Labour Supply and Optimization Frictions:

Evidence from the Danish student labour market[*]

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Abstract

In this paper I investigate the nature of optimization frictions by studying the labour market of Danish students. This particular labour market is an interesting case study as it features a range of special institutional settings that affect students' incentive to earn income and comparing outcomes across these setting effectively allow you to distinguish between different types of frictions. I find that the considered labour market is significantly affected by optimizations frictions, which masks the bunching at kink points normally associated with a positive labour market elasticity under standard theory. More concretely I find the dominate optimization friction to be individuals' inattention about their earnings during the year, while real adjustment cost and gradual learning appears to be of less importance.

1 Introduction

Labour supply elasticities – or more generally earning elasticities – are key parameters in many areas of economics, such as optimal income taxation (Saez et al., 2012). However empirical identification of these parameters remains a challenge – especially in the likely presences of optimization frictions, where Chetty (2012) shows that even a small amount of optimization frictions limits the researcher to identify only bounds on the elasticities. Bounds that in many case are so wide that it is likely to dwarf many of the econometric issued.

In this paper I shed light on the underlying nature of these frictions by studying the labour supply of Danish students. So far concrete evidence on frictions has been relatively limited in the economics literature on labour supply, which reflect that identification of optimization friction typically requires both high quality data and special institutional settings – high quality data in order not to confound optimization errors by individuals with measurement error in the data and special institutional settings that allow separation of rational behaviour from optimization errors. Kleven and Waseem (2013) is one of the few papers that fulfil both of these requirements and use this to estimate both a structural labour supply elasticity and the level of optimization frictions in a Pakistani setting, while remaining agnostic about the underlying nature of frictions.[1]

When analysing frictions the labour market of Danish students represents an interesting case study for several reasons: 1) students face a sharp kink in their budget set created by phasing out of student benefits, 2) In 2009 a reform significantly increased the income level at which students reach the kink point and 3) students face a special institutional setting, where they effectively can choose between different budget sets.

The strength of having all of these institutional settings within a well-defined labour market is that you can effectively distinguish between 3 of the main types of optimization frictions discuss on the literature – namely real adjustment costs (Attanasio, 2000), gradual learning (Mankiw and Reis, 2002 and Evans and Honkapohja, 2001) and (rational) inattention (Sims, 2003) – by examining the outcomes around each setting.

The use of the Danish student labour market as a case study further benefits from the fact that the labour market is covered by rich register data and utilizing these I find the following: First, following the 2009 reform I find an immediate and non-trivial shift in the students' earnings distribution compared to a very stable distribution both before and after the reform.

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[1] In other context such as e.g. consumption, Chetty et al. (2009) show that salience of taxes can affect demand.

Second, despite this clear evidence of a positive labour elasticity I find no sign of bunching at the kink point created by phasing out of student benefits. Finally, I find that a significant share of students fail to choose the budget set that is optimal given their final level of earnings. Taken together, these findings point to the presence of significant optimization frictions that mask the bunching at the kink point predicted by a standard labour supply model (Saez, 2010).

However, at the same time the findings do not point to real adjustment cost or learning as the main underlying frictions, as these types of frictions would lead to a more gradual transition to a new earnings distribution following the 2009 reform. Instead the findings suggest that the dominate optimization friction is students' inattention about their earnings process during the year.

After presenting graphical evidence on the above findings I proceed with a discussion of how to quantity the behavioural responses. This is not a trivial task as the lack of a clearly visible excess mass in the cross sectional setting makes it impossible to employed the standard bunching method developed by Saez (2010), and because the shift in the earnings distribution following the 2009 reforms occurs over a wider range than the range that is directly affected by the 2009 reform, which complicates a clean division of individuals into treatment and control groups.[2]

Instead I purpose a method that resembles the one used by Chetty et al. (2013) to uncover the effect of the EITC on the US income distribution and use the shift in the distribution following the 2009 reform to uncover the (local) counterfactual distribution at the kink point. Having the counterfactual distribution I use the bunching method to translate the observed responses into elasticities from which I obtain a lower bound estimate of the elasticity of 0.05-0.06

2   Optimization frictions and labour market outcomes

Before moving into the empirical analysis I start by drawing a number of hypotheses about how different types of optimization frictions affect observed labour market outcomes around different stylized policy setting. These will in section 3 be related to the actual policy setting facing the Danish students. More concretely I consider the following 3 stylized policy settings:

1.   A kink point in the budget set created by a jump in the marginal tax rate.

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[2] This method is used by among others Feldstein (1995) and Gruber and Saez (2002). See Kleven and Schultz (2013) for an application on Danish data.

2. A tax reform that changes tax rates in some parts of the income distribution.

3. Voluntary take up of benefits.

Of these, the 2 first are standard institutional settings considered in the public finance literature, whereas the 3rd needs some additional explanation.

A stylized benefit system consists of a lump sum grant that is phased out with earnings according to some schedule, and the idea here is a setting, where individuals have to decide whether or not to take up benefits. In most real life benefits systems this would from an economic perspective be a trivial choice, as the phase out stops once (net) benefits reach 0, and the budget set created by taking up benefits would thus always (weakly) dominate the budget set without benefits. However if the phase out is "prolonged" beyond the breakeven point, taking up benefits is not always optimal as illustrated in figure 1.

Figure 1
Illustration of the potential inoptimality of taking up of benefits.
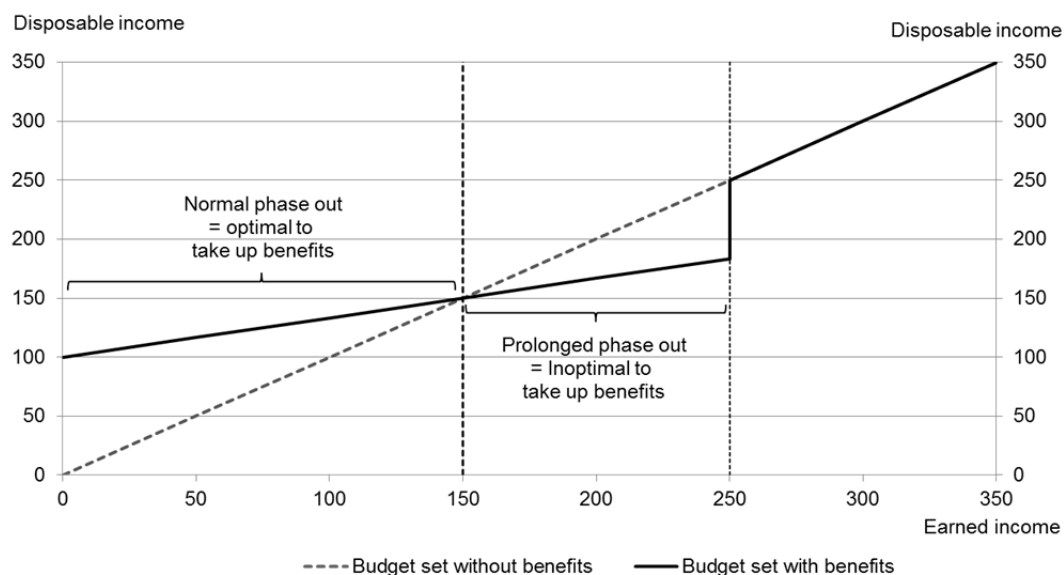
Notes: The figure shows a stylized budget set without benefits (or taxation) equal to the 45º degree line and a budget set under a benefits that gives a lump sum grant of 100, which afterwards is phased out with earnings at a rate of 75 percent. At this phase out rate net benefits reach 0 at an income of 150. If the phase out is prolonged beyond this point, it create a range of earnings were it inoptimal to take up benefits.

The figure shows a stylized budget set without benefits (or taxation) equal to the 45º degree line and a budget set under a benefit system that gives a lump sum grant of 100 that is phased out with earnings at a rate of 75 percent. At this phase out rate net benefits reach 0 at an income of 150, and take up of benefits is therefore optimal with income below this point. In contrast, it is inoptimal to take up benefits with income above this point if the phase out is prolonged.

From these 3 stylized policy settings it is possible to draw a number of hypotheses about what type of outcomes we should expect to find under the presences of different types of optimization frictions. More concretely I consider the effect of three broad groups of optimization frictions – namely:

1. Real adjustment cost on the labour market.
2. Gradual learning.
3. (Rational) inattention.

However before considering the effect of the three types of frictions I start by consider the labour market outcomes in a world without optimization frictions.

In this setting individuals would bunch at the kink point created by the jump in the marginal tax rate and thereby create clear excess mass in the earnings distribution at this point, with the excess mass being proportional to the labour supply elasticity (Saez, 2010). Following a tax reform that changes tax rates in some part of the income distribution, we should find an immediately change in earnings for the individuals who are directly affected by the change in incentives and finally, we should expect individuals to only take up benefits if it increases their disposable income – i.e. no one with earnings in the "prolonged" range shown in figure 1 would take up benefits.

Against this benchmark we start by consider the effect of real adjustment costs on the labour market (see e.g. Attanasio, 2000). Real adjustment costs imply that it is costly for individuals to change their earnings, e.g. because it requires finding a new job, which might take time and effort to search for. In this scenario, individuals are willing to accept jobs located in an earnings interval around their optimal point, as the expected benefits of renewed search do not outweigh the search costs (Chetty et al., 2011). As a consequence only a fraction of the individuals, who in a frictionless world would bunch at the kink point, do so in this setting causing the excess mass to be spread over an interval around the kink point (fuzzy bunching).

When it comes to the effect of a tax reform, the presence of real adjustment costs imply that not all individuals will find it optimal to change their income immediately. Instead they might choose to keep their current job if they e.g. expect that they in the near future have to change job for other reasons. As a consequence we should only expect to see a gradual change in the earnings distribution.

Finally, real adjustment costs on the labour market should not necessarily have anything to do with individuals being able to decide whether or not to take up benefits. As long as the administrative system is fairly simple, the economic cost of taking up benefits is trivial, and

we should therefore expect individuals to take up benefits optimally given their current job choice, even if this choice deviates from what they would have chosen in a frictionless world.

The second general class of optimization frictions that we might consider is gradual learning (see e.g. Mankiw and Reis, 2002 and Evans and Honkapohja, 2001). Gradual learning implies that individuals do not have perfect information about the institutional setting, when they are new to the system or when the system is changed. This would e.g. include knowledge of the precise position of the kink point and the design of the benefit system, and as consequence we should expect only fuzzy bunching around the actual kink point and sub-optimal take up of benefits – especially among individuals with less experience with the institutional setting.

Likewise, gradual learning would imply that the knowledge of a reform would expand gradually after its implementation and we would therefore expect to see a gradual change in the earnings distribution.

Finally we might also consider the effect of (rational) inattention (see Sims, 2003). Rational inattention builds on the idea that economic circumstances might change over time, but that it is costly for individuals to keep close attention to these changes. Changing circumstances, which in a frictionless world would have warranted a reoptimization of individual behaviour, therefore might not be noted by individuals in this scenario leaving them with ex post sub-optimal behaviour.

