

Having established that vertical integration results in a lower price level and either lower or unchanged individual prices it can be concluded that the logic of double marginalization transfers well to two-sided markets when the platform uses pure-usage pricing.

Welfare

The welfare analysis is simple under pure usage pricing with uniformly distributed transaction benefits. Integration results in lower prices on the side where it takes place. This increases demand. The increase in demand on the integrated side makes the platform more valuable, for an unchanged price, on the other side of the market. Thus, prices are (weakly) lower and demand is higher under integration and thus consumer welfare is lowest under non-integration, higher under semi-integration and highest under full integration.

With regards to producer surplus the standard argument applies. Profits are at least as high under integration since the platform can always mimic the non-integrated organization.

4 Pure Membership Pricing

The focus is now turned to the case of pure membership pricing. The platform charges a fixed fee P_i and does not charge a transaction price, i.e. $p_i = 0$.

Often, the argument for fixed membership fees is that the platform cannot observe transactions. For example, a nightclub cannot charge guests per dance and instead charges a fixed fee in the door. Likewise, Microsoft cannot observe how often a consumer uses Windows. Consequently, they charge fixed fees. Further, if transactions are not observed, a per-transaction cost does not make much sense. For this reason we assume that the platform has fixed costs of C_i per member and no variable costs. The downstream firms continue to have zero costs, besides the price they pay to the platform.

When focusing on pure membership pricing, it is commonly assumed for simplicity that end-users differ only with respect to their membership benefit β_i^k and obtain an identical per-transaction benefit α_i. Then, a consumer k on side i gets net utility

$$U_i^k = \alpha_i N_j + \beta_i^k - P_i$$

and gross utility is

$$u_i^k = \alpha_i N_j - P_i \tag{5}$$

The profit function in (2) can be written as

$$\pi^U = (P_B^U - C_B)\phi_B(u_B) + (P_S^U - C_S)\phi_S(u_S)$$

and downstream profits are given by

$$\pi_i^D = (P_i^D - P_i^U)\phi_i(u_i)$$

The downstream firms set optimal prices according to

$$P_i^D - P_i^U = \frac{\phi_i(u_i)}{\phi_i'(u_i)} \equiv \frac{1}{\sigma_i} \tag{6}$$

The platform maximizes profit, keeping in mind (6), yielding the following first order condition

$$-\phi_i(u_i) - \frac{\phi_i\phi_i' - (\phi_i')^2}{(\phi'(u_i))^2}\phi_i + \left(P_i^U - C_i\right)\phi_i'(u_i) + \alpha_j\phi_j(u_j)\phi_i'(u_i) = 0 \tag{7}$$

Combining (6) and (5) it can be seen that

$$\frac{\partial u_i}{P_i^U} = -\frac{1}{1+\sigma_i'}.$$

Now, define

$$\rho_i \equiv -\frac{\partial u_i}{P_i^U} = -\frac{1}{1+\sigma_i'}.$$

Again, ρ_i is a pass-through rate. It tells us how much of an increase in p_i^U is passed on to downstream consumers through a change in the utility u_i offered to them by the downstream firm. Notice that in (7) $\frac{\phi_i\phi_i' - (\phi_i')^2}{(\phi'(u_i))^2}$ is exactly θ_i'. Thus, we can rewrite (7)

$$p_i^U - (C_i - \alpha_j N_j) = \frac{\phi_i(u_i)}{\rho_i\phi_i'(u_i)} \tag{8}$$

which, using the definition of the semi-elasticity, can be rewritten again as

$$p_i^U - (C_i - \alpha_j N_j) = \frac{1}{\rho_i\sigma_i} \tag{9}$$

These first order conditions show great similarity to the ones found in Armstrong (2006). The only difference is again the pass-through rate multiplied in the denominator on the right hand side of the equations. The platform sets its prices according to an augmented Lerner index. Instead of subtracting costs, the platform subtracts opportunity costs. Furthermore, as with pure-usage pricing, the platform takes into account the rate at which the downstream firm passes increases in upstream prices on to end-consumers in addition to the elasticity of demand. A high pass-through rate pulls in the direction of a lower upstream mark-up.

