Capital Misallocation and Aggregate TFP

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Research Question

- Study the quantitative impact of costly external finance on aggregate productivity through reallocation
- Mechanism: Financing frictions $\Longrightarrow$ Reallocation of capital among firms $\Longrightarrow$ TFP
- Answer the question: "What is the contribution of costly external finance to the fall in reallocation"?
- I study the transitional path of the economy between two steady states SS1 and SS2
- The parameters in the initial steady state, SS1 are calibrated to match cross-sectional moments of firm distribution in the pre-recession period 1980-2007. I use the COMPUSTAT dataset (it is a large panel of listed firms for the US).
- Calibrate my model so that is able to reproduce fall in aggregate output and reallocation (as measured by OP gap) observed from data

Research Question, cont'd

- SS1: initial steady state in last quarter of 2007
- SS2: final steady state in the second quarter of 2009
- **Exercise A**: I compute the transition between SS1 and SS2, using as inputs a deterministic path of aggregate TFP shocks (calibrated to match the observed drop in GDP) and the change in the paramers of the external cost function (estimated from the micro data)
- **Exercise B**: I compute the transition between SS1 and SS2, using as inputes the deterministic path of aggregate TFP shocks but **keeping the parameters in the external cost function fixed at the pre-recession period values**
- Exercise B is the counterfactual scenario that answers the following question: "What would have been the fall in capital reallocation between firms if the degree of financial frictions had stayed constant at its pre-recession value?"

Empirical evidence on finance and investment

- Using the COMPUSTAT panel, I sort fimrs according to their finance regimes:
  1. Dividends distribution regime ($d > 0$ & $e = 0$)
  2. Equity issuance regime ($d = 0$ & $e > 0$)
  3. Liquidity constrained regimes ($d = 0$ & $e = 0$)
- What if $d > 0$ & $e > 0$? "Dividend puzzle"
- For any year (1980-2007) I compute some statistics for firms in each finance regime

Empirical evidence on finance and investment, table

1980-2007	**Equity issuance**	**Liquid.constr.**	**Div.distrib.**
Share of firms	0.23	0.297	0.474
Share of cap	0.028	0.059	0.913
Share of invest	0.039	0.057	0.904
Earnings/cap	0.567	0.275	0.355
Invest/cap	0.29	0.193	0.194
Tobin's q	3.76	1.78	2.83

▸ Data Construction

Empirical evidence on finance and investment, cont'd

- About half of the firms pay dividends
- Firms issuing equity are much more productive than the rest (earnings/capital ratio)
- These small firms (measured by capital) with high Tobin's q require external finance to finance investments
- Higher costs of raising external funds during the crisis affected most these "growth firms"

Financing frictions

- Literature that explicitely microfounds financing frictions at the firm level
- Optimal lending contract under limited enforcement: Albuquerque and Hopenhayn (2004)
- Optimal lending contract under asymmetric information: Clementi and Hopenhayn (2006)
- Here instead I follow Gomes (2001) and summarize financial market imperfections with a simple external finance cost function
- This captures the result that external funds are more costly than internal funds (see Myers and Mailuf 1984, Jensen and Meckling 1976 for theoretical explanations and Ross et al. 1993 for empirical investigation)

TFP growth and reallocation

- Huge literature that studies the relation between microeconomic productivity dynamics and aggregate productivity growth
- Several works decompose aggregate TFP growth into several parts to assess the contribution of within firm productivity growth and reallocation
- Typical TFP growth is decomposed as the sum of:
  - Within firm productivity growth
  - Between firms reallocation
  - Contribution of entry/exit
- Relevant work in this area inludes the papers of:
  - Baily, Hulten and Campbell (1992)
  - Bartelasman et al. (2013)
  - Olley and Pakes (1996)
  - Foster, Haltiwanger and Krizan (2006)

Productivity Growth Decompositions

- Baily, Hulten and Campbell (1992) postulate the following production function for plant i in period t:

$$\begin{aligned} y_{it} &= f\left(k_{it}, l_{it}, m_{it}\right) \\ &= k_{it}^{\alpha_k} l_{it}^{\alpha_L} m_{it}^{\alpha_M} \end{aligned}$$

  where k, l, m are capital, labor and intermediate inputs, respectively

- Then establishment level TFP is computed as:

$$\log TFP_{it} = \log y_{it} - \alpha_k \log k_{it} - \alpha_l \log l_{it} - \alpha_m \log m_{it}$$