Formulated in this way there is a potential big overlap between gradual learning and inattention, as e.g. inattention about changes in the institutional setting will be exactly the same as the gradual learning described above. As a consequence I will make the following distinction between gradual leaning and inattention: Gradual learning refers to learning about institutional settings that we normally would think as constant in the long run (changes in institutional settings such as tax rates only happen as a result of reforms). In contrast, inattention refers to inattention about individual economic factors that may vary even in the long run – factors such as individual wages, working requirements etc. In a world were these individual factors are partly random, individuals will never learn the true values of these by accumulated experience, but can only know them by paying close attention to their evolution.

Applied to the labour market inattention implies that individual will aim at a desired level of labour supply and earnings, but that their actual earnings will be distributed around this level due to random shock to individual economic factors over time, which the individuals fail to realize and thus offset by reoptimization. As consequence we should expect only fuzzy bunching around a kink point in the budget set. Likewise we should expect to see some individuals take up benefits even though it ex post turns out to be an inoptimal choice. Howev-

er, despite of the inattention about the evolution of individual economic factors, we should expect to see an immediate change in the earnings distribution following a tax reform, as individuals adjust their desired income to the new incentive.

Finally it should be noted that the notion of inattention as being rational rely on the presumption that the cost of paying closer attention to changes in the economic circumstances outweigh the expected benefits of smaller optimization errors. However more generally inattention might also be irrational just as the inattention might also be related to the effects of the individual's own actions – e.g. in the labour market, where individuals' labour supply and earnings may vary from month to month, while taxation is based on the cumulative earnings. In this case, knowing the effect extra earnings in one month requires the individual to keep track of (and predict) earnings in all months.

The predictions from the different hypotheses described above are summarized in table 1 and as the table shows each type of optimization friction leads to a unique set of predictions across the different institutional settings. Combining the observed outcomes across these settings therefore in principle allow you to distinguish between types of frictions.

Table 1
Hypotheses: What to expect under different types of optimization frictions?

	Bunching at the kink point	Effect of a tax reform	Take up of benefits
Benchmark:			
No frictions	Clear bunching	Immediate change	Optimal cancelling
Optimization frictions:			
Real adjustment cost	Fuzzy bunching	Gradual change	Optimal cancelling
Gradual learning	Fuzzy bunching	Gradual change	Sub-optimal cancelling
(Rational) inattention	Fuzzy bunching	Immediate change	Sub-optimal cancelling

3 Institutional setting: students' incentive to earn income

In this section I present the key features of the Danish Student benefit system and relate them to the stylized institutional settings discussed in section 2.[3]

Danish students enrolled in education above primary school (ISCED2011 level 3 and above) are eligible to state financed student benefits from the age of 18. Benefit rates vary depending on the type of education and civil status, but in 2008 the basic rate for students

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[3] A more detailed description can be found in appendix A. All relevant variables are drawn from detailed register data organized by Statistics Denmark (DST) covering the entire universe of the Danish population. A more detailed description of these registers and the variables used can be found in appendix B.

not living with their parents enrolled in tertiary educations (ISCED2011 level 5 and above) was 5,000 DKK per month (1 USD ≈ 5.5 DKK).

In addition to receiving these benefits, students are allowed to earn income of up to 6,400 DKK per month.[4] If they earn more than this baseline income limit (on a yearly basis) the excess is deducted from the amount of benefits received. Of the first 9,500 DKK 50 percent is deducted, while further excess earnings is deducted 100 percent.[5]

If a student wants to earn more than the baseline income limit they can increase the limit by cancelling one or more months of benefits. By cancelling one month of benefits a student increases the income limit by 9,500 DKK, which translates into an phase out rate of 5,000/9,500 = 52 percent. Administratively, it is fairly easy for students to cancel benefits, as it is simply done through a webpage, where students can click benefits in individual months on and off.

Taken together with the normal income tax system, which – for the incomes in the ranges considered here – imposes a marginal tax rate of 41 percent (excl. VAT) the effect of the phase out of benefits is that the effective marginal tax rate jumps from 41 to 72 percent when students' earned income excess 76,400 DKK annually.[6]

However, the effective marginal tax rate might jump even more if students fail cancel the right amount of benefits. If e.g. a student earns more than 86,000 DKK and does not cancel student benefit he faces as marginal tax rate of 100 percent. In this case it would be optimal to cancel one month of benefits in order to lower the marginal tax rate to 72 percent.

This problem corresponds to the problem of optimal take up of benefits described in section 2, which 12 months of benefits is optimal for students earning up to 76,400 DKK annually. 11 months is optimal for students with income between 76,400 and 86,000 DKK. For students earning extra 9,500 DKK 10 months is optimal etc., as illustrated in figure 2.

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[4] Income counted against the income limit is called "own income" and includes labour income, transfers other than student benefits and capital income with the exceptions of certain types of stock income, cf. appendix B.

[5] Finally, if the amount of student benefits that a student has to pay back exceeds 7,600 DKK (2008 level), the entire payback is increased by 7 percent. This implies that the marginal tax rate for excess earnings above this amount exceeds 100 percent. This is not shown in figure 2.

[6] There is a caveat to the calculation of the effective marginal tax rate, when students cancel student benefits. For most university students student benefits are limited to a period of 6 years (compared to a standard study time of 5 years) and by cancelling a number of months of benefits, the student can save them for later use. Prior to the 2009 reform students could also get higher benefits by using previously cancelled months. Some student might therefore not see the cancelling of benefits as the full loss assumed here. The probability of this does not significantly affect the conclusions drawn in the paper and are discussed in section 4 and 6 below.

Figure 2
Effective budget sets for students depending on benefits take up, 2008
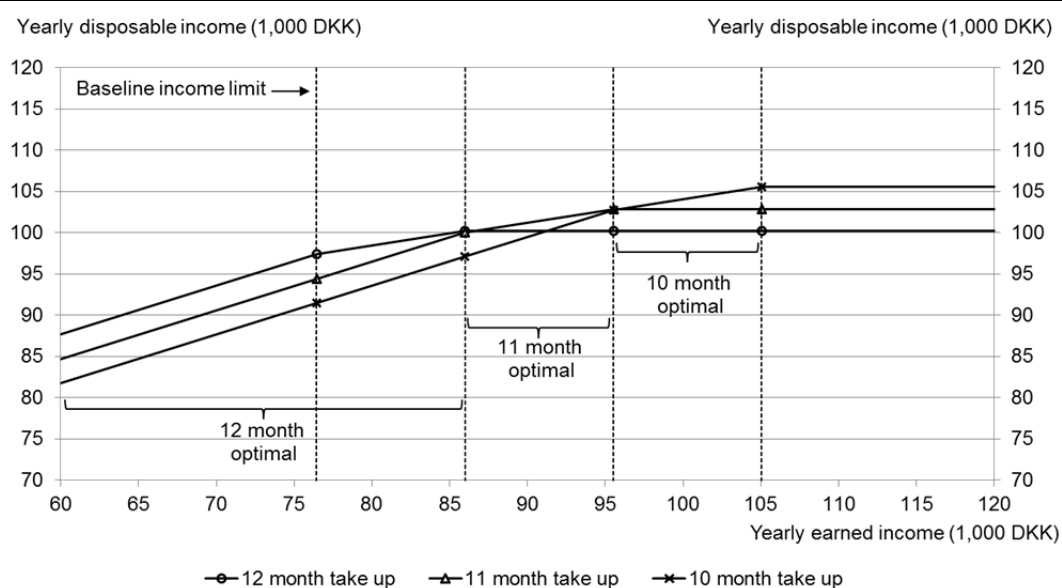
Notes: The baseline income limit is calculated as 12 x the monthly basic amount of 6,400 DKK. Yearly disposable income is calculated as first gross income consisting of 5,000 DKK x the number of months of benefits taken up plus earned income up to the income limit, which increases by 9,500 DKK for each month not taken up. Above this income limit the first 9,500 DKK in earned income is deducted in student benefits at 50 percent, while further excess is deducted 100 percent. Finally gross income is turned into disposable income based on a personal allowance of 41,000 DKK and a marginal tax rate in the normal income system of 41 percent. 5.5 DKK ≈ 1 USD.

Sources: Own calculations based on www.su.dk.

However the switch to higher effective budget sets by cancelling benefits is complicated by the fact that students have to do this actively prior to actually receiving the benefits. Cancelling benefits for a given month has to be done prior to the 15th the month before, which has to be compared with the fact that students typically receive their wage check at the end of the month or with an additional month's lag. E.g. cancelling benefits in December has to be done prior to November 15th, where students in general only have seen their wage checks up to October or September.

This time difference between, when a student has to cancel benefits and when he has the actual information about the monthly (or yearly) income imply that the students have to pay close attention to their income process during the year and to some degree predict what they will earn a couple of months into the future in order to cancel the right amount.

The student benefit system have remained basically unchanged through the period 2004-2011, which is considered in this analysis, except from a reform in 2009 that increase the baseline income limit by 25 percent for students enrolled in tertiary educations, while leaving it unchanged for lower levels of education, cf. table 2. At the same time the phase out rate

for tertiary students were also increased from 52 to 62 percent and thus causing an increase in the effective marginal tax rate from 72 to 78 percent.

Table 2
Development in the yearly baseline income limit

1,000 DKK	2006	2007	2008	2009	2010	2011
Students in tertiary education	72.1	74.1	76.4	97.7	101.6	103.5
Students in lower education	72.1	74.1	76.4	79.0	82.2	83.8

Notes: The baseline income limit refers to the income limit for students, who do not cancel any months of benefits. Tertiary education incl. university education and educations such as nurses and school teachers (ISCED2011 level 5 and above). Lower educations incl. high school (gymnasium) and vocational educations (ISCED2011 level 3-4).
Sources: www.su.dk.

In what follows all numbers related to income variables have been translated to 2008 values using the indexation implied by the baseline income limit for students in lower educations.
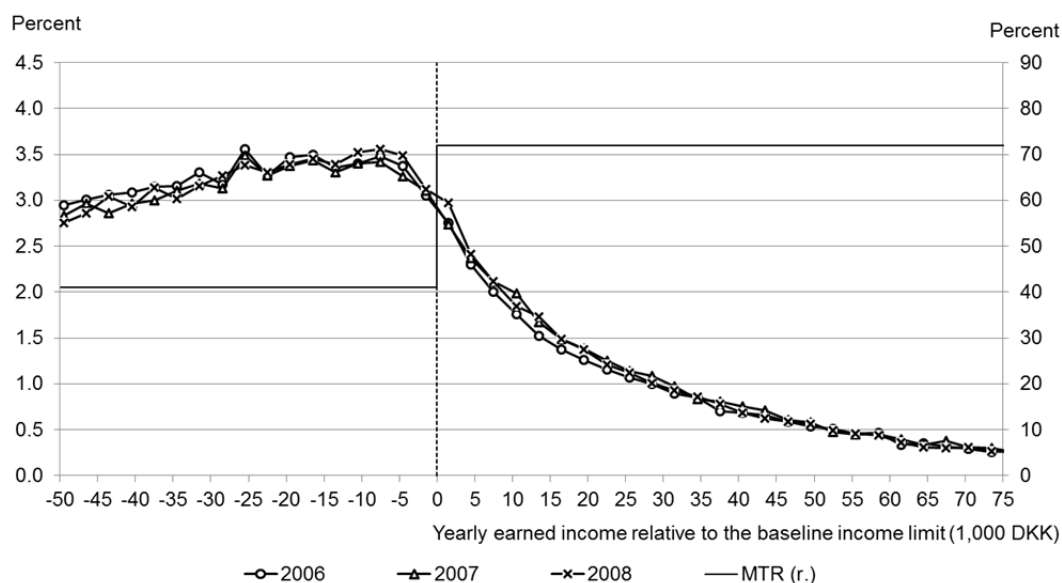
4   Graphic evidence of labour supply responses and optimization frictions

In section 3 I linked the specific features of the Danish student benefit system to the stylized institutional settings listed in section 2. In this section I examine the observed labour market outcomes around each of the institutional features and compare it with the hypothesis drawn in section 2.