Inserting the platform's price into downstream prices given by (6) and rearranging gives

$$P_B^D = \frac{1}{\sigma_B}\left(1 + \frac{1}{\rho_B}\right) + (C_B - \alpha_S N_S)$$
$$P_S^D = \frac{1}{\sigma_S}\left(1 + \frac{1}{\rho_S}\right) + (C_S - \alpha_B N_B)$$

The higher the elasticity of demand, the lower the price. Furthermore, once again, a high pass-through rate is indicative of a lower price as well. This effect, however, stems from the upstream price, which acts as the cost for the downstream firm. The downstream price does not directly depend on the pass-through rate, but does so indirectly through the upstream price. A low pass-through rate leads, other things equal, to a higher upstream mark-up. Since this is equivalent to higher costs for the downstream firm, a low pass-through rate will in turn lead to a higher downstream mark-up as well, other things equal. Lastly, the higher is the transaction benefit on some other side of the market, the higher price is charged on this side, the same effect as we saw with pure usage pricing.

For the semi-integrated organization, the platform serves side S directly and side B through downstream firm B. This gives rise to the following optimal prices

$$\tilde{P}_B^D = \frac{1}{\sigma_B}\left(1 + \frac{1}{\rho_B}\right) + (C_B - \alpha_S N_S)$$
$$\tilde{P}_S = \frac{1}{\sigma_S} + (C_S - \alpha_B N_B)$$

Lastly, the integrated organization is the same as the organization analyzed in Armstrong (2006). Consequently, optimal prices are given by

$$\bar{P}_i = \frac{1}{\sigma_i} + (C_i - \alpha_j n_j)$$

Uniformly Distributed Membership Benefits

Once again, in order to obtain a closed form solution it is assumed that benefits are distributed uniformly over the interval $\left[0, \bar{\beta}_i\right]$. Furthermore, we assume that $\bar{\beta}_S \bar{\beta}_B > 1$. This will ensure that the equilibrium is stable in the sense of Dixit (1986). Lastly, in order to ease notation we set the transaction benefit equal to one, $\alpha_B = \alpha_S = 1$. Demand on each side of the market is then

$$N_i = 1 - \frac{1}{\bar{\beta}_i} \left(P_i - N_j\right)$$

The downstream prices in the non-integrated organization are then given by

$$P_B^D = \frac{3\bar{\beta}_B + 2N_S + C_B}{4} \quad , \quad P_S^D = \frac{3\bar{\beta}_S + 2N_B + C_S}{4}$$

Prices in the semi-integrated organization are given by

$$\tilde{P}_B^D = \frac{3\bar{\beta}_B + 2N_S + C_B}{4} \quad , \quad \tilde{P}_S = \frac{\bar{\beta}_S + C_S}{2}$$

Lastly, prices in the integrated organization are given by

$$\hat{P}_B = \frac{\bar{\beta}_B + C_B}{2} \quad , \quad \hat{P}_S = \frac{\bar{\beta}_S + C_S}{2}$$

Equilibrium prices are found by solving the system of equations. Equilibrium prices in the three organizations are listed below.

The non-integrated organization

$$P_B^D = \frac{2\bar{\beta}_B\bar{\beta}_S C_B + 6\bar{\beta}_B^2\bar{\beta}_S + \bar{\beta}_B\left(\bar{\beta}_S - C_S\right) - \left(\bar{\beta}_B - C_B\right)}{2\left(4\bar{\beta}_i\bar{\beta}_j - 1\right)}$$
$$P_S^D = \frac{2\bar{\beta}_B\bar{\beta}_S C_S + 6\bar{\beta}_B\bar{\beta}_S^2 + \bar{\beta}_S\left(\bar{\beta}_B - C_B\right) - \left(\bar{\beta}_S - C_S\right)}{2\left(4\bar{\beta}_i\bar{\beta}_j - 1\right)}$$

The semi-integrated organization

$$\tilde{P}_B^D = \frac{\bar{\beta}_B\bar{\beta}_S C_B + 3\bar{\beta}_B^2\bar{\beta}_S + \bar{\beta}_B\left(\bar{\beta}_S - C_S\right) - \left(\bar{\beta}_B - C_B\right)}{2\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)}$$
$$\tilde{P}_S = \frac{\bar{\beta}_S + C_S}{2}$$

The integrated organization

$$\hat{P}_B = \frac{\bar{\beta}_B + C_B}{2}$$
$$\hat{P}_S = \frac{\bar{\beta}_S + C_S}{2}$$

It is immediately clear that, as opposed to the pure usage case, the price on side B changes following integration on side S.