- Finally aggregate productivity in period t (at the sector level) is defined as:

$$TFP_t = \sum_i \omega_{it} TFP_{it}$$

  where ω_{it} is the output (or labor) weight of plant i in the sector

Olley and Pakes gap

- Look at a measure of allocative efficiency originally proposed by Olley and Pakes (1996)
- Aggregate productivity at a given point in time can be decomposed as follows:

$$TFP_t = \frac{1}{N_t}\sum_i TFP_{it} + \sum_i \left(TFP_{it} - \overline{TFP}_t\right)(\omega_{it} - \overline{\omega}_t)$$

where TFP_{it} is firm level productivity, ω_{it} is the share of output of the firm, N_t is the total mass of active firms, and a bar over a variable indicates the unweighted average of the firm-level measure. ▸ OPgap

- Intuition: aggregate productivity can be decomposed in two terms:
  1. un-weighted average of firm-level productivity
  2. cross-term that reflects the cross-sectional efficiency of the allocation of factors (in the model, capital)

Productivity Growth Decompositions on simulated data

- Simulate model with heterogeneous firms hit by idiosyncratic productivity shocks
  - In equilibrium some firms will be expanding, some others contracting, but despite all this change at the individual level, aggregate variables will be constant
- Estimate firm-level TFP on simulated data following the same procedure as in the data

$$TFP_{it} = \log y_{it} - A_t - \alpha \log k_{it} - v \log n_{it}$$

- What about *unobserved heterogeneity*?
- My model doesn't include any unobserved structural heterogeneity
- Instead of introducing it into the model I eliminate it from the data, focusing on the estimates with *fixed effects*

Economic environment

- Firms are ex-ante identical and are subject to idiosyncratic productivity shocks denoted by z_{it};
- Firms also face a sequence of aggregate productivity levels A_t which are common across establishments and are known with *perfect foresight*.
- Firms use capital and labor as factor inputs and produce output by operating a decreasing returns to scale production function; the operating profit function (in data: cash flow from operations) is:

$$\pi\left(k_{it}, A_t, z_{it}\right) = \max_{n \geq 0} \left\{A_t z_{it} F\left(k_{it}, n_{it}\right) - w n_{it}\right\}$$

- Even though firms are ex-ante identical they differ ex-post since they experience different histories of idiosyncratic productivity shocks.

Economic environment, cont'd

- Firms can finance investment either with internal funds or borrowing from the financial market (by raising new equity)
- Only way firms can save is by accumulating capital: I choose to rule out firm savings in cash holdings or other financial assets
- External funds are more costly than internal funds: see Mayers and Mailuf for agency cost theory
- This last element is captured in a reduced form way following Gomes (2001)

Economic environment, cont'd

- Flow of funds identity:

$$D = \pi(k, A, z) - I(k, k') - AC(k, k') - \lambda(k, k', z) \quad (1)$$

where $I(k, k') = k' - (1 - \delta)k$ and $\pi(k, z) = \max_{n \geq 0} \{zAk^{\alpha}n^{v} - wn\}$, where $0 < \alpha + v < 1$. I need a profit function that is strictly concave to generate a well-defined firm scale.

- The counterpart of equation (1) in the data is the following:

$$\begin{aligned} \text{Div} &= \text{operating profit - capital expenditures-} \\ &\quad \text{cost of raising external funds (new shares)} \end{aligned}$$

- The function $C(k, k')$ represents standard quadratic adjustment costs (needed to match volatility of invest. rates):

$$AC(k, k') = \frac{\gamma}{2}\left(\frac{I}{k}\right)^2 k = \frac{\gamma}{2}\left(\frac{k' - (1 - \delta)k}{k}\right)^2 k$$

External finance

- How does the firm finance investment?
- Ideally I would prefer to model financial intermediation endogenously, but this would make much more demanding the quantitative analysis
- Rather I attempt to capture the basic idea that external funds are costly with a simple functional form:

$$\lambda\left(k, k', z\right) = \lambda\left[I\left(k, k'\right) + AC(k, k') - \pi\left(k, z\right)\right]$$

- where the difference $I\left(k, k'\right) + AC(k, k') - \pi\left(k, z\right)$, if positive, denotes the amount of external funds the firm needs to raise
- I start with minimal assumptios about its form:
- in particular I assume that these costs are positive and increasing if the firm uses external funds.
- If no external funds are required, these costs are zero.

External finance, cont'd

- Definition

  The financing cost function $\lambda(x)$ satisfies: (i) $\forall x \leq 0$, $\lambda(x) = 0$; (ii) $\forall x > 0$, $\lambda(x) > 0$, $\lambda'(x) > 0$

  - This assumption can accomodate several specification of the cost function, possibly motivated by alternative interpretations of the nature of the financial imperfection figure
  - Consistent with Pecking Order Hypothesis (Myers and Majluf (1984)):
    - Firms first use internal finance and if they don't have enough, then issue debt, and as a last resort equity.The pecking order hypothesis can account for the stylized facts that retentions and then debt are the primary sources of finance.