Evidence from bunching at the kink point

Figure 3 shows the earnings distribution for students enrolled in tertiary educations before the 2009 reform around the baseline income limit. Only students, who are fully eligible for student benefits the entire year is included in this figure, however inclusion is not conditional on actually receiving student benefits (i.e. students are allowed to cancel benefits). Under the assumption that students cancel the right among of benefits, their effective marginal tax rate jump from 41 to 72 percent at the baseline income limit as described in section 2.

Figure 3
The income distribution for tertiary students, 2006-2008

Percent

4.5   90

4.0   80

Yearly earned income relative to the baseline income limit (1,000 DKK)

─○─ 2006   ─△─ 2007   ─×─ 2008   ── MTR (r.)

Notes: Students have to be fully eligible for student benefits (but necessarily receive student bene-fits) and have earnings above 6,500 DKK to be included in the calculation of densities. The marginal tax rate (MTR) is calculated under the assumption that students always cancel the optimal amount of student benefits. In that case MTR = 1 – (1-t)•(1-q), where t = 0.41 and q = 0 below the baseline income limit and q = 0.52 above. The baseline income limit was 76,400 DKK in 2008. Bin size = 3,000 DKK.

Sources: Own calculations based on DST

This figure shows that the earnings distribution was extremely stable during the 3 years prior to the reform and with no clear sign of excess mass around the kink point. In a frictionless world this would imply that the labour supply elasticity was negligible, but from the cross sectional evidence alone – which most bunching studies rely on – we are not able to determine whether this outcome is truly driven by a zero labour supply elasticity or whether optimization frictions prevent the formation of a clear excess mass at the kink point. Naturally, we cannot neither distinguish between different types of optimization frictions.

Evidence from the 2009 reform

When comparing the pre-reform earnings distribution with the distributions after the 2009 reform, we see in figure 4 a clear shift in the distribution with mass moving from below the initial kink point to a range above. Given the fact that the distribution was extremely stable in the years prior to the reform this shift constitutes compelling graphical evidence for a

positive labour supply elasticity, suggesting that the lack of bunching at the kink points is due to optimization frictions.[7]

Figure 4
The income distribution for tertiary students before and after the 2009 reform
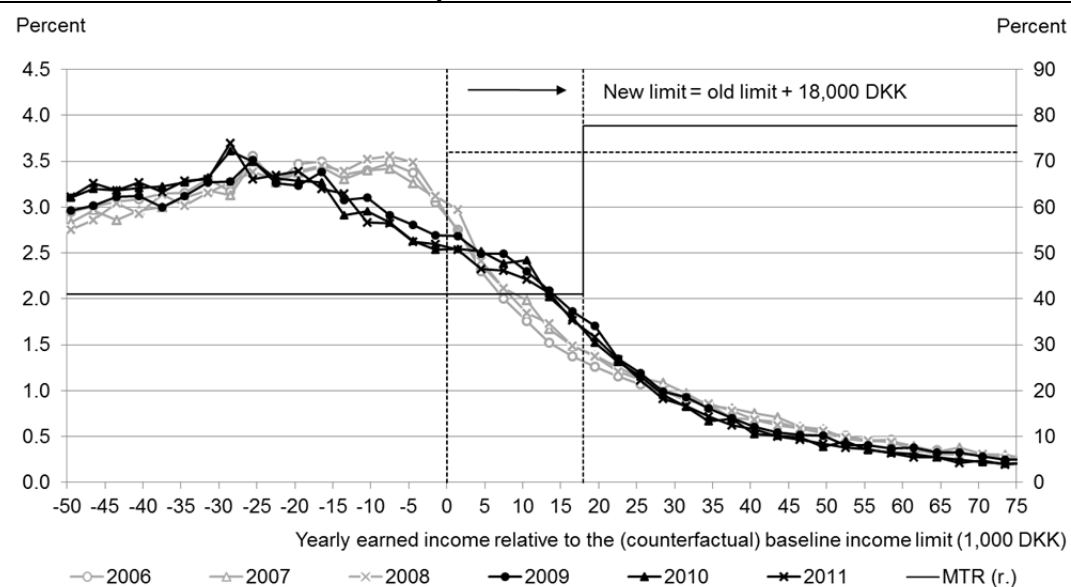
Notes: See notes to figure 3. For the years 2009-11 income is measured relative to the baseline income limit without the 2009 reform. This corresponds to the baseline income limit for students in lower education listed in table 2.

Sources: Own calculations based on DST

Furthermore, the fact that shift in the distribution appears to happen instantaneously from 2008 to 2009 speaks against both real adjustment cost and gradual learning as the dominate frictions. Taken together, the two first pieces of empirical evidence thus points to inattention as the dominate optimization frictions in this labour market.

**Evidence from the cancelling of student benefits**

Turning to the cancelling of student benefits I consider the earnings distribution for students conditional on the amount of student benefits they cancel. In figure 5 this is done for students who have cancelled exactly 1 month and thus taken up 11 month of benefits.

By cancelling 1 month of benefits these students increased their income limit to 86,000 DKK (before the 2009 reform) and we should not expect to find students with earnings 9,500 DKK above this amount (where they reach the 100 percent marginal tax rate). If they wished to earn more they should have cancelled an extra month of student benefits in order

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[7] The interpretation of the shift in the earnings distribution as an indication of a positive labour supply response to the 2009 reform is also supported by the fact that the earnings distribution of students in lower educations, who was unaffected by the 2009 reform, remained stable.

to increase the income limit and thereby lower their effective marginal tax from 100 to 72 percent.

Figure 5
The income distribution for tertiary students who cancel 1 month of student benefits
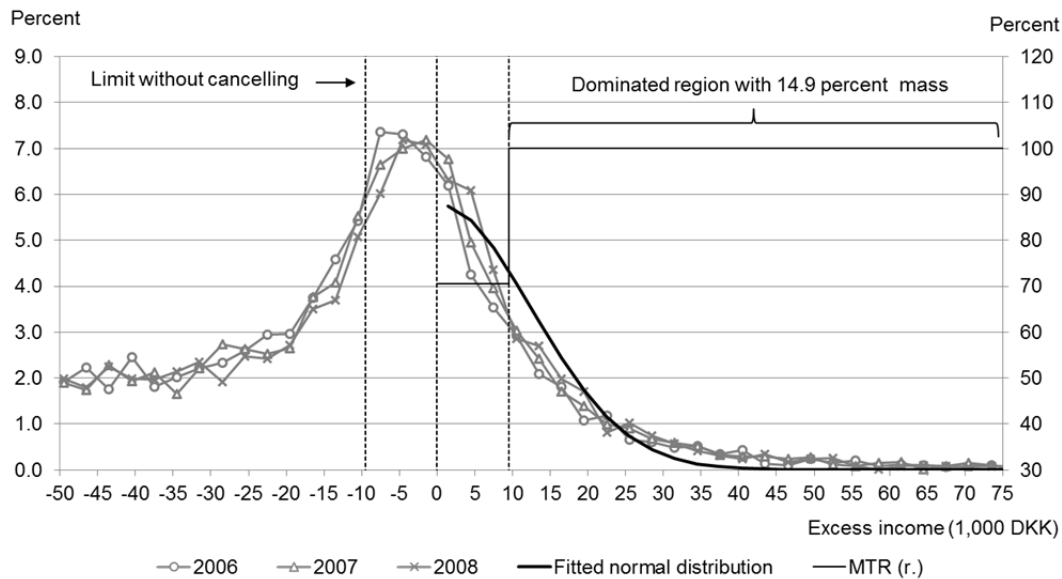
Notes: Excess income is defined as the yearly earning income relative to the actual income limit that the individual is facing. The marginal tax rate (MTR) is calculated using the formula MTR = $1 - (1-t) \cdot (1-q)$, where $t = 0.41$ and $q = 0.50$ for the first 9,500 DKK above and $q = 100$ above this level. Bin size = 3,000 DKK. The fitted normal distribution is chosen to minimize the squared error with the right side of the actual distribution (excess income > 0). This yields a standard error of 12,500 DKK.
Sources: Own calculations based on DST

From figure 5, however, we see that, even though the earnings distribution for this group of students is centred on the actual income limit that they faced after cancelling 1 month of benefits, a significant proportion of student deviate from this earnings level.[8]

Considering e.g. the upper part of the distribution, 14.9 percent of the students, who have cancelled exactly 1 month of benefits, earned more than 9,500 DKK above their actual income limit and thus hit the effective marginal tax rate of 100 percent. As a consequence these students could with relatively little effort have cancelled another month of benefits and thereby increased their disposable income. For the 6.3 percent, who had an excess of more than 20,000 DKK, the increase in disposable income would have been at least 3,000 DKK by cancelling additional month(s) of benefit.

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[8] When interpreting the distribution in figure 5 as a result of optimization frictions it is important to eliminate measurement errors from the data, as these will otherwise result in an upward bias of the amount of frictions. An assessment of the amount of measurement errors and the results robustness to these are presented in appendix B and C.

Considering the lower part of the distribution we also see a significant proportion (70 percent) of students, who earned less than the actual income limit. In principle these students cancelled benefits without the need to do so and therefore received fewer benefits than they could have, however there might be some intertemporal considerations that rationalize this behaviour. As student benefits are limited to typically 6 years, student might find it optimal to student benefits for later use by cancelling some months even in years, where their earnings are below the income limit. In contrast to the upper part of the distribution, it is therefore less straight forward to take this as firm evidence of sub-optimal cancelling.

Above the amount of optimization frictions is quantified by the share of students in the dominated region. However this metric is problematic as it depends crucially on the part of the sample that is included in the calculation. Considering e.g. the students, who do not cancel benefits, only 5.0 percent end up in the dominated region (compared to 14.9 percent above), but this is of course due to the inclusion of a large number of students, who are well below and not trying to target the income limit.

When considering inattention a more natural way to quantify the amount of frictions is to ask how much variance in their final earnings relative to their desired earnings individuals are will to accept.

One way to quantify this is to assume that everybody to the right of the income limit (either with excess income above 0 or above 9,500 DKK) target the income limit and calculate a standard error from this part of the distribution.[9] Doing this for the "raw" distribution yields a standard error around 20-24,000 DKK, cf. table 3 – a quite large standard error, which is due to the fact that the distribution have relatively fat tails. If I instead fit a normal distribution to the considered part of the distribution (the fitted line is shown in figure 5), the resulting standard error is reduced to 11-14,000 DKK corresponding to 1-2 months of earnings for the students around the kink point.