Equilibrium demand in the three organizations is given by

The non-integrated organization

$$N_B = \frac{(\bar{\beta}_S - C_S) + 2\bar{\beta}_S(\bar{\beta}_B - C_B)}{2(4\bar{\beta}_B\bar{\beta}_S - 1)} \quad , \quad N_S = \frac{(\bar{\beta}_B - C_B) + 2\bar{\beta}_B(\bar{\beta}_S - C_S)}{2(4\bar{\beta}_B\bar{\beta}_S - 1)}$$

The semi-integrated organization

$$\tilde{N}_B = \frac{(\bar{\beta}_S - C_S) + \bar{\beta}_S(\bar{\beta}_B - C_B)}{2(2\bar{\beta}_B\bar{\beta}_S - 1)} \quad , \quad \tilde{N}_S = \frac{(\bar{\beta}_B - C_B) + 2\bar{\beta}_B(\bar{\beta}_S - C_S)}{2(2\bar{\beta}_B\bar{\beta}_S - 1)}$$

The integrated organization

$$\hat{N}_B = \frac{(\bar{\beta}_S - C_S) + \bar{\beta}_S(\bar{\beta}_B - C_B)}{2(\bar{\beta}_B\bar{\beta}_S - 1)} \quad , \quad \hat{N}_S = \frac{(\bar{\beta}_B - C_B) + \bar{\beta}_B(\bar{\beta}_S - C_S)}{2(\bar{\beta}_B\bar{\beta}_S - 1)}$$

Since the denominators are all positive by stability we get the following conditions to ensure non-negative demand

1. $(\bar{\beta}_j - C_j) + 2\bar{\beta}_j(\bar{\beta}_i - C_i) \geq 0$
2. $(\bar{\beta}_j - C_j) + \bar{\beta}_j(\bar{\beta}_i - C_i) \geq 0$

This will be assumed to hold for the remainder of the section.

The price changes on side B when going to either semi or full integration are given by

$$\Delta P_B^{N \to SI} = \frac{-\bar{\beta}_B\bar{\beta}_S}{2} \frac{[(\bar{\beta}_B - C_B) + 2\bar{\beta}_B(\bar{\beta}_S - C_S)]}{(4\bar{\beta}_B\bar{\beta}_S - 1)(2\bar{\beta}_B\bar{\beta}_S - 1)} < 0$$
$$\Delta P_B^{N \to I} = \frac{\bar{\beta}_B}{2} \frac{[(\bar{\beta}_S - C_S) + 2\bar{\beta}_S(\bar{\beta}_B - C_B)]}{(4\bar{\beta}_B\bar{\beta}_S - 1)} > 0$$
$$\Delta P_B^{SI \to I} = \frac{\bar{\beta}_B}{2} \frac{[(\bar{\beta}_S - C_S) + \bar{\beta}_S(\bar{\beta}_B - C_B)]}{(2\bar{\beta}_B\bar{\beta}_S - 1)} > 0$$

Clearly, by the restrictions laid upon the parameters by stability and non-negative demand, the change in the price facing consumers on side B is negative when the platform integrates with the downstream firm on the other side of the market. This means that consumers on side B actually experience an increase in prices following integration on the other side of the market. The remaining price changes are all positive, meaning that prices decrease as a result of integration on own side.

The changes in prices on side S are given by

$$\Delta P_S^{N\to SI} = \Delta P_S^{N\to I} = \frac{\bar{\beta}_S\left[(\bar{\beta}_B - C_B) + 2\bar{\beta}_B(\bar{\beta}_S - C_S)\right]}{2\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)} > 0$$

$$\Delta P_S^{SI\to I} = 0$$

Whether the vertical structure on side S is removed jointly with the vertical structure on side B or not, the price decreases with the same amount. Therefore, when going from the semi-integrated organization to the fully integrated organization the price on side S does not change.