External cost function: specification

- Typically two types of costs associated with raising external finance:
  - Informational costs
  - Transaction costs
- Due to data limitation, I focus only on transaction costs
- Large body of empirical research quantified the costs of issuing securities: consensus of significant economies of scale
- Decreasing average cost of financing: external funding can be very expensive for very small operations
- These empirical findings suggest the linear specification of the external cost function:

$$\lambda = \lambda_0 + \lambda_1 \cdot \text{New issues}$$

Firm's decision problem

- Firm maximizes expected discounted value of net cash flow, taking real wage w and interest rate r as given. The single's firm maximization problem can be written as:

$$V(k, z; w) = \max_{I, k'} \left\{ d(k, k', z) - \left(\lambda_0 + \lambda_1 \left| e(k, k', z) \right| \right) 1_{\{e>0\}} + \beta E_{z'|z} \left[V(k', z'; w) \right] \right\}$$

s.t.

$$d = \pi(A_t, k, z; w) - I - \frac{\gamma}{2} \left(\frac{I}{k} \right)^2 k + e,$$

$$k' = (1 - \delta) k + I, \; d \geq 0$$

where d denotes dividends and e new equity issuances.

- The firm maximizes the present discounted value of cash flows; if cash flows in the current period (given by operating profits minus capital expenditures) are negative, then the firm must bear the equity finance cost.

Firm's decision problem, cont'd

- Instead the simpler problem without costly external finance is:

$$V(k,z;w) = \max_{I,k'}\left\{\pi(A_t,k,z;w) - I - \frac{\gamma}{2}\left(\frac{I}{k}\right)^2 k + \right.$$
$$\left. +\beta E_{z'|z}\left[V(k',z';w)\right]\right\}$$

s.t.

$$k' = (1-\delta)k + I.$$

- As discussed before, firm can only save through real assets
- Allowing for other forms of saving (e.g. cash holdings) would give firms more means of transferring funds across periods, and as a result may alleviate firm's financing constraints
- Why do I consider only convex adjustment costs? Both convex and non-convex adjustment costs are identified in various studies (see Cooper-Haltiwanger 2006 and Bloom 2009). However in my model a fixed adjustment cost cannot be identified together with fixed costs of issuing new equity

Invariant Distribution

- The solution to the firm's optimization problem delivers the *policy functions*

$$k'(k,z),\ I(k,z),\ n(k,z),\ y(k,z)$$

- For a fixed wage, the shock process z and the policy functions generate a stationary distribution of firms over k and z, $\mu(k,z)$, that satisfies:

$$\mu'(k',z') = \sum_k \sum_z 1_{\{g(k,z)=k'\}} \cdot Q(z,z')\,\mu(k,z). \qquad (2)$$

Household

- Since I consider the model in the stationary equilibrium with interest rate r_t, wage rate w_t and aggregate quantities constant over time, the household's problem can be simplified in this following *static* problem: ▸ detail

$$\max_{C,N} U(C,N)$$

s.t.

$$C = \int d(k,z;w)\, d\mu(k,z) + wN$$

The optimality condition with respect to labor supply is

$$-\frac{U_n(C,N)}{U_c(C,N)} = w$$

- Solving the household's problem I get the hosehold's decision rules for consumption $C(w;\mu)$ and labor supply $L^s(w;\mu)$

Aggregation

- I call μ^* the invariant distribution (i.e. the fixed point of equation 2). Given the invariant distribution $\mu^*(k,z)$ I can compute the aggregate variables:
- Aggregate investment:

$$I(w;\mu^*) = \sum_{k,z} I\left(k, k'(k,z); w\right)\mu^*(k,z)$$

- Aggregate labor demand:

$$N^d(w;\mu^*) = \sum_{k,z} n(k,z;w)\,\mu^*(k,z)$$

- Aggregate output supply:

$$Y(w;\mu^*) = \sum_{k,z} y(k,z;w)\,\mu^*(k,z)$$

- Aggregate adjustment costs:

$$AC(w;\mu^*) = \sum_{k,z} \frac{\gamma}{2}\frac{I(k,z;w)^2}{k}\mu^*(k,z)$$

Equilibrium definition

- Aggregate equity issuance costs:

$$E\left(w;\mu^{*}\right)=\sum_{k,z}\lambda\left(k,k'\left(k,z\right),z\right)\mu^{*}\left(k,z\right)$$