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[9] Intuitively, this is done by mirroring the right side of the distribution around the income limit and calculate the standard from this constructed distribution.

Table 3
Summary of the evidence for income uncertainty, 2006-08

	Mass[1]		Raw standard error[2]		Fitted standard error[3]	
Cancelled:	E > 0	E > 9,500	E > 0	E > 9,500	E > 0	E > 9,500
0 months	11.1	5.0	20.3	24.4	11.1	13.3
1 month	30.3	14.9	22.7	24.1	12.5	13.8

Notes: E = Excess income (Earned income – income limit).
1)   Share (percent) of sample with an income above the income limit or the income limit + 9,500 DKK respectively.
2)   Standard error (1,000 DKK) calculated from the part of the distribution either above the income limit or above the income limit + 9,500 DKK.
3)   Standard error (1,000 DKK) calculated by fitting a normal distribution to the distribution either above the income limit or above the income limit + 9,500 DKK.
Sources: Own calculations based on DST

While the sub-optimal cancelling of benefits – as argued above – speaks against real adjustment cost as the dominate type optimization friction present in this labour marked, it might be consistent with both gradual learning and inattention, cf. table 1. However a key difference between these two explanations is that under gradual learning we should expect that sub-optimal cancelling of benefits primarily to be found among new students.

In order to investigate this, I show in figure 6 the distribution from figure 5 split into two sub-samples of students, who have either be a student for 2 or more years or had a high income the year before – with the idea being that these two sub-samples should have better information about the structure of the student benefits system.

Figure 6
Cancelling depending on student history for tertiary students who cancel 1 month of student benefits
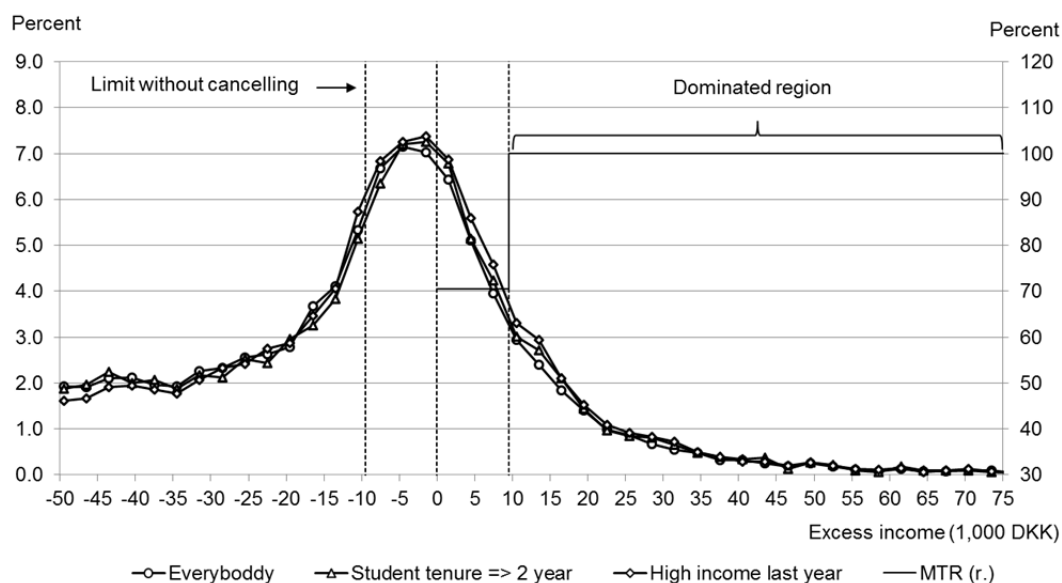
Notes: See notes to figure 5. Student tenure is measured from the start of the student's first tertiary education. High income last year is defined as having an income no less than 20,000 DKK below the baseline income limit.
Sources: Own calculations based on DST

As this figure shows, there is fundamentally no difference between the distributions, and evidence does therefore not support that the sub-optimal cancelling is caused by gradual learning among the students.

5   The nature of inattention

The graphical evidence in section 4 points to inattention about their earnings process as the dominate optimization friction in the labour market for Danish students. However, because there is a time lag of 1-2 months between when students have to decide whether or not to cancel benefits and when they have precise information about their current accumulated earnings, the sub-optimal cancelling we observe in figure 5 might simply reflect income surprises in the end of the year. In this case we should expect to find a positive correlation between positive individual income surprises and the amount of income exceeding their income limit.

In order to investigate this I use monthly income register data available from 2008 and define an end of the year income surprise as the difference between the sum of November and December pay and the sum of the September and October pay. Plotting this measure against the individual excess income gives the picture presented in figure 7.

Figure 7
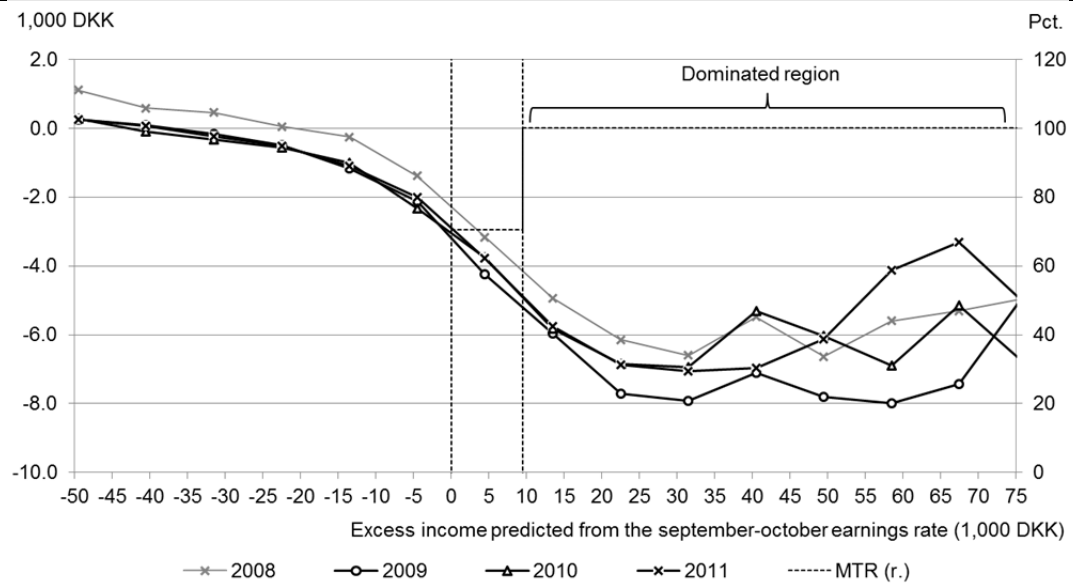**Average end of year income surprise over the income distribution, 2008-11**
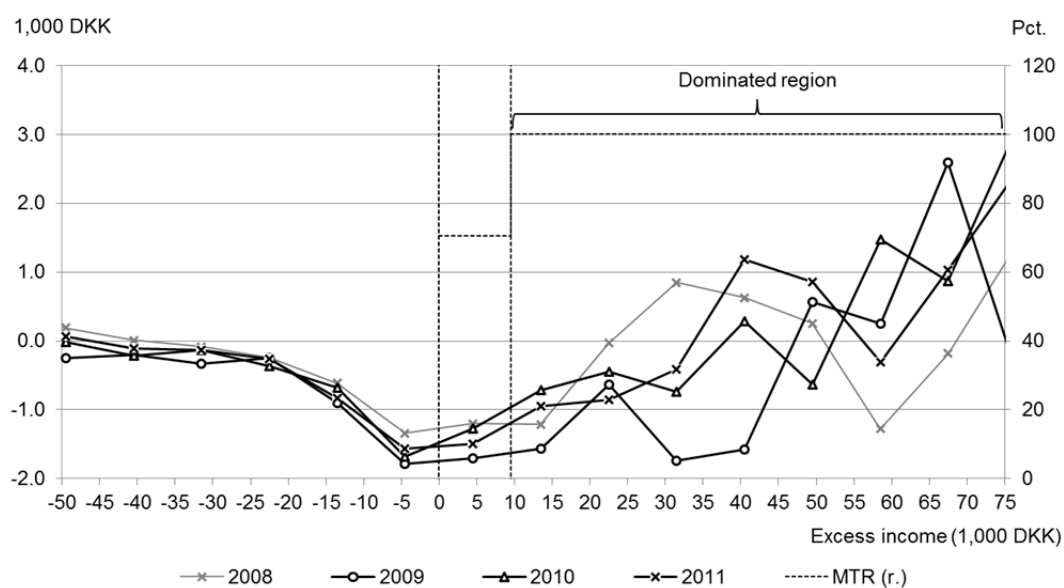
Notes: The figure only includes individual who cancel either 0 or 1 month if student benefits. The individual end of year income surprise is calculated as as the difference between the sum of November and December pay and the sum of the September and October pay.Only labour income is included in this data and months without employment are treated as 0 income. Bin size = 9,000 DKK.
Sources: Own calculations based on DST

From this figure it is clear that there is a tendency to find larger end of year income surprises among the individuals who end up with larger excess income. However the magnitude the effect is not enough to explain the level of sub-optimal cancelling. Going e.g. from an excess income of 10,000 DKK to 50,000 DKK the average income surprise only increases by around 4,000 DKK, which therefore only explain 10 percent of the excess.

The figure, however, seems to reveal another interesting feature from the monthly income data. It seems to be the case that students reduce their earnings when they approach the income limit. This behaviour is more clearly visible when plotting the average end of year income surprise against the level of earnings that the students would have had without the income surprise – i.e. the yearly level of earnings if the November and December pay had equalled the earnings in September and October (called predicted income), cf. figure 8.

Figure 8
Average end of year income surprise over the predicted income distribution, 2008-11

Notes: See notes to figure 7. Predicted excess income is the excess income that the individual would have had without the end of year income surprise – i.e. the actual earned income minus the difference between the sum of the November and December pay and the sum of the September and October pay. Bin size = 9,000 DKK.

Sources: Own calculations based on DST

From figure 8 we see a consistent drop in the average end of year income surprise of magnitude of 6-8,000 DKK for individuals, who at their September-October earnings rate were in risk of exceeding their income limit by the end of the year.

This drop could of course just be due to mean reversion after following a positive income shock in September-October, but note further that this drop is the same in the pre-reform year 2008 as in the post-reform years despite that the baseline income limit has been increased by 25 percent. That the drop occurs over the same range of excess income therefore reflect that the behaviour has moved up in the earnings distribution. Indeed, drawing the distributions 2008-2011 for the predicted income without the end of year income surprise removes much of the post-reform shift observed in figure 4.

This type of behaviour is not straight forward to reconcile with standard rationale inattention. Under risk neutrality standard rational inattention would suggest that individuals choose a job, which in expectation would give them their desired level of earnings. In the labour market considered here it appears that individuals take a job, which in expectation gives them a level of earnings above their desired level. Something that they first realize in the end of the year and instead of cancelling an extra month if student benefits – which would be a relative easy way to avoid the 100 percent effective marginal tax rate – they seek to reduce their labour supply and thus earnings.