From the changes in individual prices we can draw the immediate conclusion that the price level, i.e. the sum of prices charged on the two sides, decreases when going from the non-integrated organization to the fully integrated organization. However, it is not immediately clear what happens to the price level when there is integration on one side only and the downstream unit remains on the other side of the market. When the price decreases on side S but increases on side B what is the overall effect? The change in the price level when integration occurs on side S only is given by

$$\begin{aligned}\Delta P^{N\to SI} &\equiv \left(p_B^D + p_S^D\right) - \left(\tilde{p}_B^D + \tilde{p}_S\right)\\ &= \frac{\bar{\beta}_S}{2}\left[\frac{\left\{2\bar{\beta}_B\left(\bar{\beta}_S - C_S\right) + \left(\bar{\beta}_B - C_S\right)\right\}\left(2\bar{\beta}_B\bar{\beta}_S - 1 - \bar{\beta}_B\right)}{\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)}\right]\end{aligned}$$

The price change can be either positive or negative since the term $\left(2\bar{\beta}_B\bar{\beta}_S - 1 - \bar{\beta}_B\right)$ can be either positive or negative while the denominator is positive by the stability assumption and the term in the first bracket is positive by the non-negative demand restrictions on parameters. Thus, we can conclude that vertical integration in this set-up can actually increase the overall price level. Though vertical integration will always lower the price *on the side where it takes place*, it can lead to a higher price level by increased prices on the other side of the market. A price increase on side B is more likely the higher is β_B. Intuitively, the more consumers value memberships on side B the more able is the platform to increase the price on side B without seeing a large drop in demand.

Welfare

The fact that prices can increase as a result of vertical integration means that the welfare results of integration are not straight forward. Following an integration on side S only the direct effect on welfare from the price changes is positive on side S but negative on side B. However, welfare is affected by more than the change in price. Indeed, since consumers derive utility from interaction with members on the other side of the platform the demand on the other side of the market will affect welfare as well. If more people join the platform on side S following the price decrease, this will have a positive effect on welfare on side B. Likewise, if fewer members join on side B following the price increase, this will have a negative impact on welfare on side S.

For the case of full integration the prices decrease on both sides of the market and the effect on welfare will be unambiguously positive. However, the case of semi-integration on side S will have to be examined more closely.

We know that the price decreases on side S and increases on side B. What about demand on the two sides? As can be seen below, it turns out that demand increases on both sides of the market following integration on side S.

$$\delta_B^{N\to SI} \equiv N_B^{SI} - N_B^N = \frac{\bar{\beta}_S}{2} \frac{\left(\bar{\beta}_B - C_B\right) + 2\bar{\beta}_B\left(\bar{\beta}_S - C_S\right)}{\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)} > 0$$
$$\delta_S^{N\to SI} \equiv N_S^{SI} - N_S^N = \bar{\beta}_B\bar{\beta}_S \frac{\left(\bar{\beta}_B - C_B\right) + 2\bar{\beta}_B\left(\bar{\beta}_S - C_S\right)}{\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)} > 0$$

For consumers on side S the effects on welfare then pull in the same direction: the price decreases and there are more consumers joining on side B. For consumers on side B the effects pull in opposite directions. The price increases but more consumers are joining on side S. For the effect on welfare on side B we therefore have to calculate consumer surplus before and after integration. Since demand is linear, consumer surplus amounts to the area of the triangle below the demand curve. Consumer surplus on side B under non-integration is given by

$$CS_B^N = \frac{\bar{\beta}_B}{8}\left\{\frac{\left(-2\bar{\beta}_S\left(\bar{\beta}_B - C_B\right) - \left(\bar{\beta}_S - C_S\right)\right)^2}{\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)^2}\right\}$$

and consumer surplus on side B in the semi-integrated organization is given by

$$CS_B^{SI} = \frac{\bar{\beta}_B}{8}\left\{\frac{\left(-\bar{\beta}_S\left(\bar{\beta}_B - C_B\right) - \left(\bar{\beta}_S - C_S\right)\right)^2}{\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)^2}\right\}$$