- The equilibrium consists of constant wage w, constant interest rate r, policy functions $n\left(k,z\right)$, $I\left(k,z\right)$, $y\left(k,z\right)$ that solve the firm's problem, decision rules C and N^{s} that solve the household problem and markets clear:

$$L^{s}\left(w;\mu\right)=N^{d}\left(w;\mu^{*}\right)$$

and

$$C\left(w;\mu\right)+I\left(w;\mu^{*}\right)+AC\left(w;\mu^{*}\right)+E\left(w;\mu^{*}\right)=Y\left(w;\mu^{*}\right)$$

Computation - Steady State

- The algorithm follows Aiyagari(1994) and Huggett(1993). I start by guessing a value for the wage w. For the given wage I solve the firm's decision problem by value function iteration on a discrete grid. Then I compute the invariant distribution of firms over capital and productivity. As a last step I check whether the labor market equilibrium condition holds. If not, I update the wage.
- More in detail:
- Step 1 - Make a guess for equilibrium wage w.
- Step 2 - Given w, solve the firm's problem by value function iteration on a discrete grid. Even if slow, it is the most robust method (better to use this because policy functions are non linear due to the fixed equity cost). Get policy function $k' = g(k, z)$ and the other decision rules.

Computation, cont'd

- Step 3 - Using the policy function $g(k, z)$ computed in step 2 and the exogenous Markov chain for productivity shocks, compute the invariant distribution $\mu^*(k, z)$ by iterating on (2)
- Step 4 - Using the stationary distribution $\mu^*(k, z)$ obtained in step 3, compute aggregate labor demand $N^d(w) = \sum_{k,z} n(k, z) \mu^*(k, z)$. Then check if equation[1]

$$-\frac{U_n\left(C, N^d(w)\right)}{U_c\left(C, N^d(w)\right)} = w$$

is satisfied. If it is, stop; otherwise update the wage and go back to step 2. An alternative way is to compute explicitely the excess demand function for labor: $N^d - N^s$.

- Iterate until convergence.

[1] Aggregate consumption can be computed from the resource constraint.

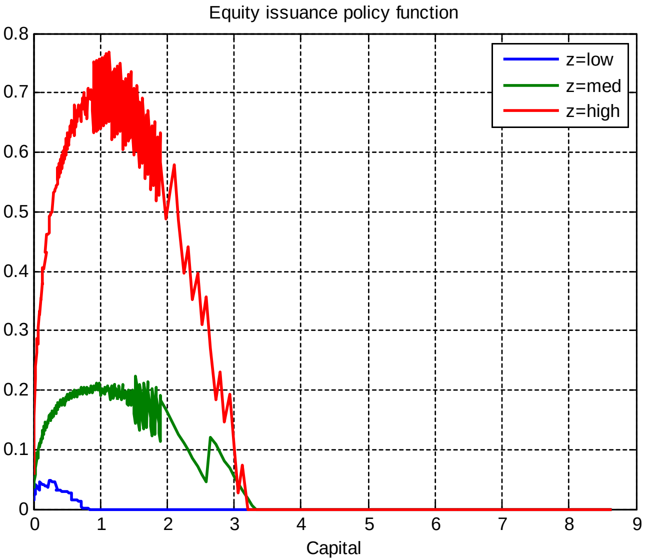
Finance regimes

- Firms can be in three different finance regimes (this why heterogenity is important)
    1. $d = 0$, $e > 0$: equity issuance regime
    2. $d > 0$, $e = 0$: dividend distribution regime
    3. $d = 0$, $e = 0$: liquidity constrained regime
- Regime will change over time according to shocks and capital accumulation

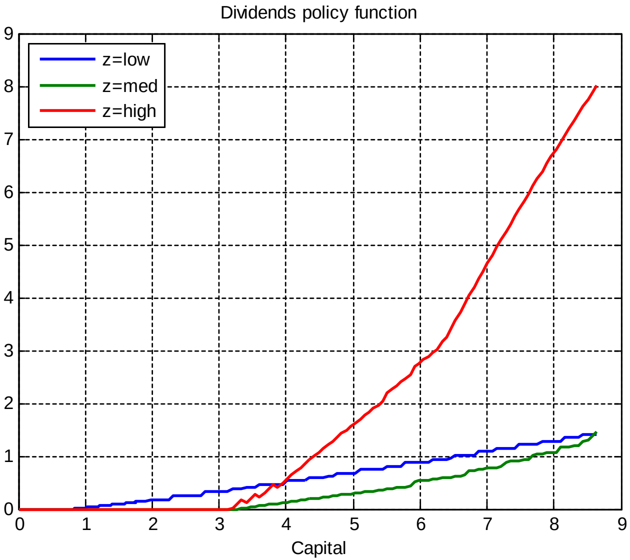
Importance of firm heterogeneity

- With a representative firm in steady state all investment can be financed without equity issuance
- Finance regime is constant
- The impact of financial frictions on reallocating capital is different for *growth firms* and *mature firms*