One way to rationalize this is to think that individuals are relatively risk adverse and thus take a job that with a high probability will give them their desired level of earnings, but once this level has been achieved they react to the reduced earnings incentives created by the phasing out of student benefits and reduce labour supply. However, perhaps more realistically the inattention that individuals exhibit in this labour market is not fully rational.

6   Estimation of the labour supply response

After having shown in the sections above the likely presences of significant optimization frictions in the Danish student labour market, I proceed in this section with a discussion of how this is likely to affect the way labour supply elasticities are normally estimated.

Considering the labour supply responses observed in section 4 it clear that the two "standard" methods for estimating labour supply responses in public finance – the Saez (2010) bunching method and the Feldstein (1995) difference-in-difference (DiD) method – may fail to undercover the true elasticity.

When applying the bunching method researchers typically calculate the excess mass by fitting a high order polynomial to the distribution around the kink point excluding a range, where there is "visible bunching". However, in the student labour market considered here there is no visible bunching and a credible counterfactual distribution using this method in the purely cross sectional setting would therefore in practise follow the actual distribution yielding a zero excess mass and elasticity.

Likewise, when applying the DiD method, the labour supply elasticity is estimated by comparing individuals who are treated by (tax) reforms to different extent, where treatment status typically are assigned based on pre-reform earnings.[10] In the case considered here, this would imply that students with earnings between the pre-reform and the post-reform kink point would be assigned a lower marginal tax rate and the students above the-post reform kink point a slightly higher marginal tax rate. However, from figure 4 it is clear that the shift in the distribution happens over a much wider range than would be assign treatment status using this method and as a consequence the assigned treatment groups would consist of a mix of the actual the treatment and control groups.

To undercover a labour supply elasticity in this setting I therefore instead employ a method that resemble the method use by Chetty et al. (2013) and utilize the shift in the distribu-

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[10] In practices the estimation procedure is more advanced using the treatment status based on pre-reform earnings as an instrument and controlling for underlying income dynamics such as mean reversion. See Weber (2014) for a recent discussion of the DiD method.

tion created by the 2009 reform to undercover the (local) counterfactual distribution and hence the excess mass created by the pre- and post-reform kink.[11] Finally, I turn this excess mass into a labour supply elasticity using the Saez (2010) bunching formula.[12]

Figure 9 shows the average income distribution over the 3 pre- and post-reform years considered in this analysis, which illustrates the shift in the distribution after the reform also seen in figure 4. From this figure we can identify two areas with excess mass: Taking the post-reform distribution as a (local) counterfactual we find an excess mass 3.1 percentage points below the pre-reform kink point. Likewise, taking the pre-reform distribution as a (local) counterfactual we find an excess mass of 2.1 percentage points below the post-reform kink point

Figure 9
Identifying excess mass using the 2009 reform
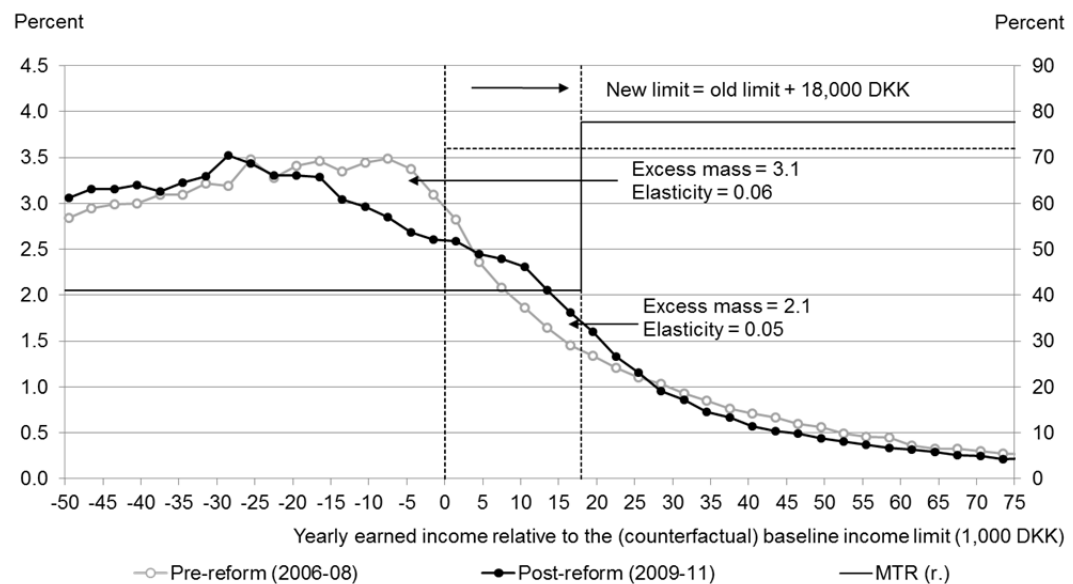
Notes: See notes to figure 4. For the calculations of the elasticities see table 4.
Sources: Own calculations based on DST

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[11] The method resembles the method used by Chetty et. al. (2013) except that the source of the variation in the distribution here does not come from differences in knowledge about the tax schedule in a cross sectional setting, but from the time series variation created by a reform.

[12] One caveat has to be mentioned in connection to the translation of the excess mass into a labour supply elasticity. The formula for this translating derived by Saez (2010) rely theoretically on the marginal indifference individual, who bunch at the kink point, to change his earnings the same amount found when comparing two fully linear tax systems. In the presences of inattention, where individual not necessarily hit their desired income, this will no longer be the case and it is therefore not trivial that the formula is still valid in this setting. Saez (1999) performs simulations of the income distribution and assess the amount of bunching under various model setups, incl. income uncertainty, but he does not evaluate the performance of the bunching estimate in these simulations. As a robustness check I therefore preform a more structure estimate of the labour supply elasticity in appendix D.

To translate these excess masses into elasticities I use the Saez (2010) bunching formula, where the change in earnings in responses to a tax change (dz) can be expressed as:

$$dz = \frac{B}{f(z)} \tag{1}$$

Where B is the excess mass and $f(z)$ is the counterfactual density at the kink point (z).[13]

Using this formula the elasticity (ε) can be calculates as:

$$\varepsilon = \frac{dz}{d(1-t)} \frac{1-t}{z} = \frac{d\log(z)}{d\log(1-t)} \tag{2}$$

Which yields an elasticity of 0.06 for the pre-reform kink point and 0.05 for the post-reform kink point, cf. table 4.

Table 4
Calculating the labour supply elasticity for the tertiary students

	Pre-reform kink point	Post-reform kink point
Excess mass	3.1	2.0
Counterfactual density	2.6	1.4
Kink point	76,400 DKK	97,700 DKK
dlog(z)	0.047	0.045
dlog(1-t)	0.744	0.975
Elasticity	0.06	0.05

Notes: Bin size = 3,000 DKK.
Sources: Own calculations based on DST

This elasticity estimate is perhaps surprisingly small compared to the consensus in the literature of around 0.25 according to Saez et. al. (2012) and considering that the many students might have a large degree of flexibility in increasing their earnings if desired.[14] However there are a couple of reasons why the estimated elasticity is a lower bound.

First of all taking the post-reform distribution as the (local) counterfactual for the pre-reform distribution (and vice-versa) rely on the assumption the post-reform distribution at the pre-reform kink point is unaffected by the post-reform kink point. This would be true in a frictionless world, but with the fuzzy bunching created by optimization frictions this will not necessarily longer hold.

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[13] The counterfactual density is estimated as the average density in the two bins around the relevant kink point divided by the bin size.

[14] Working in the other direction is the fact that students might use a student job to gain valuable job experience, in which case the low intratemporal elasticity reflect future career concerns. However, dividing student job into non-relevant jobs (retail, waitering and postal service) and relevant jobs (everything else) does not give different elasticity estimates, which indicate that the future career concerns are not the prime reason for the low estimates.

Examining figure 9 it indeed seems to be the case that the excess mass around the post-reform kink point start to build up already at the pre-reform kink point and thereby biasing both the pre-reform and the post-reform excess mass downwards.

Secondly, as student benefits are limited to typically 6 years, some students might not see it as a full loss to cancel benefits in the way it is assumed above. If students expect to use the saved benefits later the real loss is only in terms of the difference in present value.

This implies that the phase out rate used so far – and hence the size of the kink point – is an upper bound of the actual phase out rate and thereby further implying that the estimated elasticity is a lower bound. Assuming e.g. that 20 percent of the students in a given year is indifference between receiving benefits within the year or saving them for later imply that the average kink point size will be 20 percent lower than the one used above. Scaling down $d\log(1-t)$ by this amount, increases the elasticities to 0.08 and 0.06, respectively.

7 Conclusion

In this paper I have investigated the nature and impact of labour market optimization friction using the Danish student labour market. This labour market is an interesting case study as it features a number of special institutional settings, which allow you to distinguish between different types of optimizations frictions.

Examining labour market outcomes across these institutional settings I find clear evidence of a positive labour supply response following a reform in 2009 that substantially increase the earnings level at which student benefits starts to being phased out. Yet, despite of this clear evidence of a positive labour supply elasticity, I find no visible bunching at the kink point created by the phase out of benefits in contrast to what standard theory suggest (Saez, 2010).

I take this as evidence of the presences of significant optimization frictions that mask the labour market outcomes suggested by standard theory – a finding that might be surprising given that student labour markets in general are associated with a lot of job turnover and part time workers and thus expected to have a high level of flexibility. However this is not at odd, as a closer examination of the observed outcomes also speaks against real adjustment costs or gradual learning about the institutional settings as the dominate optimization frictions. In particular because the positive labour supply responses after the 2009 reform materializes immediately. Instead, the evidence appears to be consistent with inattention about their earnings process during the year as the dominate friction among the individuals in the considered labour market.

Of course, the relative strength of the different types of frictions might not be directly transferable to other labour markets and in particular you would probably expect real adjustment to play a bigger role in the regular labour market, where workers in general tend to be more specialized full time employees. However, the finding that inattention in itself can create enough optimization frictions to mask the labour supply responses to a kink in the tax schedule predicted by standard theory is interesting even for the broader labour market.

Following the investigation into the relative importance of the different optimization frictions I discuss the implications for identifying the underlying labour supply elasticity and purpose a method that utilizes the shift in the earnings distribution created by the 2009 reform to uncover the local counterfactual distribution around the kink points created by the phase out of student benefits. Having this counterfactual distribution I use the Saez (2010) bunching formula and estimate a lower bound labour supply elasticities in the range of 0.05-0.06.

This method is in many ways a compelling method for estimating labour supply elasticities, but at the same it time poses high requirements on the data being used. Indeed, as the presences of optimization frictions causes a mixing of treatment and control groups in the way they are typically assigned in the commonly used Feldstein (1995) difference-in-difference method, you are forced to rely more heavily on only the time variation in labour supply and this is only credible if the earnings distribution is relatively stable in the non-reform year. This is potential a problem in labour markets where real adjustments or gradual learning plays a more dominate role, as this would cause the labour supply responses to be more gradual following a reform – a gradual responses that often will be difficult for the researcher to credibly attribute to the reform.