Consequently, the change in consumer surplus when the platform integrates with the downstream firm on side S is

$$\begin{aligned}\Delta^{CS} &\equiv CS_B^{SI} - CS_B^N \\ &= \frac{\bar{\beta}_B\bar{\beta}_S\left[\left(\bar{\beta}_B - C_B\right) + 2\bar{\beta}_B\left(\bar{\beta}_S - C_S\right)\right]}{8}\left\{\frac{\left[2\bar{\beta}_S\left(\bar{\beta}_B - C_B\right) + \left(\bar{\beta}_S - C_S\right)\right]}{\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)^2\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)} + \frac{\left[\bar{\beta}_S\left(\bar{\beta}_B - C_B\right) + \left(\bar{\beta}_S - C_S\right)\right]}{\left(4\bar{\beta}_B\bar{\beta}_S - 1\right)\left(2\bar{\beta}_B\bar{\beta}_S - 1\right)^2}\right\}\end{aligned}$$

Under the restrictions laid upon the parameters by stability and non-negative demand, the above expression is positive. This means that consumer surplus increases on side B following integration on side S. Hence, maybe surprisingly, vertical integration on side S is actually welfare improving for consumers on side B despite the fact that these consumers now pay a higher price. The reason is that the effect on consumer participation on side S is greater than the effect on the price on side B.

To sum up, the effect on prices is not as clear-cut as with pure-usage pricing. It is possible that prices increase following integration. However, prices can only increase on one side of the market and when this occurs, the price will always fall on the other side of the market. Furthermore, consumers always benefit from integration, whether this is a partial or full integration.

As with usage pricing, profits must be higher under integration of any kind as well. Thus, integration eliminates the double marginalization problem an unambiguously increases welfare. The standard results of double marginalization carry over quite well with the caveat that vertical integration can raise the price level.

5 Conclusion

Two-sided markets have special characteristics and optimal price structures differ from those of standard markets. As a consequence, standard antitrust results cannot be transferred directly to two-sided markets. The logic needs to be worked out in a two-sided set-up.

This paper analyzed the effects of vertical integration in a two-sided market. Optimal prices were derived, focusing on a very general organizational structure which allowed for a downstream firm on both sides of the market. The platform's optimal price structure closely resembles that of Rochet and Tirole (2006) and Armstrong (2006). However, in addition to opportunity costs and elasticity of demand, the platform also takes into account the rate at which the downstream firms pass on increased costs to end-consumers. The higher the pass-through rate the lower the upstream mark-up, other things equal.

Vertical integration always lowers the price on the side where it takes place. However, it is possible that asymmetric integration, leaving a vertical structure on one side of the market, results in a higher price level due to a higher price charged on the other side of the market.

Welfare is affected by price and - via the network externality - by the number of participants on the other side of the market. Consequently, it is possible that consumers benefit by a price increase, as long as this price increase is the result of a price decrease on the other side of the market, leading to more participants here. Indeed, it turns out that consumer welfare always increases on both sides of the market following any kind of integration. Even when this leads to an increase in the price, consumers still prefer integration since this brings more members to the other side of the market. Therefore, it seems that the logic of double marginalization transfers to two-sided markets at least with respect to the welfare results.

It would be interesting to see how these results are affected if the downstream firms are not monopolists. There are two possibilities when introducing competition downstream. Either the downstream firms compete for consumers on both sides of the market. This could be the case in the example of Visa where one could imagine banks competing for both cardholders and merchants. Or, there could be some number of downstream firms on each side of the market, serving that side of the market only. In the first case, the downstream firms would themselves be platforms. This naturally makes the analysis more complex. If instead downstream firms compete solely on their own side of the market, competition would result in lower mark-ups than the ones prevailing under monopoly. In the case of perfect competition the downstream firms do not impose a negative externality on the platform and the double marginalization problem would not occur.

Lastly, it would be interesting to analyze the results on prices and welfare for other distributions of transaction- and fixed benefits.

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