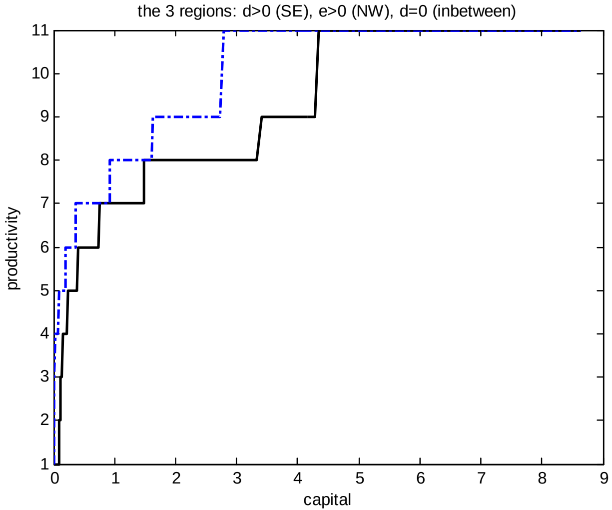
Policy Functions



Policy Functions, cont'd



Distribution among finance regimes



Intuition

- Firms that are either very small or very productive issue new equity and do not distribute dividends
- They are in *equity issuance regime*
- Firms that are either very large or less productive use internal funds to finance investment and also distribute dividends
- They are in the *dividend distribution regime*
- Remaining firms do not distribute dividends and do not issue new equity
- They are in *liquidity constrained regime*

Calibration

- Baseline scenario: I calibrate parameters to match some selected cross-sectional moments of the firm distribution, period 1980-2007
- Dataset: Compustat North American industry annual file, period 1980-2007
- I consider only manufacturing firms (SIC code between 2000 and 3999)

Pros and Cons of Compustat

- Cons: small firms are underepresented
- Pros: provides detailed info about financial variables such as: debt, equity issuance, interest expenses
- A richer dataset for US manufacturing such as LRD lacks info about financial variables

Calibration

Baseline	Parameter	Calibration target
Exponent on capital	α	TFP process
Exponent on labor	v	Labor share (macro lit.)
Depreciation rate	δ	Average I/K
Discount factor	β	Interest rate 4%
Weight on leisure	h	time spent on market work
Adjustment cost	ψ	std I/K
Shock persistence	ρ	TFP process
Shock standard deviation	σ_ε	TFP process
Fixed cost	λ_0	micro data
Slope	λ_1	micro data

Calibration - Technology and Shock

- Cobb-Douglas production function with decreasing returns to scale

$$y = A_t F\left(z_{it}, k_{it}, n_{it}\right) = A_t z_{it} k_t^{\alpha} n_t^{v}$$

with $0 < \alpha + v < 1$

- Productivity shock follows

$$\log z_{it} = \rho \log z_{it-1} + \varepsilon_{it}$$

- Under these Hp, profit function is

$$\pi\left(A, z, k, n\right) = \left(1 - v\right)\left(\frac{v}{w}\right)^{\frac{v}{1-v}} A\left(zk^{\alpha}\right)^{1/(1-v)}$$

Technology and Shock, cont'd

- Bring to the data the equation for profits:

$$\pi(A, z, k, n) = A_t \times k_{it}^{\alpha/(1-v)} \times z_{it}^{1/(1-\alpha)} \times const.$$

- Run a regression of log real profits (OIBDP in Compustat) on log real capital (PPENT in Compustat):

$$\log \pi_{it} = a + b \log k_{it} + \delta_t + e_{it} \quad (3)$$

where $b = \alpha/(1-v)$ and $e_{it} = \frac{1}{1-\alpha} z_{it}$

- Incorporate time fixed effects to capture variation in aggregate productivity (term A_t in the model)
- Firm fixed effects? Instrument capital with lagged capital?