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Appendix A: The Danish student benefit system

Student benefit rates

   Danish students enrolled in almost all educations above the primary school (ISCED2011 level 3 and above) are eligible to state financed student benefits from the age of 18. Benefit rates vary depending on the type of education and civil status with the main rates (2008 level) listed in table A1. Benefits for students aged 18-19 in lower educations (ISCED2011 level 3-4) furthermore depend on their parents' income.

Table A1
Overview over basic student benefit rates, 2008

Monthly rate (DKK)	Baseline rate	Reduced with parents' income[1]	Minimum rate
Lower education and aged 18-19			
Living with parents	2,489	8.76 / 1,000 DKK	1,108
Not living with parents[2]	5,007	4.45 / 1,000 DKK	3,211
Tertiary education or lower education and aged 20+			
Living with parents	2,489	0 / 1,000 DKK	2,489
Not living with parents	5,007	0 / 1,000 DKK	5,007

Notes: Tertiary education incl. university education and educations such as nurses and school teachers (ISCED2011 level 5 and above). Lower educations incl. high school (gymnasium) and vocational educations (ISCED2011 level 3-4).

1) Student benefits to student in lower education below age 20 depend on the parents' income in the way that the baseline rate is reduced by the listed amount for parent income exceeding 273.644 DKK until the minimum rate is reached. An extra allowance for the parents' income of 29.046 DKK is given for each sibling under the age of 18.

2) Students in lower education below age 20 have to apply for the higher benefits even if they are not living with their parents.

Sources: www.su.dk

   On top of these basic rates it is possible to obtain a number of supplement payments summarized in table A2.

Table A2
Overview over supplement student benefit rates, 2008

	DKK per month
Supplement for single parents	5,007
Supplement if both parents are on student benefits	2,000
Disability supplement on tertiary educations	7,120
Supplement for tuition fees (maximum)	1,954

Notes: The 2008 special rates were no longer available online. The rates listed here are therefore based on the 2009 rates indexed back using the increase in the basic rates.

Sources: www.su.dk

   The criteria for the different rates can be updated on a monthly basis and individual rates may therefore change during the year. This is likely to be a source of error in the predicted of final student benefits described in appendix B given that the demographic information in the registers only is available on a yearly basis.

On top of these rates students in some circumstances has the possibility to "double clip", which means that the students receives a double benefit rate for that month. Prior to the 2009 reform this was possible in three situations:

1. During the last 12 month of the education if the student have cancelled student benefits in previous months.
2. The last month before paid internship (where it is not possible to get student benefits).
3. In connection with childbirth or adoption.

After the 2009 reform only the two last situations still apply.

On most educations student benefits are limited to the standard study time, except on university educations where student benefits are limited to 72 "clip" = 6 years, which is 1 year extra compared with the standard study time on most university educations.

Student loans

While receiving student benefits students also has the possibility to take up a state administrated subsidized loan that payout 2,562 DKK per month (2008 level). The loan cannot be received if the student cancelled student benefits and student loans might therefore give an additional incentive not to do so. The loans are paid back after the student leaves the educational system according to a fixed schedule.

Income control

When students receive student benefits they are subject to an income test. The test is automatically done after the end of the income year by the student benefit administration, who draw the relevant information from the tax authorities income register of which most is 3rd party reported (see Kleven et al, 2011 for details). Based on this information the student benefit administration calculate a so-called "own income" (in Danish: egenindkomst), which consist of all gross income components except from the student benefits themselves, mandatory and employer administrated pension contributions and income taxed under the stock income tax scheme (dividends and capital gains).

The own income is compared to an individual income limit, which is generated as the sum of monthly amounts depending on the student's actions:

- In months where the student is eligible and receives student benefits a low amount of 6.370 DKK is added to the income limit.
- In months where the student is eligible, but does not receive benefits (the student has cancelled benefits) a medium amount of 15.908 DKK is added.

- In months where the student is ineligible for student benefits a high amount of 30.619 DKK is added.

On top of these amounts the income limit for parents is further increased by a yearly amount of 23.008 DKK per child below 18.

As the analysis in the paper only focuses on students who are fully eligible for student benefits the entire year the key variation in the individual income limits comes from the students' cancelling of benefits, which moves them from the low to the medium amount and thereby increase their income limit by 15,908 – 6,370 = 9,538 DKK per month cancelled relative to a baseline amount of 12 • 6,370 = 76,440 DKK per year.

If a students' own income exceeds his/her income limit the excess has to be paid to the student benefit administration according to the following formula: of the first 9,538 DKK (= Medium – Low amount[15]) 50 percent has to be paid back, while further excess fincome is paid back 100 percent. Finally if the amount that is to be paid back exceeds 7,569 DKK (= basic student benefit rate for student not living with their parents + student loan payout) the payback is further increase by 7 percent. In the register the payback – except the 7 percent increase – is treated as a reduction in the received student benefits.

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[15] After the 2009 reform the low amount for the lower education is used for the tertiary educations.

Appendix B: Calculating own income and determining eligibility

The data are constructed by drawing from a number of register data sets organized by Statistics Denmark (DST). In particular income data from the tax return (INDH), education information (UDDA) and weekly information about recipient status for public transfers (DREAM) along with standard demographic information (BEF). Finally, the individual monthly earnings are draw from the E-income register (BFL) available from 2008. All of these registers contain entire universe of the Danish population and can be linked using a unique identification number.

Student benefits and income limit

Eligibility for and pay out of student benefits are determined from the DREAM data set, where the first challenge is to aggregate the weekly information into monthly information (the interval at which student benefits are paid out). This is done by first allocating weeks to months based on the position of Wednesday and then counting the number of weeks where student benefits have been paid out (code 651) and the number of weeks where an individual has been eligible for student benefits without receiving them (code 652).

In a month with 4 weeks 3 or more weeks with pay outs are coded as a month were the individual has received student benefits. Similarly 3 or more weeks with eligibility for student benefits without receiving them is coded as an eligible month (the individual has cancelled student benefits). In months with 5 weeks the number of weeks has to be 4 or more.

These numbers are coupled with the educational and demographic registers to determine the benefit rate the each individual is eligible for and the income limit that the individual faces. The key variables here are the level of the current ongoing education (UDD) and the civil status (FM_mark), which can be used to determine whether individuals are not living with their parents (code 6).

Finally, the number of children, which affected both the income limit and the benefit rate are calculated from the number of children below 18 in the household (variable PLADS, code 3) for the individuals who are not them self a child in a household (individuals not living with their parents).

With the above variables the individual income limit is calculated as:

$$\text{Income limit} = \text{Amount}_{Low} \bullet \text{No}_R + \text{Amount}_{Medium} \bullet \text{No}_E + \text{Amount}_{Child} \bullet \text{No}_C \quad \text{(B1)}$$

Where No_R is the number of months, where the individual receives student benefits. No_E is the number of months where the individual is eligible for student benefits without receiving them (student benefits have been cancelled), and No_C is the number of children below 18

years. The amounts are the corresponding contributions to the income limit described in appendix A.

Own income

When it comes determining "own income", the income registers unfortunately do not contain the own income variable constructed student benefit administration and this variable therefore has to be constructed. A challenge in this respect is that the registers only contain pre-aggregated income variables and not the full set of information available on the tax return and it is therefore not possible simply to apply the code used by the student benefit administration. Instead the own income variable is constructed by adding together labour income, capital income (earned interests) and transfers other than student benefits (excluding child related transfers) defined from the variables listed in table B1.

Table B1
Variables used in the constructed of own income

Variables that is always included	
Labour income excl. labour market contribution	LOENMV – SLUTBID
Capital income (earned interest)	RENTEINDK
Other transfers	QMIDYD
Additional variables that sometimes is included	
Business income	NETOVSKYD
Capital income from investment funds	PEROEVRIGFORMUE – AKTIEINDK – SKATFRIYD
Other types of income (scholarships etc.)	RESUINK_GL

Notes: A more detail description of the variables (in Danish) can be found at www.dst.dk/times.

These three income components, however, do not fully cover the income that is included in the student benefit administrations definition of own income. In particular, business income among self-employed students, capital income from investment funds[16] and other types of income such and certain types of scholarships are included in the student benefit administrations definition but not in the three main components included here.

The additional income components can in principle be found in the register data from the variables listed in table B1, but these variables do not precisely correspond to the variables that the student benefit administration uses – either because they are calculated net of certain deductions (NETOVSKUD) or because they include additional income components. A general inclusion of these variables therefore adds as much error to the own income variable as leaving them out. Instead I imply the following strategy for determining the individual own income.

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[16] But not direct dividend payments and capital gain taxes under the stock income scheme.

First I calculate each individual's own income based on the three main income components and the individual income limit based on the number of months of student benefits and the level of his current study and the number of children. The difference between the own income and the income limit identify the excess income that is to be deducted according to the formula described in appendix A in the benefits that the student benefit administrations initially have paid out.

Second I identify the actual deduction based on the difference between the student benefits that initially have been paid out and the final level of student benefits registered in the tax returns (variable: STIP). For the individuals with positive deductions I can uniquely identify the excess income that would correspond to the observed deduction.

Finally, if difference between the excess income calculated in step 1 and the excess income calculated in step 2 exactly corresponds (+/- 2 DKK) to a combination of the 3 additional income components listed in table B1, I add these income components to the own income variable for that individual.

Of course this procedure is potential problematic as it only add to the precision of the variable for the individuals who exceeds the income limit and because the procedure risk adding wrong income components that simply by chance matches the difference between the excess income calculated in step 1 and step 2.

However, given that the additional income components have to exactly match the differences in own income it seems safe to assume that risk of addition wrong components is minimal and given that the amount of frictions in section 4 is identified from the individuals exceeding the income limit, I choose to do this adjustment to the own income definition. Over the 6 years 2006-2011 the adjustment is applied to 32,000 individuals or 5 percent of the student sample in tertiary educations.

Assessing the accuracy of the own income variable and income limit

With the above construction of the own income variable it is possible to assess the accuracy of the construction – especially because measurement error in the outcome variable will create an upward bias in the estimations of optimization frictions.[17]

In order to do this, I calculate each individual's predicted student benefits based on the number of month the individual have received student benefits during the year, their income limit and their own income. If the predicted student benefits lies within +/- 10 DKK of the actual student benefits I define it as a "hit".

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[17] This is in contrast to "regular" regression analysis, where measurement error in the outcome variable only will lead to higher standard errors.

There is however two problems with this way of assessing the accuracy of the own income and income limit. First of all a hit also depend on an accurate modelling of the student benefit rates – a potentially large source of error given the number of rates described in appendix A, but this type of error of less important for the analysis of labour supply responses in the paper. Secondly – and more problematic – (small) errors in the own income variable only affected the predicted student benefits, if the own income excess the income limit. The assessment of the accuracy of the own income variable is therefore only precise above the income limit.

Table B2 summarizes the proportions of hits (the hit rate) for different parts of the sample. At an aggregate level the procedure accurately predicts the student benefits for 2/3 of the sample with better hit rate for the tertiary students (80 percent hit rate) than for the students in lower educations (40 percent hit rate), which is probably due to larger variety in the benefits rates for students in lower educations.

Table B2
Assessing the accuracy of the own income definition

Hit rate (percent)	2007	2007	2008	2009	2010	2011
Aggregate his rate	66.4	66.0	65.1	65.6	67.8	67.9
- Lower educations	40.7	42.2	41.1	35.1	42.1	42.2
- Tertiary education	81.6	80.4	80.1	83.3	84.7	84.4
Among the tertiary students						
- below the income limit	85.1	84.2	83.4	85.1	86.1	85.6
- above the income limit	58.6	58.1	61.3	59.9	65.1	65.1

Notes: A "hit" of the own income definition is definitions as a predicted student benefits within +/- 10 DKK of the actual final student benefits received. 99.9 percent of the hits are with +/- 1 DKK, which can be attributed to rounding errors. Tertiary education incl. university education and educations such as nurses and school teachers (ISCED2011 level 5 and above). Lower educations incl. high school (gymnasium) and vocational educations (ISCED2011 level 3-4).

Source: Own calculations based on DST.

Among the tertiary students the hit rate is naturally higher for the student below the income limit, where the marginal errors in the own income does not affected the predicted student benefits. Some of these errors can be attributed to errors in the applied student benefit rate due to e.g. student moving from their parents during the year, child birth and "double clipping" prior to 2008, however trying to control for these types of errors does not significantly improve the hit rate – especially for the individuals above the income limit.

As a consequence of this potential measurement error in either the own income and/or the income limit I conduct a robustness test in appendix C by replicating the key graphs in the paper only with the part of the sample, where I can accurately predict the final student

benefits. As the appendix shows this sample restriction does not affect the conclusions significantly.

The monthly income data (E-income)

The monthly income data is collected from the E-income statistics from 2008, which is collected by the Danish tax authorities. It is mandatory for all firms to report their wage payments to this register.

From this statistics I draw the variable AJO_SMALT_LOENBEGREB, which corresponds to the labour income variable used in table B1 gross of labour market contribution. As the labour market contribution is 8 percent the variable is made net by multiplying by 0.92. With this correction the yearly income in the E-income statistics almost exactly matches the labour income in the yearly income register. Put into numbers, a regression of labour income on yearly E-income yields a parameter estimate of 0.997 with a R^2 of 0.989.

Sample size

With the data drawn from the registers I get the breakdown of the size of the Danish student population shown in table B3. The core sample consists of the fully eligible employed student that numbers around 90,000 per year.

Table B3
The size of the Danish student population

1,000 persons	2006	2007	2008	2009	2010	2011
Everybody aged 18-30	813.5	820.3	833.4	841.1	855.4	869.2
In education	327.5	333.1	334.4	338.6	354.4	379.3
Of these:						
- Lower education	183.6	188.5	190.4	194.2	202.1	214.2
- Tertiary education	143.9	144.6	144.1	144.4	152.3	165.1
Among the tertiary students						
- Fully eligible	97.6	97.8	97.7	98.2	105.8	115.9
-- Employed[1]	85.0	86.2	86.1	84.4	88.8	95.6
--- Also the year after	52.3	52.9	53.7	53.0	55.4	

Notes: Tertiary education incl. university education and educations such as nurses and school teachers (ISCED2011 level 5 and above). Lower educations incl. high school (gymnasium) and vocational educations (ISCED2011 level 3-4).
1)   Employed is defined as having a positive labour income.
Sources: Own calculations based on DST

Appendix C: Robustness check wrt. measurement error

As shown in appendix B it is not possible to precisely predict the student benefits received for the entire sample of students. In the case these errors are a result of errors in the coding of the benefit rates it will not affect the analyses conducted in the paper, however if the errors stems from errors in the coding of the individual income limits or individual own income it poses a threat, as these measurement errors will make some individuals behaviour appear sub-optimal.

As a robustness check to the analyses in the paper I therefore repeat the key figures in the paper (figure 3-5) using only the part of the sample, where I can actually predict their final student benefits.

Figure C1 corresponds to figure 3 in the paper and shows the same general patterns as the original figure, except from a slightly steeper drop in the density at the kink point. However this steeper drop is natural, as the predicted student benefits only depend on marginal changes in the own income and the individual income limit above the baseline income limit. Small errors in these component will therefore only lead to an exclusion from the sample above this limit and thereby create the steeper drop.

Figure C1
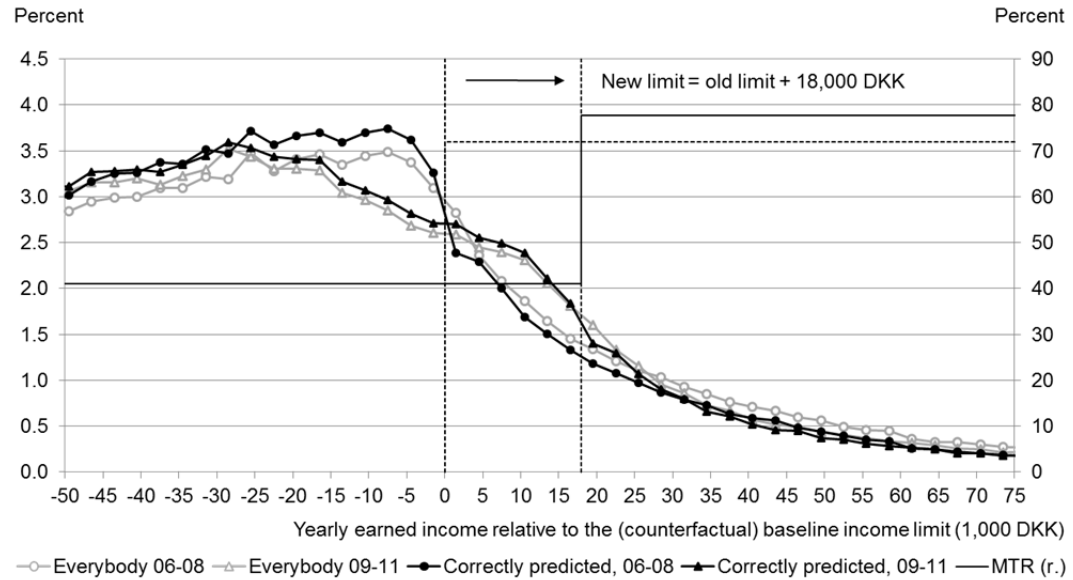**The income distribution for tertiary students, 2006-2008**
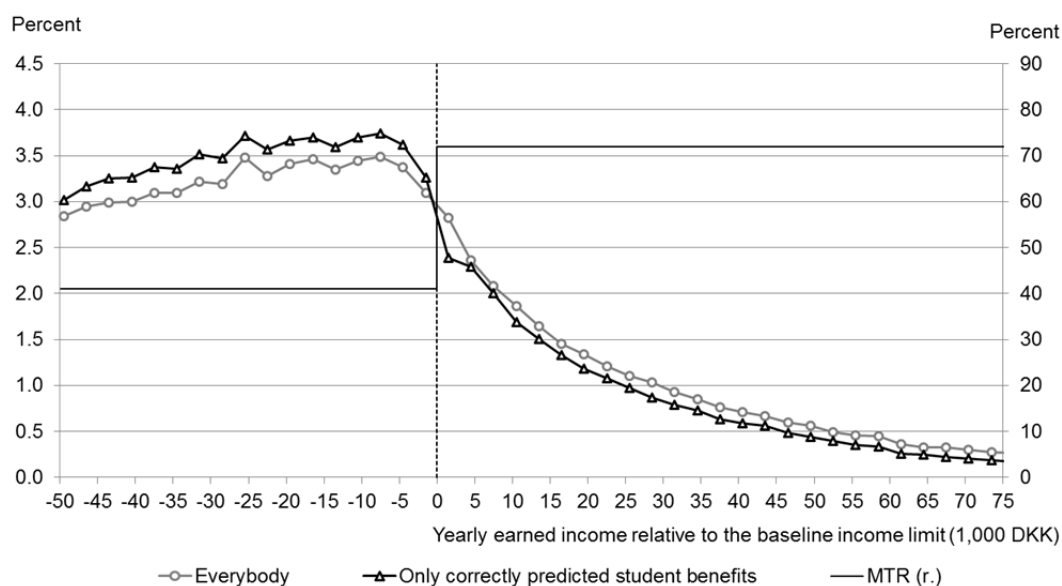
Notes: See notes to figure 3. The line for "Everybody" corresponds to average over the years in figure 3. "Only correctly predicted student benefits" only includes individuals with predicted student benefits with +/- 10 DKK of the actual student benefits received.
Sources: Own calculations based on DST

Similar the exclusion of the individuals, where I cannot accurately predict student benefits, does not significantly affect the conclusions drawn from the other key figures, cf. figure C2 and figure C3.

Figure C2
The income distribution for tertiary students before and after the 2009 reform

Notes: See notes to figure 4 and figure C1.
Sources: Own calculations based on DST

Figure C3
The income distribution for tertiary students who cancel 1 month of student benefits
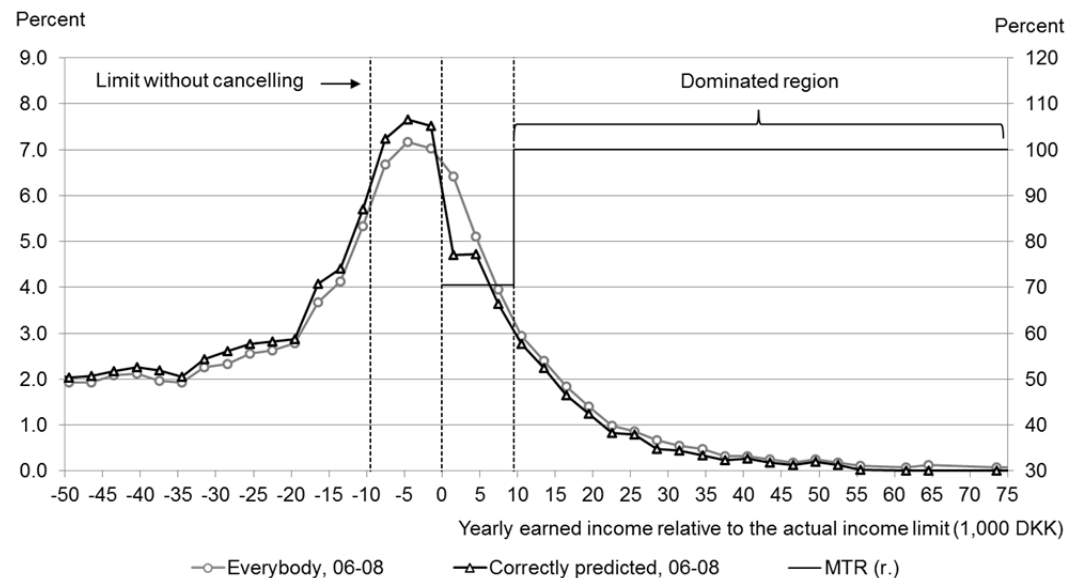
Notes: See notes to figure 5 and figure C1. For the correctly predicted sample the mass in the domi-
nated region is 11.5 percent, while the estimated standard errors for excess income > 0 and
excess income > 9,540 are 12,700 and 11,200 DKK, respectively.
Sources: Own calculations based on DST

Appendix D: GMM estimation of the labour supply of students

As a supplement to the non-parametric estimates of the labour supply elasticity presented in the paper I present in this appendix a more structural approach that jointly identifies the labour supply elasticity and the amount of variance in their final earnings relative to their desired earnings individuals are will to accept.[18]

The idea behind the structural approach is to formula a model of labour supply under income uncertainty and the Danish student benefit system, simulate the effect of a reform similar to the 2009 reform described in the paper and estimate the two parameters by minimizing the squared difference between the simulated changes in the earnings distribution and the observed change in the distribution shown in figure 9. In this way the approach falls into the frame of GMM (Generalized method of moments) estimation.