Technology and Shock, cont'd

- I thus recove the exponent of capital as

$$\alpha = \widehat{b} \cdot (1 - v)$$

fixing $v = 0.65$,in line with macro literature. I get $\alpha = 0.3$

- I use the residuals from the regression to measure the shock process, by fitting an AR(1) to $\eta_{it} = (1 - \alpha)\, e_{it}$:

$$\eta_{it} = \rho \eta_{it-1} + \sigma \xi_{it}$$

- I estimate ρ and σ which are the input to give to the Tauchen routine to get the Markov chain for z

$$\widehat{\rho} = 0.76,\ \widehat{\sigma} = 0.21$$

Calibration - financing costs

- In my model firms with high productivity but with little capital issue new equity
- When the costs of issuing new equity increase, some of these small-highly-productive firms become cash constrained
- This causes a fall in firm-level TFP
- To assess quantitatively productivity losses I use this decomposition

$$TFP = \frac{\left(E\left[z^{1/(1-v)}\right] E\left[k^{\alpha/(1-v)}\right] + cov\left[z^{1/(1-v)}, k^{\alpha/(1-v)}\right]\right)^{1-v}}{\left(E\left[k\right]\right)^{\alpha}}$$

which the model counterpart of the Olley-Pakes decomposition in the data

- The covariance term represents the reallocation effect, which captures the fact that capital may move among firms with different productivity shocks
- If there were no reallocation effect, the covariance term would be zero

Calibration - Financing costs

- Two types of costs associated with external finance:
  1. Informational costs
  2. Transaction costs
- *Informational costs* are related to the bad signal the firm may transmit to the market when trying to raise funds (see agency cost theories, Mayers and Majluf 1984)
- Very hard to quantify
- *Transaction costs* are given by compensation to intermediaries, legal and accounting costs associated to debt or equity issuance

Calibration - Financing costs, cont'd

- Since in data external finance takes the form of debt rather than equity finance, I regress interest expenses on debt issuance using Compustat data:

$$Int_{it} = \lambda_0 + \lambda_1 Debt_{it} + \beta' X_{it} + d_t + f_i + \varepsilon_{it}$$

- Note that d is time effects, f is firm fixed effect (using panel on firms, estimate the equation with fixed effect estimator to control for unobserved time invariant heterogeneity), X_{it} is a vector of firm controls (age, leverage, etc)

Calibration - Financing costs, cont'd

- Estimate in steady state: use pooled sample for 1970-2007. This allows me to calibrate the two parameters for the steady state of my model (pre-recession period, before great recession starts)

Pooled regression with time dummies and firm fixed effects

	(1)
VARIABLES	xint
dltis	0.103***
	(0.0239)
Constant	-13.46***
	(3.903)
Observations	127,780
Number of gvkey	7,686
R-squared	0.429

Robust standard errors in parentheses

Calibration - Financing costs, cont'd

- Get an estimate of the *variation* of the cost of external finance during the recession (2007-2009). I re-estimate the empirical equation on the recession years (2007-2009).

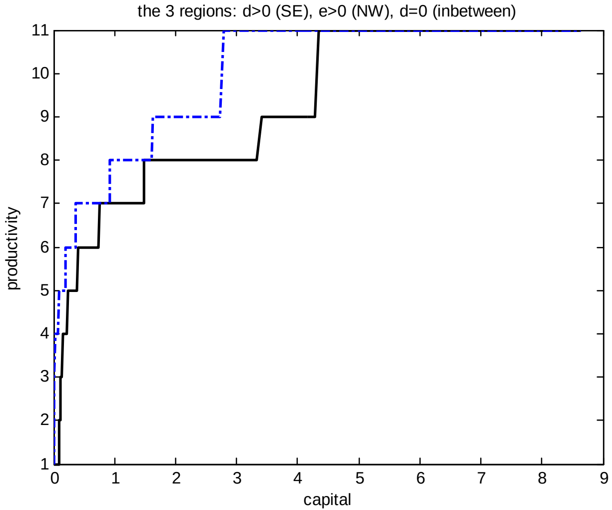
	(1)
VARIABLES	xint
dltis	0.1387**
	(0.0486)
Constant	35.48
	(22.21)
Observations	6,050
R-squared	0.396