The model

Following the norm in most recent empirical papers in public finance I start with a simple quasi-linear utility function (see e.g. Saez et. al., 2012):

$$u_i = c_i - \frac{\mu n_i}{1 + \mu} \left(\frac{\hat{z}_i}{n_i}\right)^{\frac{1+\mu}{\mu}} \tag{D1}$$

where c is private consumption and $\hat{z}$ is the income level that the individuals target. μ and n is parameters of the utility function that can be interpreted as the labour supply elasticity and potential earnings, respectively. Final earnings (z) is stochastic and given by:

$$z_i = \hat{z}_i + \varepsilon_i \tag{D2}$$

where ε_i is an iid. error term.

The budget constraint that the students are facing can be written as follows:

$$c_i = (1 - t)[SU - qT_i \cdot 1(T_i > L) + z_i \cdot 1(z_i \leq T_i) + T_i \cdot 1(z_i > T_i)] \tag{D3}$$

This equation states that if students raise their announced income target (T) above the baseline income limit (L) the base student benefits (SU) is phase out at a rate q. Next, given the announced income target the students are allowed to keep any income below this target, while any excess income is taxed at 100 percent. The announced income target thus effectively constitutes an income ceiling for the student. Finally, both student benefits and earned

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[18] I do not model inattention endogenously, but simply assume that individuals cannot observe/reoptimize their earnings during the year. In this way the estimated end-of-year earnings variation should be interpreted as the underlying earnings variance net of reoptimization during the year.

income is subject to the ordinary tax system, which here is summarized by the (marginal) tax rate t.

In order to simplify the optimization I assume that the students are risk neutral and that ε_i is normal $N(0, \sigma)$ distributed. In this setting maximizing expected utility only depends on income through expected consumption, which given (D3) can be written as:

$$E(c_i) =$$
$$(1 - t)\left[SU - qT_i \cdot 1(T_i > L) + \left(\hat{z}_i - \sigma \frac{f(\theta_i)}{F(\theta_i)}\right) \cdot F(\theta_i) + T_i \cdot \left(1 - F(\theta_i)\right)\right] \quad \text{(D4)}$$

where $\theta_i = \frac{T_i - \hat{z}_i}{\sigma}$.

Optimal behaviour implies the follows two first order conditions for T and $\hat{z}$ respectively:

$$\frac{\partial E(u_i)}{\partial T_i} = 0 \Leftrightarrow \frac{\partial E(c_i)}{\partial T_i} = 0 \Leftrightarrow 1 - F\left(\frac{T_i - \hat{z}_i}{\sigma}\right) = q, \text{ for } T_i > L \quad \text{(D5)}$$

$$\frac{\partial E(u_i)}{\partial \hat{z}_i} = 0 \Leftrightarrow \frac{\partial E(c_i)}{\partial c_i} = \left(\frac{\hat{z}_i}{n_i}\right)^{\frac{1}{\mu}} \Leftrightarrow \hat{z}_i = \left((1 - t)F\left(\frac{T_i - \hat{z}_i}{\sigma}\right)\right)^{\mu} n_i \quad \text{(D6)}$$

Both conditions have a straightforward economics interpretation. When it comes to raising the announced income target students have to balance the decrease in the probability that their marginal income will hit the income ceiling with the phase out of student grant. Second, given the announced income target the students choose a target income (labour supply) as a function of not only the standard tax rate (t) but also the implicit tax rate created by the risk of hitting the income ceiling. The strength of the responses to these tax rates depend on labour supply elasticity (μ). Finally, note that the students in the absence of taxes and phase out of student benefits in this model will target an income of n_i, which therefore can be interpreted as potential (expected) earnings.
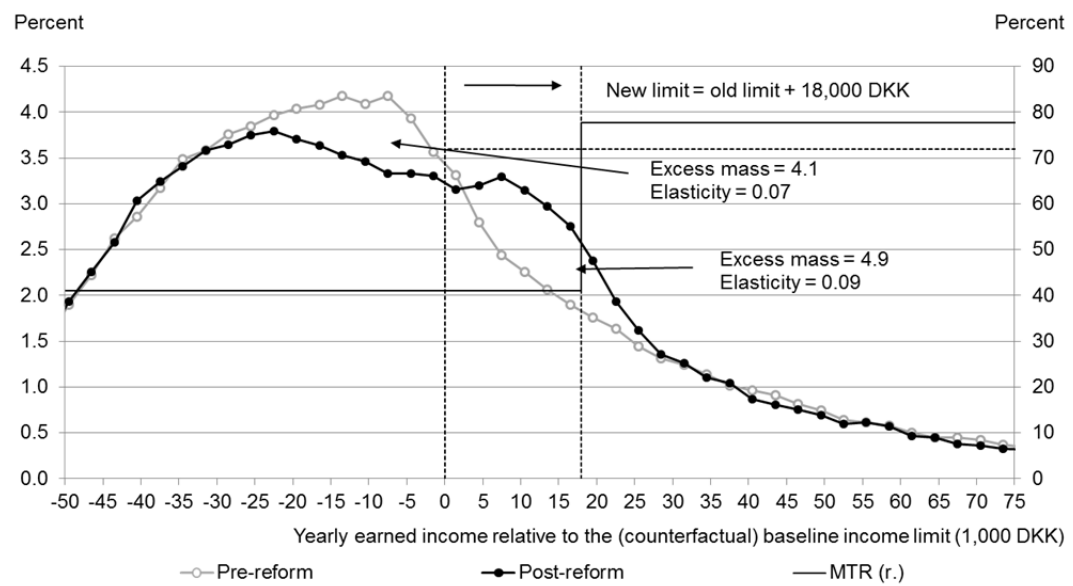
Simulation

Before moving into the actual estimation, I present the performance of the model based on a simulation with fixed parameter values. The simulation is done by solving the model for a large number of individuals with different drawn of the distribution of potential earnings and with different realizations of the stochastic component of income (ε). More concretely I draw log potential earnings (measured in 1,000 DKK) form a normal distribution with mean 4.3 and standard error 0.5 and set the labour supply elasticity (μ) to 0.1 and the standard error of the stochastic component of income (σ) to 7.

In this setting I implement both the pre-reform policy setting $(L = 76.4, q = 0.525)$ and the post-reform setting $(L = 94.4, q = 0.623)$. The tax rate (t) is in both cases set to 0.41.

The resulting earnings distributions are show in figure D1, which shows the same shift in mass from below the pre-reform kink point to a range above as in figure 4 in paper. The figure also reports elasticity estimated using the same non-parametric method as in section 6. The method is able to recover the true elasticity with a small downwards bias, which stems from the fact that the post-reform distribution that is used as the local counterfactual distribution at the pre-reform kink point, is affected by the post-reform kink due to the optimization frictions as also discussed in the section 6.

Figure D1
Simulated earnings distribution before and after the 2009 reform.

Notes: Simulated distribution of realized earnings based on a draw of 100,000 individuals with log earnings normally distributed with mean 4.3 and standard error 0.5. $\mu = 0.1$ and $\sigma = 7$.
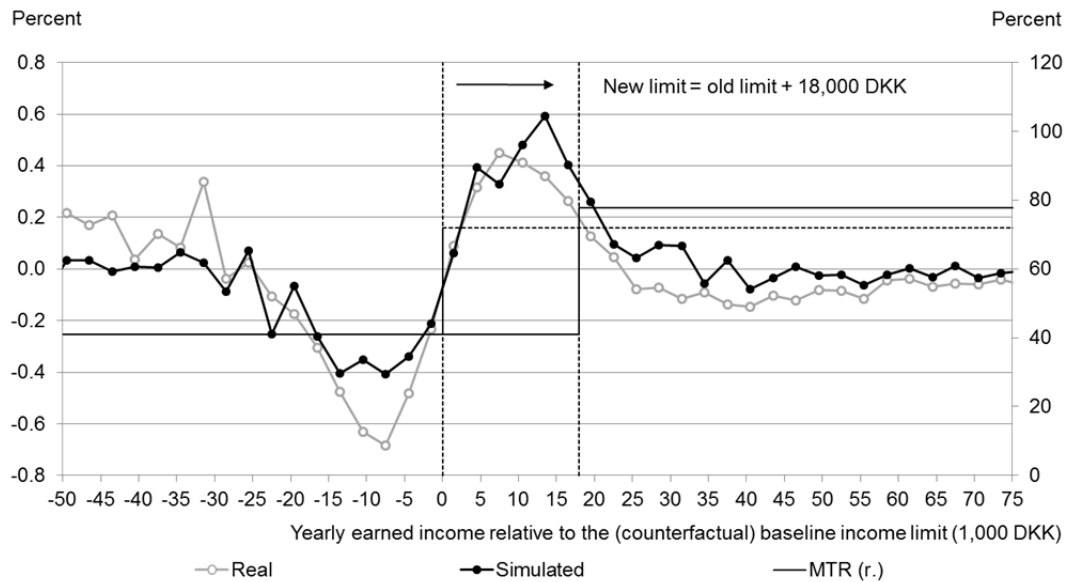Sources: Own calculations

GMM estimation

From the simulation above I can calculate a change in the frequency in each bin and map this to the actually changes seen in figure 4 and from there, choose the parameter values of μ and σ that minimizes the sum of squared errors between the actual and simulated data. This procedure yields an estimate of the labour supply elasticity of 0.06 and standard error of ε of 6,000 DKK. [19] The estimated labour supply elasticity is in other word more or less the same as the non-parametric estimate in the paper, while the standard error of individuals' final

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[19] The estimation is done as a grid search going from μ=0.01 to μ=0.20 in steps of 0.01 and from $\sigma = 1$ to $\sigma = 20$ in steps of 0.5. If the objective function is defined as the change in the distribution relative to the pre-reform distribution I obtain μ =0.09 and $\sigma = 8,000$ DKK.

error is of an order 2 lower. Given these parameter estimates I obtain a simulated change in the earnings distributions compared to the actual change as shown in figure D2.

Figure D2
Simulated and actual change in the earnings distribution following the 2009 reform.

Notes: Simulated distribution of realized earnings based on a draw of 100,000 individuals with log earnings normally distributed with mean 4.3 and standard error 0.5. $\mu = 0.05$ and $\sigma = 6.5$.
Sources: Own calculations