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

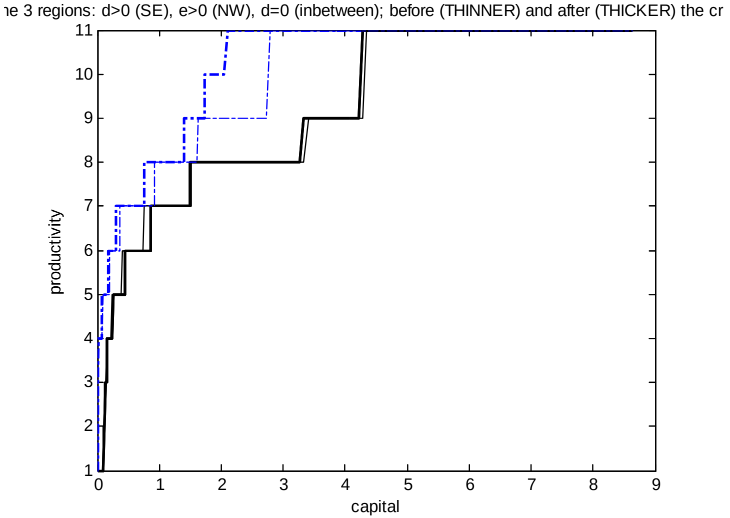
Baseline scenario

Variable	Data	Model
Average I/K	0.17	0.17
std I/K	0.156	0.156
Autocorr. of I/K	0.596	0.64
Cov(k,z)	0.438	0.534
Equity issuance	0.23	0.248
Liquidity constr.	0.297	0.259
Dividend distrib.	0.474	0.493

Pre-crisis distribution



After crisis distribution



Counterfactual experiment

	% ΔY (post/pre)	% ΔTFP	% ΔCov(k,z)
Data (US)	-4.3	-1.97	-1.4
Exercise A	-4.3	-1.08	-0.34
Exercise B	-3.75	-0.56	-0.165

- The shock exercise A is calibrated to reproduce % drop in aggregate output.
- Cov(k_{it},z_{it}) is the empirical counterpart of the Olley-Pakes gap
- What is missing? role of entry-exit, other frictions...

Distribution of firms across finance regimes

	Equity iss.	**Liquid.constr.**	**Div.distrib.**
Baseline	0.248	0.259	0.493
Exercise A	0.201	0.342	0.457
Exercise B	0.248	0.259	0.493

Conclusions

- Main result is that financial crisis changes the distribution of firms across different finance regimes
- More firms find themselves in the liquidity constrained regime
- Less firms can access equity market
- Raising costs of external funds affects most small and high productive firms who have larger investment-to-capital ratio

How I constructed investment rates

- Firms record capital stock at book value rather than the more useful economic concept which is replacement value
- Use perpetual inventory model (Salinger and Summers 1983) to convert book value of capital into replacement value for every firm-year
- The recursive formula for $k_{i,t}$ is:

$$k_{i,t} = \left[k_{i,t-1} \frac{P_t}{P_{t-1}} + I_{i,t} \right] \left(1 - \frac{2}{L_i} \right)$$

  for $t = 1, 2, ..$, where P_t is deflator for non-residential investment. Start from $k_{i,0} = Bk_{i,0}$, where $Bk_{i,0}$ is the book value of capital in the base year
- L_i is the time average of $L_{i,t} = \frac{Bk_{i,t-1} + I_{i,t}}{Depr_{i,t}}$

Productivity Growth Decompositions

- According to Baily, Hulten and Campbell (1992), productivity growth ΔTFP_t can be decomposed as

$$
\begin{aligned}
\Delta TFP_t &= \sum_i \omega_{i,t-1} \Delta TFP_{it} + \sum_i \left(\omega_{it} - \omega_{i,t-1}\right) TFP_{it} \\
&+ \left(\sum_{i \in N} \omega_{i,t} TFP_{it} - \sum_{i \in X} \omega_{i,t-1} TFP_{it-1} \right)
\end{aligned}
$$

- The first term captures the contribution of *within-firm productivity growth*
- The second term refects the contribution of *reallocation of output shares among incumbent firms*
- The last term is the contribution of *entry and exit*
- The decompostion proposed by FHK is similar

Derivation of the OP formula

- Some easy algebra:

$$
\begin{aligned}
TFP_t &= \sum_i \omega_{it} TFP_{it} \\
&= \sum_i (\overline{\omega}_t + \omega_{it} - \overline{\omega}_t) \left(\overline{TFP}_t + TFP_{it} - \overline{TFP}_t \right) \\
&= N_t \overline{\omega}_t \overline{TFP}_t + \sum_i (\omega_{it} - \overline{\omega}_t) \left(TFP_{it} - \overline{TFP}_t \right) \\
&= \overline{TFP}_t + \sum_i (\omega_{it} - \overline{\omega}_t) \left(TFP_{it} - \overline{TFP}_t \right)
\end{aligned}
$$

where $\overline{TFP}_t = \frac{1}{N_t} \sum_i TFP_{it}$

OP gap in the literature

- Olley and Pakes (1996) found that the covariance term increased substantially in the US telecommunications equipment industry following the deregulation of the sector in the early 1980s
- They argued that this was because the deregulation permitted inputs to be reallocated more readily from less productive to more productive firms
- Bartelsman et al. (2013) analyzed a cross-country sample and found that the OP term increased substantially in Eastern European countries in the 1990s while the increases were generally less marked in Western Europe
- This is consistent with the view that the transition to a market-based system has allowed Eastern European countries to improve their allocation of resources

Return

Extension with borrowing and lending

- I denote debt by b: a positive value denotes debt, a negative value denotes cash.
- For example after a positive productivity shocks that calls for a burst in investment the firm can finance the difference $I+(1+r)\,b-\pi(k,z)$ either by raising external equity finance (thus paying the cost $\lambda(k,k',z)>0$) or by issuing debt (thus choosing $b'-(1+r)\,b>0$)
- To sum up in this case the Bellman equation can be written as:

$$v(k,b,z)=\max_{k',b'}\left\{\begin{array}{c}\pi(k,z)-pI(k,k')+\\ b'-(1+r)\,b-\lambda(k,k',z)+\\ \beta\sum_{z'}v(k',b',z')\,P(z'|z)\end{array}\right\}$$

s.t.

$$b'\leq \underline{z}k^{\theta}+sk'$$

Output weighted TFP in the model

- Let $\mu(k,z)$ denote the stationary distribution of firms over capital and productivity
- The output-weighted productivity in the model is computed as:

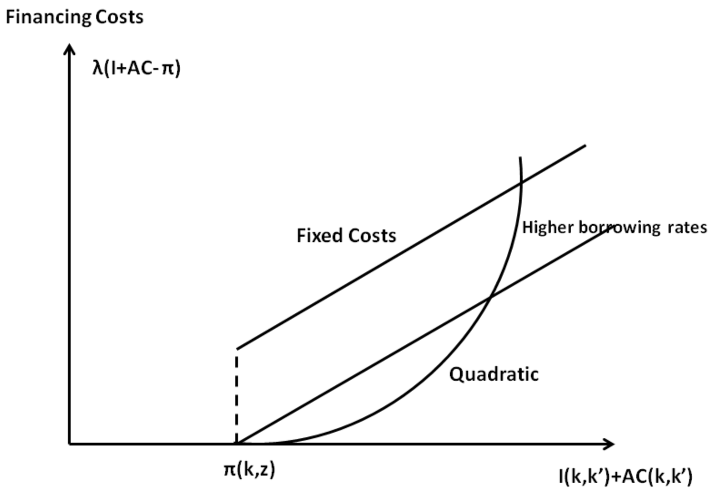
$$TFP = \int_z \int_k \omega(k,z)\, e^z$$

where

$$\omega(k,z) \equiv \frac{y(k,z)\,\mu(k,z)}{\int_z \int_k y(k,z)\, d\mu(k,z)}$$

◂ Return

Financing costs - specification



Return

Household

- Preferences:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

- Budget constraint:

$$C_t + \int P_t \theta_{t+1} d\mu_t + b_{t+1} - (1 + r_t) b_t = w_t N_t + \int (d_t + P_t) \theta_t d\mu_t$$

where θ_t denotes the shares owned by hh, b_t denotes bond holdings. In equilibrium $\theta_t = 1$ and $b_t = 0$.

- First-order conditions wrt N_t and b_{t+1}:

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = w,$$

$$U_c(C_t, N_t) = \beta U_c(C_{t+1}, N_{t+1})(1 + r_{t+1}).$$

Calibration - Financing costs

- Two types of costs associated with external finance:
  1. Informational costs
  2. Transaction costs
- *Informational costs* are related to the bad signal the firm may transmit to the market when trying to raise funds (see agency cost theories, Mayers and Majluf 1984)
- Very hard to quantify
- *Transaction costs* are given by compensation to intermediaries, legal and accounting costs associated to debt or equity issuance
- Since in data external finance takes the form of debt rather than equity finance, I regress interest expenses on debt issuance using Compustat data:

$$Interest_{i,t} = \lambda_0 + \lambda_1 DebtIssues_{i,t} + \varepsilon_{i,t}$$

  getting $\lambda_1 = 0.028$ (similar to Gomes 2001); λ_0 (sensitive to units of measure) is estimated with SMM

Data Construction

- Source: COMPUSTAT annual dataset
- Delete financial or regulated firms
- Delete observations (firm-year) with negative or zero values for book value of capital (PPENT), sales (SALE) or assets (AT)
- Delete observations with missing data for assets, book value of capital sales, operating income before depreciation (OIBDP), investment (CAPXV)
- I choose not to balance the panel: a lot of equity issuance is done by small firms who stay only a few years in the sample covered.
- To reduce the impact of outliers, I winsorize (at 99 percentile) investment over capital and earnings over capital
- Return